A LONG BASE LINE RICH WITH A 27 kton WATER TARGET AND RADIATOR FOR DETECTION OF NEUTRINO OSCILLATIONS

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A Long Base Line RICH with a 27 kton Water Target and Radiator for Detection of Neutrino Oscillations

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Abstract:

A 27 kton water volume is considered as a target for a long base line neutrino beam from CERN to Gran Sasso. Charged secondaries from the neutrino interactions produce Cherenkov photons in water which are imaged as rings by a spherical mirror.

The photon detector elements are 14400 photomultipliers (PMs) of 127 mm diameter with single photon sensitivity. A coincidence signal of ≈ 300 PMs in time with the SPS beam burst starts readout of the PMs in bins of 1 ns over a period of 100 ns. This defines the effective detector granularity to be 1.44 Mpixels, quite sufficient for the maximum expected event size of ≈ 2·10^4 photon hit points.

Momentum, direction and velocity of hadrons and muons are determined from the width, center and radius of the rings, respectively. Momentum is measured if multiple scattering dominates the ring width, as is the case for most of the particles of interest. Thresholds in water for muons, pions, kaons and protons are 0.12, 0.16, 0.55 and 1.05 GeV/c, respectively.

Momentum resolutions of ≈ 10%, mass resolutions of 5-50 MeV and direction resolutions of < 1 mrad, are achievable.

Electrons and gammas can be measured with energy resolution $\sigma_E/E = 7%/\sqrt{E}$(GeV) and with direction resolution ≈ 1 mrad.

The detector can be sited above ground because it is directional and the SPS beam is pulsed thus it has excellent rejection of cosmic ray background.

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1. Introduction

Even though the paper delivered by T.Y at Uppsala was called "RICH Outlook" and surveyed our recent work in this area, we have decided here to concentrate on one important part of that work (Long Base Line RICH) because it is timely and even by itself fills the available space. For those interested in our CsI-Fast-RICH test beam results see [1] and for imaged aerogel, R&D on HPDs and LHC-B see [2].

1.1 Long Base Line RICH

The question of neutrino masses and mixing remains one of the most important unsolved problems of particle physics. Experiments in this field use either accelerator neutrinos, solar neutrinos or atmospheric neutrinos, each sensitive to a different range of neutrino masses and mixing angles. CERN and Italy are now considering a neutrino beam traversing 731 km of earth to arrive at the Laboratorio Nazionale Gran Sasso (LNGS) where long base line experiments can be sited. The possibility for such experiments was already among the physics goals of the Gran Sasso Project and special care was taken to build the experimental halls aligned towards CERN [3]. The advantage of long base line neutrino experiments is, of course, their increased sensitivity to small mass differences.

For this purpose, we have proposed [4] the large water radiator and RICH detector shown schematically in Fig. 1. The water serves as neutrino target and Cherenkov radiating medium and is cheap and safe enough to allow large mass. The photodetectors will be visible light photomultipliers (PMs). Note that the hardware elements of this device are completely proven (clean water, phototubes, mirrors and a swimming pool of olympic volume) and require no additional R&D.

Of course, this is the technique long used in the pioneering IMB and Kamiokande proton decay, solar neutrino and atmospheric neutrino experiments. It is also the technique which will be used in the new Super-Kamiokande (50 kton, 20 kton fiducial) experiment.

Our technique, however, differs in one essential aspect, namely its use of a mirror to give focused images which allow momentum, velocity and mass determination via Cherenkov images which are multiple scattering dominated (MSD). Without the mirror, image widths are dominated by track length aberrations which are of little direct physical interest but prohibit observation of multiple scattering aberrations which allow to determine momentum.

1.2 Basic Cherenkov Relations

The Cherenkov emission angle $\theta$ relative to the particle direction is given by the Cherenkov relation i.e $\cos\theta=1/\beta$. The number of detected photoelectrons $N$ is given by the integral of the Frank-Tamm relation i.e
\[ N = N_0 Z^2 L \sin^2 \theta \] (1)

where \( L \) the particle pathlength in the medium, \( Z \) the particle charge and \( N_0 \) is the detector response parameter defined as

\[ N_0 = (\alpha/hc) \int (QTR)dE = \left(370 \text{eV}^{-1} \text{cm}^{-1}\right)TRq_{\text{int}} \] (2)

where \( \alpha \) is the fine strucure constant, \( E \) is the photon energy, \( q_{\text{int}}=\int QdE \) is the energy integral of \( Q \) the quantum efficiency, \( T \) is the radiator transmission and \( R \) is the mirror reflectivity. A glass window, visible light PM has \( Q \) varying from 4 to 28\% for \( E \) from 2 to 3.5 eV. Precise integration finds \( q_{\text{int}}=0.32 \) eV thus, for \( R=0.95 \) and \( T=1 \) thus we expect \( N_0=112/\text{cm} \) and for 20\% PM coverage \( N_0=22/\text{cm} \).

1.3 Momentum from RICH

It is perhaps well known that a ring image determines particle direction \((\theta_p, \phi_p)\) from the ring center and the particle velocity \( \beta \) from the ring radius [5]. From the defining relation for momentum, \( p=m\beta \gamma \), we obtain for the momentum error

\[ \frac{\sigma_p}{p} = \sqrt{\left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_\beta}{\beta}\right)^2} \] (3)

where \( m \) is assumed measured with error \( \sigma_m \). The velocity resolution obtainable from a RICH is \( \frac{\sigma_\beta}{\beta}=k_r=\tan \theta \sigma_\theta /\sqrt{N} \) where \( \theta \) is the measured angular radius, \( \sigma_\theta \) is the angular width and \( k_r \) is the RICH constant [5] thus

\[ \frac{\sigma_p}{p} = \sqrt{\left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_\theta}{\sqrt{N}}\right)^2} \] (4)

Obviously, \( m \) must be measured in order that momentum be determined moreover, the above relation shows that the resolution from \( \beta \) (the 2\text{nd} term) degrades as \( \gamma^2 \) thus is rapidly limited at high momenta. Neither the momentum \( p \) or the momentum error \( \sigma_p/p \) can be found without knowing the mass \( m \), however to find the mass we must use the same kinematical relation \( m=p/\beta \gamma \) and, by the same argument, obtain Eq. 4 in a different form i.e.
\[
\frac{\sigma_m}{m} = \sqrt{\left(\frac{\sigma_p}{p}\right)^2 + \left(\gamma^2 \tan \theta_\text{eq} \frac{1}{\sqrt{N}}\right)^2}
\] (5)

Obviously, to determine \(m\) we need a direct measurement of \(p\) along with the RICH measurement of \(\beta\).

1.4 Multiple Scattering Dominance (MSD)

When multiple scattering is the dominant angular error then momentum can be determined from the ring image width [6]. Since this method is new (or sufficiently old that it has been forgotten), we give here a short review of the method and its capabilities.

Historically, some early measurements of pion and muon masses were based on this effect in nuclear emulsions. In the experiment of Goldchmidt et al. [7], secondary particles produced by primary cosmic ray protons in emulsion were tracked (by human scanners looking through microscopes) and range was measured to find the particle kinetic energy via the Bethe-Bloch relation. They simultaneously measured the r.m.s angular deflection of the track to find momentum. Similar results were obtained by Camerini et al. [8] from multiple scattering and grain counting (i.e \(dE/dx \approx 1/\beta^2\) at low energies). However, most of the early measurements of pion and muon masses were from range and grain counting.

The quadratic \(\gamma\) dependance of Eq.'s 4, 5 is reduced to first order if the angular width of the ring is multiple scattering dominated (MSD) i.e

\[
\sigma_\theta = \sigma_\theta(\text{ms}) = \frac{k_{\text{ms}}}{\gamma \beta} \sqrt{\frac{L}{X_0}}
\] (6)

where \(k_{\text{ms}}=\left(\frac{13.6}{\sqrt{2}}\right)\text{MeV}=9.6\text{MeV}\) and \(X_0\) is the radiation length of the radiator [5]. Clearly \(\sigma_\theta\) is proportional to \(1/\beta\) and is formally similar to particle bending in a magnetic field (the bending angle is also \(\propto 1/\beta\)). The strict similarity dissapears when we insert the \(\theta\) dependance of \(\beta\) and the \((\theta, N)\) dependance of \(L\).

Combining Eq.'s 1, 4, 6 we obtain the momentum error of an MSD-RICH from the \(\beta\) error (i.e the 2\text{nd} term of Eq. 4)

\[
\frac{\sigma_p}{p}_{\beta} = \frac{nK \cos^2 \theta}{m\sqrt{\sin^2 \theta_m - \sin^2 \theta}}
\] (7)
where $\cos \theta_m = 1/n$ and $K = n k_{ms}/\sqrt{N_0 X_0}$. Thus if mass is known, Eq. 7 gives the momentum error arising from the $\beta$ measurement.

1.5 Momentum from ring radius and width when the pathlength L is known

In general, momentum is determined from multiple scattering by solving Eq. 6 for $p = (k_{ms}/\beta \sigma_0(m))\sqrt{L/X_0}$. Expressed in terms of the experimental variables ($\theta$, $\sigma_0$) we get

$$p = \frac{K' \cos \theta}{\sqrt{(\sigma_0^2 - \sigma_{\theta 0}^2)}}$$  \hspace{1cm} (8)

where $K' = n k_{ms}/\sqrt{(L/X_0)}$ whereas $\sigma_0(m) = \sqrt{(\sigma_0^2 - \sigma_{\theta 0}^2)}$ where $\sigma_0$ is the experimental angular width and $\sigma_{\theta 0}$ is the width due to all non-momentum dependant sources. These include chromatic ($E$), pixel ($xyz$), emission point ($z_\theta$) and impact parameter ($x_\theta$) errors but not the energy loss or slowing error ($s_l$) which is momentum dependant.

Here we require that $K'$ be constant thus $L$ must be known (or measured). This is the case for most RICH detectors where the track pathlength $L$ is determined by external trackers. But for long base line RICH, the pathlength $L$ is not (and cannot be) directly measured. This case is treated in section 1.6.

The mass defining kinematical relation $m = p/\beta \gamma$ may now be written in terms of the experimental variables ($\theta$, $\sigma_0$) as

$$m = n k' \cos \theta \sqrt{\frac{\sin^2 \theta_m - \sin^2 \theta}{\sigma_0^2 - \sigma_{\theta 0}^2}}$$  \hspace{1cm} (9)

Thus, from Eq.'s 8, 9 we evaluate the momentum and mass errors as

$$\frac{\sigma_p}{p} = \sqrt{\frac{\varepsilon^2}{2} + \left(\tan \theta \sigma_0\right)^2}$$  \hspace{1cm} (10)

$$\frac{\sigma_m}{m} = \sqrt{\frac{\varepsilon^2}{2} + \left[\left(\gamma^2 + 1\right)\tan \theta \sigma_0\right]^2}$$  \hspace{1cm} (11)

where $\varepsilon = \sigma_0^2/\left(\sigma_0^2 - \sigma_{\theta 0}^2\right)^2$ for MSD but becomes large and dominant when MSD no longer applies. These derivations require an estimate [9] of the width error i.e...
\[
\sigma_{\sigma_\theta} = \frac{\sigma_\theta}{\sqrt{2N}}
\]  

(12)

1.6 Momentum from ring radius and width when N is measured but L is not known

Here we treat the long base line RICH case when the pathlength L is not measured but its value is obtained from Eq. 1 i.e \( L = \frac{N}{N_0} \sin^2 \theta \). Now the momentum defining relation, Eq. 6, written in terms of the measured variables \(( \theta, \sigma_\theta, N )\) becomes

\[
p = \left( \frac{K}{\tan \theta} \right) \sqrt{\frac{N}{\sigma_\theta^2 - \sigma_{\theta_0}^2}}
\]  

(13)

where again \( K = \frac{n k_m s}{\sqrt{N_0 X_0}} \). The mass equation \( m = \frac{p}{\beta \gamma} \) now expressed in terms of the same measured variables is

\[
m = \left( \frac{nK}{\tan \theta} \right) \sqrt{N \left( \frac{\sin^2 \theta_m - \sin^2 \theta}{\sigma_\theta^2 - \sigma_{\theta_0}^2} \right)}
\]  

(14)

thus from Eq.'s 13, 14 we obtain for the momentum and mass errors

\[
\frac{\sigma_p}{p} = \sqrt{\frac{1 + \frac{e^2}{4} + \left( \frac{\tan \theta \sigma_\theta}{\sin^2 \theta} \right)^2}{N}}
\]  

(15)

\[
\frac{\sigma_m}{m} = \sqrt{\frac{1 + \frac{e^2}{4} + \left[ \left( \frac{\gamma^2 + \frac{1}{\sin^2 \theta}}{\tan \theta \sigma_\theta} \right)^2 \right]}{N}}
\]  

(16)

It may be noted that the resolutions of Eq. 's 10-11 are only marginally better than Eq. 's 15-16 thus, little is lost by not having a tracker inside the water radiator volume. In fact, we have not found any reasonable way to implement such a tracker without seriously compromising the RICH imagery. Luckily, Eq.'s 15-16 show that the impossible is also unnecessary. With \( N = 750 \) points on the image (or even half that many) the \( 1/\sqrt{N} \) term of Eq.'s 10-11 and 15-16 is small i.e 4-5\% thus indicating that good resolutions are possible.
2. Experimental Layout

The layout of Fig. 1 shows the proposed radiator, mirror and detector array. The mirror center of curvature C determines the origin (0, 0, 0) of the ZXY coordinate system. The water volume starting at z=6 m extends to z=36 m and transversely to x=±15 m, y=±15 m. The reflecting spherical mirror is placed at the downstream end of the cube at z=36 m.

The PM detectors at z=16 m are arrayed on a flat grid 20 m upstream of the mirror and 10 m into the water target. A pixel size of 127 mm ø with 20% coverage of the 30 x 30 m² surface (i.e. 180 m²) will suffice hence 14400 PMs (of 5' ø) on a uniform 900 m² grid. A coincidence of ≈ 300 PM hits during the 6 µs SPS burst window (see section 5.1) will signal an interesting event and start readout. The PMs will be readout in bins of 1 ns for a period of ≈ 100 ns thus increasing the effective detector granularity to 1.44 Mpixels, quite enough to image events of maximum expected size N = 2·10⁴.

The mirror radius (r_m=36 m) is chosen relatively small so that the image radius \[ r_{im}=\frac{f_1}{r_0}=(r_m/2)(0.73)=13 m \] is mostly contained inside the 30 m wide detector array. Because the detector array is 80% transparent, the water radiator can be extended into the good optics region 10 m upstream of the PM plane. Photons from this region will have 16% rather than 20% imaging efficiency.

Even with only 20% detector surface coverage, an imaged hadron will have N=750 points (see section 3.5) from an 85 cm long track in water (i.e one nuclear absorption length=1 λ). The size of N will be reduced by a factor .5 (or .7) if a photon absorption length of 50 (or 100) m is attained. Absorption losses can, of course, be compensated by additional PM coverage.

The momentum range for MSD extends up to about 4.5 GeV/c, a range which includes almost all hadrons produced by 12 GeV neutrinos via quasi-elastic and deep inelastic charged and neutral current interactions (see section 5.4). Note that the threshold momentum for Cherenkov radiation in water is \( p=1.12\text{m} \) thus 0.12, 0.16, 0.55 and 1.05 GeV/c for muons, pions, kaons and protons, respectively. About 40% of the protons from quasi-elastic interactions are above the proton threshold (1.05 GeV/c). Generally, all above threshold hadrons should be well measured in long baseline RICH i.e direction, momentum, velocity, mass and Ze (see section 4.3).

Electrons and gammas will also be well measured because an electromagnetic shower in water (radiation length \( X_0=0.36\text{ m} \)) contains many shower electrons which radiate if \( p > 0.57 \text{ MeV/c} \). Thus, a Cherenkov shower is shorter than a normal (dE/dx sensitive) shower and is less affected by low energy fluctuations. Since it fully contained in < 5 m (see section 4.4) we can take as the fiducial target length 25 m i.e 23 ktons. The shower gives a somewhat more diffuse (but still identifiable) ring than normal with about 3000 points/GeV in the image (see section 4.4). This means that water is an excellent EM calorimeter with a small stochastic term \( \sigma_E/E=7%/\sqrt{E(\text{GeV})} \).
In addition, the initiating shower particle direction is accurately determined (≤ 1 mrad) from the center of the ring. Neutral pions and neutrons have not yet been simulated thus the capabilities of long base line RICH in this regard are still to be established.

Muons will also be well measured because at low momenta when the muon range is ≤ 85 cm (i.e. 1 hadronic λ) then the MSD mass resolution (3-6 MeV) is good enough to distinguish muons and pions. Atmomenta p > 1.5 GeV/c it becomes more difficult to distinguish muons and pions by mass but then the muon range is then so long that image "lights up like a neon sign" i.e N > 5000 is a good muon signature (compare to N=750 for pions). The momentum determination of muons in water is also good (i.e. \(\sigma_p/p ≤ 6\%\) for \(p ≤ 4.5\) GeV/c). It becomes limited by emission point errors \(\sigma_\theta(z_e)\) caused by the long muon pathlength in water. Extension to higher momentum (\(\sigma_p/p ≤ 10\%\) for \(p ≤ 15\) GeV/c) can probably be obtained by application of a suitable software algorithm (see section 4.5).

To accurately measure muon momenta up to 40 GeV/c will require a separate RICH gas counter 20 m thick placed downstream of the water target, as shown in Fig. 1. It has a mirror radius of 40 m centered at C' located at z=22 m and the photodetector surface of radius 20 m measured from C'. The radiator gas, NTP C₂F₆ (\(\eta=24.5\)), detects muons above 3 GeV/c and measures momentum to 2% at 10 GeV/c, 6% at 20 GeV/c, 9% at 25 GeV/c, 13% at 30 GeV/c and 23% at 40 GeV/c. Mass is also measured as shown in section 4.1. The RICH gas counter acts exactly like an analyzing magnet of a spectrometer in that it measures the momentum of the secondary particles emerging from the water target (mostly muons). The photodetectors cover 20 % of the 850 m² surface area thus 170 m². Here, the photodetector pixel size must be 20 x 20 mm² thus 425 kpixels are needed. We propose to use 3300 HPDs [10] of 250 mm diameter each with 128 Silicon pixels of 20 x 20 mm². This counter is optional and would only be installed if there are strong physics arguments requiring precision measurement of high energy muons (i.e. muon disappearance type experiments). Several large area trackers installed at the exit of the water radiator would measure the outgoing muon direction and an iron filter and tracker placed after the gas RICH would be used to further identify the muons.

3. Properties of ring images

Every charged particle above Cherenkov threshold will produce a ring image. Neutrals which decay into charged pairs will also form images. Electrons and gammas will shower and produce somewhat more diffuse images (see section 4.4). Gammas from \(\pi^0\) or \(\eta\) decays will also be imaged but we have not yet ascertained if these particles can be identified and reconstructed. High energy neutrons will n-p scatter and
produce proton recoils which are imaged if they are above threshold. A large effort of simulation and analysis will be required to determine the limits of RICH imagery.

The optics of spherical mirrors combined with the fact that Cherenkov light rays in any plane containing the track form parallel bundles and thus can be focused to a point. Contributions from all these planes form a ring on the mirror focal surface. In addition, it is easy to show that parallel particles form the same ring image thus the ring center determines the particle direction \((\theta_p, \varphi_p)\) \[5\].

An important advantage of RICH is that the Cherenkov angle distribution is Gaussian without the Landau tail which characterizes \(dE/dx\) devices.

3.1 Parameters of the image

The parameters of the ring image are the photon detection point \((x, y, z)\), the photon emission point \((z_e, x_e)\), the particle direction \((\theta_p, \varphi_p)\) and the photon energy \(E\). The photon emission point \(z_e\) is measured along the particle track and the impact parameter \(x_e\) is the perpendicular distance to the track from the mirror center of curvature \(C\) as shown in Fig. 2. Note that two coordinate systems are in use: The \(ZXY\) system is the fixed space coordinate system as in Fig.1; the second \(PQR\) coordinate sytem is defined from \(C\) such that \(P\) is parallel to the track and \(Q\) intersects the track thus \((z_e, x_e, 0)\) is the photon emission point in the \(PQR\) system.

Six of the eight variables \((z, x, y, x_e, \theta_p, \varphi_p)\) can be arbitrarily well measured with the group \((x, y, z, \theta_p, \varphi_p)\) depending only on the precision of the photon detector while \(x_e\) depends on the tracker. The photon emission distance \(z_e\) has an irreducible error

\[
\sigma_{z_e} = \frac{\Delta z_e}{\sqrt{12}} \tag{17}
\]

which can only be improved by reducing the radiating pathlength \(L=\Delta z_e\). The error in \(\theta\) due to errors in any of the variables \(v_i=(z_e, x_e, z, x, y, \theta_p, \varphi_p)\) may be expressed as

\[
\sigma_{\theta}(v_i) = \left(\frac{\partial\theta}{\partial v_i}\right) \sigma_{v_i} \tag{18}
\]

where \(\partial\theta/\partial v_i\) is calculated from the reconstruction relation \(\theta=\theta(v_i)\). The energy error for a square detector response is

\[
\sigma_E = \frac{\Delta E}{\sqrt{12}} \tag{19}
\]

\[\sigma_{\theta}(v_i)=\left(\frac{\partial\theta}{\partial v_i}\right) \sigma_{v_i} \tag{18}\]

where \(\partial\theta/\partial v_i\) is calculated from the reconstruction relation \(\theta=\theta(v_i)\). The energy error for a square detector response is

\[
\sigma_E = \frac{\Delta E}{\sqrt{12}} \tag{19}\]
and can only be improved by reducing the energy bandwidth $\Delta E$. The corresponding Cherenkov angle error is

$$
\sigma_\theta(E) = \left( \frac{\partial \theta}{\partial n} \right) \left( \frac{\partial n}{\partial E} \right) \sigma_E
$$

where $n(E)$ gives the radiator dispersion. The errors ($\sigma_{Z_e}$, $\sigma_E$) determine the RICH resolution limits. An analytic evaluation of $\sigma_\theta(v_i)$ via the reconstruction relation $\theta=\theta(v_i)$ may be found in [5].

### 3.2 Impact parameter $x_e$, interaction point $z_{ev}$ and emission point $z_e$ determination

If tracker data were available and a minimum of two tracks were observed then an interaction vertex $(x_v, y_v, z_v)$ can be found. Assume that at least two tracks come from a vertex point $(z_v, x_v, y_v)$ and that this point is measured as are the track directions $(\theta_p, \phi_p)$. The impact parameter, along the Q axis in the plane $\phi=\phi_p$ (see Fig. 2), is then determined as

$$
x_e = z_v \sin \theta_p - \rho_v \cos \theta_p
$$

where $\rho_v = \sqrt{(x_v^2 + y_v^2)}$. Similarly the vertex distance along the track is

$$
z_{ev} = z_v \cos \theta_p + \rho_v \sin \theta_p
$$

where $z_e=0$ is the point where the Q axis intersects the track (normally).

Obviously, the photon emission point $z_e$ cannot be measured but the precise emission point is much less critical (focusing is defined by the condition $\partial \theta / \partial z_e = 0$).

The best we can do is assume it to be equidistant between $z_{ev}$ and $z_{et}$.

$$
z_e = \frac{z_{ev} + z_{et}}{2}
$$

where for muons which reach the mirror $z_{et} = \sqrt{(r_m^2 - x_e^2)}$ and for a hadron $z_{et} = z_{ev} + \lambda$.

Long base line RICH, however, will not have a tracker but $(\theta_p, \phi_p)$ will be known from the ring image center hence the plane of the impact parameter is known i.e $\phi=\phi_p$. The magnitude of the impact parameter $x_e$ can be found by measuring time. We find from geometry and optics that the arrival time $t_i$ of the $i$th photon of the ring is
\[
\begin{align*}
\frac{ct_i}{n} &= -z_{e \nu} \cos \theta - x_e \sin \theta \cos \phi_i \\
&\quad + 2\sqrt{r_m^2 - r_e^2 + (z_e \cos \theta + x_e \sin \theta \cos \phi_i)^2} \\
&\quad - \sqrt{r^2 - r_e^2 + (z_e \cos \theta + x_e \sin \theta \cos \phi_i)^2}
\end{align*}
\]

where \( t_i \) is measured from the time of track creation at point \( z_{e \nu} \). Since this absolute time is not known we can only measure the time difference between \( t_i \) and some reference photon \( t_0 \) of the ring (say at \( \phi=0 \)). We have verified, numerically, that to 10\% accuracy

\[
x_{e_i} = \frac{c(t_i - t_0)}{n \sin \theta (1 - \cos \phi_i)}
\]

i.e each photon point gives a measure of \( x_e \). The full image therefore gives the average

\[
x_e = \frac{1}{N} \sum_{i=1}^{N} x_{e_i}
\]

from \( N \) time measurements.

If two images are observed and their impact parameters are found then a single constraint relates these impact parameters and, when satisfied, guarantees that the tracks come from a common vertex. In fact if \( M \) images are seen, the number of contraints relating their impact parameters is \( C=2M-3 \).

The unit vector \( \hat{a} \) along the track and parallel to the \( P \) axis has components \( (a_z, a_x, a_y) \) where \( a_z=\cos \theta_p, a_x=\sin \theta_p \cos \phi_p, a_y=\sin \theta_p \sin \phi_p \). The unit vector \( \hat{b}=(b_z, b_x, b_y) \) along the impact parameter axis \( Q \) (of Fig. 2) has components \( b_z=-a_x, b_x=\sin^2 \phi_p+\cos \theta_p \cos^2 \phi_p, b_y=(\cos \theta_p-1) \cos \phi_p \sin \phi_p \). The unit vector normal to \( \hat{a} \) and \( \hat{b} \) is \( \hat{c}=\hat{a} \times \hat{b} \) has \( c_z=-a_y, c_x=b_y, c_y=\cos^2 \phi_p+\cos \theta_p \sin^2 \phi_p \). For two images (labeled 1 and 2) we can show that the distances along \( \hat{a} \) to the vertex point (in PQR system) are

\[
\begin{align*}
z_{ev_1} &= \alpha_{22} x_e - \alpha_{21} x_e \\
z_{ev_2} &= \alpha_{12} x_e - \alpha_{11} x_e
\end{align*}
\]

where \( x_{e1} \) and \( x_{e2} \) are impact parameters found from timing (i.e Eq.'s 25, 26) and \( \alpha_{22}=(a_{22} b_{2x} - a_{2x} b_{2z})/D, \alpha_{21}=(a_{22} b_{1x} - a_{2x} b_{12})/D, \alpha_{12}=(a_{12} b_{2x} - a_{1x} b_{22})/D, \alpha_{11}=(a_{12} b_{1x} - a_{1x} b_{12})/D \) with \( D=a_{22} a_{1x} - a_{2x} a_{12} \). The constraint relation
with the relations

\[ Q = Q_e + \arcsin(\epsilon_1^2 v^2) - 2 \arcsin(\epsilon_2^2 v^2) \]

Knowing \( Q \) and \( Q_e \) we can then reconstruct the original generalized emission angle \( Q_e \) (see Fig. 2) by numerical inversion of the equation

\[ \cos Q = \frac{(z + x z_e + x_x e + y y_e) - (z + x x_e + y y_e)}{r e} \]

will be satisfied if the tracks come a single vertex (or decay or interaction) point. If so, that point \((z_v, x_v, y_v)\) is given by the vector equation

\[ \mathbf{r}_v = x_v \mathbf{e}_2 + z_v e^2 \mathbf{e}_2 + x_v \mathbf{e}_1 + z_v e^2 \mathbf{e}_1 \]

Note that for \( M=3 \) images there are \( C=3 \) constraints thus a \( \chi^2 \) minimization will strongly improve the vertex determination (even the \( C=1 \) constraint will help). We estimate that the vertex point will be determined with \( \text{cm} \) like accuracy although this has not yet been demonstrated by simulation.

The choice of \( z_e \) is obtained as before from Eq. 23 et. seq.

### 3.3 Reconstruction of the Cherenkov angle

A general method to reconstruct the Cherenkov angle \( \theta \) from the hit point \((x, y, z)\) requires that the particle impact parameter \( x_e \) and the emission point \( z_e \) along the track be known [5]. Using the values of \((x_e, z_e)\) found (see section 3.2) with the detected photon coordinates \((x, y, z)\), with \( r = \sqrt{x^2 + y^2 + z^2} \), we obtain the generalized detected angle \( \Omega \) (see Fig. 2) from the relation

\[ \cos \Omega = \frac{(z z_e + x x_e) z_p + (x z_e - z x_e) x_p}{r e} \]

where \( r_e = \sqrt{(x_e^2 + z_e^2)} \) (by construction \( y_e=0 \)), \( z_p = \cos \theta_p \) and \( x_p = \sin \theta_p \). We then find the generalized emission angle \( \Omega_e \) (see Fig. 2) by numerical inversion of the equation

\[ \Omega = \Omega_e + \arcsin\left(\frac{r_e \sin \Omega_e}{r}\right) - 2 \arcsin\left(\frac{r_e \sin \Omega_e}{r_m}\right) \]

with \( \Omega \) from Eq. 30. Knowing \( \Omega \) and \( \Omega_e \) we can then reconstruct the original Cherenkov emission angles \((\theta, \phi)\) from the relations

\[
\begin{align*}
\frac{x_{e_2}}{x_{e_1}} &= \frac{\alpha_{11} a_2 y - \alpha_{21} a_1 y + b_1 y}{\alpha_{12} a_2 y - \alpha_{22} a_1 y + b_2 y} \\
\end{align*}
\]
\begin{align*}
\cos \theta &= \frac{(z z_p + x x_p) \sin \Omega \sin \Omega' \cos \Omega}{r \sin \Omega} - \frac{z \sin \Omega' \cos \Omega}{r_e \sin \Omega} \\
\sin \theta \cos \phi &= \frac{(x x_p - z x_p) \sin \Omega \sin \Omega' \cos \Omega}{r \sin \Omega} - \frac{x \sin \Omega' \cos \Omega}{r_e \sin \Omega} \\
\sin \theta \sin \phi &= \frac{y \sin \Omega \sin \Omega'}{r \sin \Omega}
\end{align*}
\tag{32}

where $\Omega' = \Omega_e - \Omega$.

In Fig. 3 we show an event generated by a 40 GeV/c $\nu_\mu$ in an Argon gas radiator ($\gamma_t = 10$) with 25 m pathlength giving $= 2000$ points on the images (taken from an earlier detector setup [6] and used here to illustrate a point). This interaction produced only three above threshold particles as labeled in Fig. 3. Pattern recognition is easy, even visually. The generated $\theta$ distributions are shown in Fig. 4 along with the reconstructed $\theta$ distributions (from Eq.'s 30-32) for tracks 2 and 3 of Fig. 3. Note that the reconstructed distributions are as sharp as the generated distributions (or even sharper due to pattern recognition cuts). Note also that track 2 has a very wide $\theta$ distribution (MSD) whereas track 3 is intrinsic (beyond MSD).

3.4 Number of points on the image

As shown in section 1.2 and Eq. 5, the response factor $N_0 = (R)(f)(q_{\text{int}})(370$ eV$^{-1}$ cm$^{-1}$). For $f = 20\%$ surface cover and an $R = 95\%$ mirror and visible light PMs with $q_{\text{int}} = 0.318$ eV thus $N_0 = 22$ cm$^{-1}$. For a $\beta = 1$ particle in water with $L = 85$ cm (i.e. $1\lambda$) we expect $N = 830$ photoelectrons points per image. This will be reduced to $= 400$ for the average photon pathlength of 35 m and a water absorption length of 50 m.

4. Long base line RICH performance

4.1 Particle momentum resolution

We evaluate resolutions $\sigma_p/p$ and $\sigma_m$ for tracks with $\theta_p = 0$ in the detector layout of Fig. 1. The pixel sizes are $\Delta x = \Delta y = 125$ mm, $\Delta z = 1$ mm, $\Delta t = 2$ ns corresponding to $\Delta x_e = 20$ mm. The refractive index and dispersion function $n(E)$ of water are taken from [11].

In Fig. 5 we show the various contributions to the angular error $\sigma_\theta$ vs $x_e$ for a 1 GeV/c pion with 85 cm pathlength ($1\lambda$). We note that the dominant contributor is multiple scattering $\sigma_\theta(\text{ms}) = 15$ mrad (i.e. MSD) while chromatic $\sigma_\theta(\text{E}) = 3.6$ mrad, pixel $\sigma_\theta(\text{xyz}) = 1.9$ mrad and energy loss $\sigma_\theta(\text{sl}) = 0.4$ mrad (i.e. slowing) are less important. As seen in Fig. 5, only $\sigma_\theta(x_e)$ and $\sigma_\theta(z_e)$ vary with $x_e$ but are not yet significant even for $x_e$ as large as 15 m.
Fig. 6 shows the momentum resolution $\sigma_p/p$ from Eq. 15 and photon number $N$ vs $x_e$ for a 1 GeV/c pion. The loss in momentum resolution with $x_e$ is due to the loss of imaged points $N$, in accord with Fig. 5 which shows that the $x_e$ errors are small.

In Fig. 7 we give the resolution $\sigma_p/p$ vs $p$ for $\mu$, $\pi$, $K$, $P$ for muon pathlengths of 15 m and for hadron pathlengths of 0.85 m (i.e 1 $\lambda$). The solid curves are from Eq. 15 (for MSD) whereas the dot-dash curves are from Eq. 7 (i.e from the $\beta$ measurement when mass is known). Note that the solid curve resolutions are excellent i.e between 1 and 6% for $p \leq$ 5 GeV/c. For kaons and protons the dot-dash curves are everywhere < 1% and better than the solid curves whereas for pions and muons they are better only below 1.3 and 0.5 GeV/c, respectively.

Fig. 8 shows the mass resolution from combined $m$ and $\beta$ measurements (Eq. 16). For $p \leq$ 1.25 GeV/c the resolution $\sigma_m$ ~ 5-7 MeV is sufficient for $\mu/\pi$ ID. Above 1 GeV/c the muon range becomes so long that the muon is identified by the large value of $N > 5000$. The $K$ resolution, $\sigma_m$=20-30 MeV, is sufficient for $\pi/K$ ID and the proton resolution, $\sigma_m$=50-60 MeV, suffices for $K/P$ ID. Therefore, the combined $m$ and $\beta$ measurements allow to identify all stable particles and so we are free to choose the best resolution curves of Fig. 7 (solid or dot-dash).

Fig. 9 shows the performance of the 20 m thick RICH gas counter (NTP C$_2$F$_6$, $\gamma$=24.5) for muons of 3 to 40 GeV/c via Eq. 7. The mass resolution shows that $\sigma_m$=10 MeV (Eq. 16), this is marginal for $\mu/\pi$ ID however very few hadrons or electrons get out of the water. An additional iron filter placed after the RICH gas counter would guarantee adequate muon ID. The momentum resolution $\sigma_p/p$ is 2% at 10 GeV/c, 9% at 25 GeV/c and 23% at 40 GeV/c thus, gas RICH is equivalent to a powerful magnet.

4.2 Particle direction determination

The polar angles ($\theta_p$, $\phi_p$) of a particle producing a ring image are determined with high precision from the ring center i.e $\sigma_{\theta_p}=$$\sigma_{\phi_p}=$$\sigma_\theta$/\sqrt{N}. For a 1 GeV/c pion track in water with Cherenkov pathlength of 85 cm we have $\sigma_\theta$=15 mrad and $N$=400 hence $\sigma_{\theta_p}=$$\sigma_{\phi_p}$=0.75 mrad. The direction error for electrons, gammas and muons should be at least as good because $N$ is considerably greater.

4.3 Hadron images

In Fig.'s 10 we show images of 1 GeV/c pions for an on-axis vertex at $z_v$=20.75. The conditions of the simulation were $\theta_p$=0 and $x_e$=0 in the geometry of Fig. 1. All resolution errors $\sigma_\theta$ were set to zero except $\sigma_\theta(z_e)$ and these were generated by choosing values of $z_e$ uniformly distributed between $z_{ev}$ and $z_{ev}+\lambda$. The transverse radiator size was taken to be $x$=$y$=$\pm$20 m to allow displaced images to be seen in full. The binning of the imaged points $\Delta x$=$\Delta y$=0.4 m is, in fact, bigger than the long base line RICH pixel
size $\Delta x = \Delta y = 0.125$ m and approximates the expected ms error. Similar images were obtained at $z_{ev}=10.75$ and $15.75$ m but are not shown. As can be seen the image is sharp and we found that radius of the image changed little for a $z_{ev}$ variation of $10$ m. The imaged radii are $12.63$, $12.49$ and $12.66$ m for $z_{ev}=10.75$, $15.75$ and $20.75$ m, respectively.

In Fig. 11 we see the simulated image of an on-axis $1$ GeV/c pion created at $z_{v}=20.75$ m with $\theta_{p}=300$ mrad thus $x_{e}=6.1$ m (see Eq. 21). The image would have been cutoff at $y=15$ m in the geometry of Fig. 1 but only $\approx 20\%$ of the image would have been lost. We note that the image is still sharp and suitable for pattern recognition. This is the essential criterion because if $x_{e}$ and $z_{e}$ are measured (section 3.2) then the reconstruction procedure (section 3.3) removes all "out of focusness" and leaves only essential abberations (i.e Fig. 5) and restores MSD.

The position of a RICH image in the detector is determined by the particle direction $\theta_{p}$ thus a displacement $\Delta x=\pm 15$ m would still leave half the ring within detector grid. Therefore, the angular range of acceptance is $\tan(\Delta \theta_{p})=x/z_{d}$ which for $x=\pm 15$ m and for the detector grid at $z_{d}=16$ m implies that our acceptance $\Delta \theta_{p}$ is $=750$ mrad.

This acceptance could be further increased if the walls of the cube (say at $\pm w$) are made reflective. This, however, introduces a pattern recognition problem since the part of the ring which is reflected would have focused outside $w$ at point $x=w+d$ and detected at $x_{d}=w-d=x_{d}$ (similarly for $y$). This pattern problem may be resolvable but for the moment and pending further study, we assume no reflections. Of course, the practical problem of making all the walls reflective may be the decisive element.

4.4 Electron and gamma images

We made a full GEANT simulation of electron showers in water for $1$, $3$, and $5$ GeV/c electrons starting at $z_{v}=25.2$ m with $\theta_{p}=0$ and $x_{e}=0$ in the geometry of Fig. 1. We took the GEANT output for the shower electrons as the input to our program which simulates ring images with, of course, ms capability.

Fig. 12 shows images of $10$ electrons of $5$ GeV (we reduced N by $10$ for each event so $10$ events gives the right value i.e $N=16800\pm 500$ but smooths the distributions). The image is quite easily recognizable.

In Fig. 13 we see the radius distribution with a FWHM=200 cm thus $\sigma_{r}=48$ mrad i.e much larger than for hadron images. Moreover the tails at small radii are significant and will be valuable for further identification of electrons or gammas.

Fig.14 shows a single event (with N reduced by 10). Note that its width and tails agree with the smoothed distribution of Fig. 13.

Fig. 15 is a visual image of the showering tracks in depth.

In Fig. 16 we show the number of hits from showering electrons vs depth $Z$. It is surprising to note that almost all of the shower is largely spent in $5$ m (i.e $13.9 \times_{0}$) and
completely spent in 6.2 m (i.e. 17.2 $X_0$). Thus we will require that neutrino interactions occur in the first 25 m (fiducial) leaving the last 5 m to contain an EM shower.

The simulation gave values of $N=3150 \pm 300, 10200 \pm 300$ and $16500 \pm 500$ for electron energies $E=1, 3$ and $5$ Gev, respectively. The ratios $N/E$ are $3150 \pm 300, 3400 \pm 100$ and $3360 \pm 100$, respectively hence the average $N/E=3350$. Values for the stochastic term $(\sigma_E/E)^2N$ are $9.5\%, 5.2\%$ and $6.8\%$, respectively thus the average EM energy resolution in water is

$$\frac{\sigma_E}{E} = \frac{7.2\%}{\sqrt{E(GeV)}}$$

4.5 Muon images

Muon images have not been simulated but from analytical calculations we know that at 2.25 GeV/c the image widths $\sigma_\theta(ms) = \sigma_\theta(z_e) = 26$ mrad. Beyond this point and up to 4.5 GeV/c the ms contribution is still large enough that $\sigma_\theta(z_e)$ can be subtracted in quadrature leaving a still significant ms value (see Fig. 7).

Above 4.5 GeV/c the ms contribution becomes so small that $\sigma_\theta(z_e)$ dominates ($\approx 36$ mrad for 15 m pathlength) and we must adopt a different strategy. We know that any track with $N > 5000$ (without the width and tails of an EM shower) must be a muon thus, its mass is determined. If the track is accompanied by a second track and both impact parameters are found by timing (Eq.'s 25-26) then their common vertex point can also be found (Eq.'s 27-29). We then can find the muon pathlength to the mirror (N also gives a good estimate of this length) Taking $z_e$ at the track center (Eq. 23) and $x_e$ as described above, we reconstruct the Cerenkov angle distribution from Eq.'s 30-32. Since $\sigma_\theta(z_e)$ is dominant, the reconstructed $\theta$ distribution will order the photon emission points along the track. This process effectively splits up a long track into many shorter ($\approx 0.85$ m long) segments which are individually not dominated by $\sigma_\theta(z_e)$. When the photons of a segment are reconstructed using its central $z_e$ value, each $\theta$ distribution narrows and we obtain a momentum resolution dominated by intrinsic errors. Statistically this is equivalent to $15/0.85=18$ separate measurements of momentum each of $= 50\%$ accuracy for an overall expected value $\approx 12\%$ at 15 Gev/c.

5. Beams and sites

5.1 The SPS extracted beam

The SPS beam operates at 200 MHz thus with a 5 ns periodicity. This means that succeeding rf bunches are separated in the water target by only 1.5 m hence absolute timing cannot determine the interaction vertex point (this would be possible if the rf bunches were separated by 100 ns). In other words, at any given time within an SPS burst there will be 20 rf bunches inside the water target.
One, two or three SPS beam bursts can be extracted every SPS cycle of 10 sec. If one burst is extracted it is 23 μs long and contain $1.3 \times 10^{13}$ p. If two bursts are extracted they are 10 μs long separated by 50 ms and each with $1.3 \times 10^{13}$ p for a total of $2.6 \times 10^{13}$ p/SPS cycle. When three bursts are extracted they are 6 μs long separated by 50 ms with a total of $3.9 \times 10^{13}$ p. Assuming a 30% SPS duty factor we expect $10^6$ SPS cycles/yr thus from 1.3 to $3.9 \times 10^{19}$ protons on target (p.o.t.)/yr [12].

5.2 The Neutrino beam and event rates

The neutrino beam is designed to give a flux of 500 events/kton-yr for $10^{19}$ p.o.t./yr [13]. Since we can expect up to $3.9 \times 10^{19}$ p.o.t./yr (see above), the flux rises to $\approx 2000$ events/kton-yr. Since long base line RICH will contain 23 fiducial ktons of water, we will amass up to 45000 events/yr (for no oscillations i.e for $\Delta m^2=0$, $\sin^22\theta=1$).

5.3 Where to site long base line RICH

As is clear from Fig. 1, the Gran Sasso tunnel would have to be 43 m in diameter to contain a 30 m cube. This is twice the diameter of the present and future Gran Sasso tunnels so we have investigated the operation of long base line RICH above ground.

As a worse case scenario we take the full cosmic ray flux (including the soft component) given by the PDG [14] of 180/m²-s. For the long base line RICH surface area (900 m²) we obtain a rate 0.16 MHz. Thus, during a beam burst of 6 μs we expect $\approx 1$ muon to traverse long base line RICH. Since each 6 μs burst delivers $\approx 10^{13}$ p.o.t., the probability for a real event is $(500/10^6) \times 27=.0135$ i.e one real event (S) per 74 openings. During this opening we expect $\approx 1$ through-going muon (B) thus $S/B=1$. The other 73 openings produce only obvious background which cannot be confused with the signal since it lies in another burst.

Several methods are available to reduce this background to a negligible level. Firstly by optically shielding the PMs so that they view only the mirror. Secondly by timing, since to determine $x_0$, the PM hits will be binned in buckets of 1 or 2 ns width. This will readily distinguish background from signal photons which arrive with $\leq 100$ ns dispersion (in 6 μs) thus a potential background reduction factor of $\approx 60$. Thirdly from the track patterns, since the background is mostly vertical (and does not form images) whereas the signal is mostly longitudinal (and does form images). Lastly by shielding, even though long base line RICH will be above ground it should be placed behind the mountain (when viewed from CERN) thus screening out the more horizontal muon tracks. All of these reduction factors may not be necessary and further study will be required to choose the right mix.
5.4 Event Distributions

In Fig.'s 17-20 we show the $p$ and $\theta_p$ distributions for pions and protons (from the generator Pythia 5.7) for 12 GeV/c $\nu_\mu$s interacting via charged and neutral currents. The solid curves show all particles and the dotted curves show those above threshold.

The momentum distribution of pions is shown in Fig. 17. The most probable pion momentum is $= 0.5$ GeV/c but with tails out to 5 or 6 GeV/c. This momentum range is well measured by long base line RICH (see Fig. 7).

The $\theta_p$ distribution for pions is shown in Fig. 18. Most of these pions lie within our acceptance of (750-900 mrad) but a substantial tail extends out to 1.57 radians. We have not yet characterized the source of these large angle pions.

The proton momentum distribution is shown in Fig. 19. It extends out to 8 to 9 GeV/c where our momentum resolution is stupendous i.e. $\sigma_p/p < 1\%$ for $p \leq 10$ GeV/c. Of course we lose those below threshold however, 40% of the quasi-elastic protons are still above threshold.

The $\theta_p$ distribution of protons is shown in Fig. 20. Almost all protons lie within our acceptance of (750-900 mrad).

6. Summary and future developments

We have shown how a RICH counter can measure momentum and have applied this method to investigate long base line neutrino oscillation experiments. This method allows large mass targets but still with measurement of momentum, direction, velocity, mass and absolute charge for charged hadrons and muons. In addition, electrons and gammas can be measured by calorimetry with excellent energy and direction resolution. Another advantage of this method is that it uses only proven hardware i.e phototubes, mirrors and clean water.

What would such a detector cost. The major elements of this detector are the PMs. Here costs are rather firm i.e. $10^3$ SF each (the SNO price) thus $= 15$ MSF total for the PMs. Note that PMs are reusable and even though they require an initial investment, they still have value at the end of the experiment. The cost for an above ground swimming pool of Olympic volume is estimated as 2-3 MSF. Readout electronics is about 100 SF per channel thus 1.4 MSF. Mirrors are $10^3$ SF/m$^2$ and we need 900 thus 1 MSF. Water purifying and circulating system is perhaps 1 MSF. The sum comes to 21 MSF to which we add a 20% contingency for a total of 25 MSF.

We do not estimate the cost of the RICH gas counter as we are not sure it is needed and we do not yet have firm costs for HPDs. In any case, we would not install this detector in an initial stage.
Referances


Figure Captions

Fig. 1. The layout of the 27 kton water target and radiator between z=0 to z=36 with x=±15 m, y=±15 m. A mirror of curvature r_m=36 m is at position z=36 m. Behind the water radiator is a 20 m thick gas RICH to measure the momentum and velocity of high energy muons.

Fig. 2. The geometry of a mirror focused ring image is defined by the mirror center of curvature C, the particle direction p and the Z axis along the neutrino beam. The PQR axes are centered on C and defined so that P is parallel to \( \hat{p} \) and Q intercepts the track, thus \( \hat{p} \) is in the PQ plane and has PQR coordinates \((z, x, 0)\).

Fig. 3. Ring images of a 40 GeV neutrino interaction \( \nu_\mu p \rightarrow \mu^- \pi^+ \eta' \pi^0, \rho^+ \rightarrow \pi^+ \pi^0, \Delta^+ \rightarrow \pi^+ \pi^0, \eta^- \gamma \pi^0 \rightarrow \gamma \pi^+ \pi^- \) in 25 m of an Argon gas radiator with \( \gamma_t=10 \). The three above threshold tracks are labeled 1, 2, 3 along with their identity and energy.

Fig. 4 (a, b) Distributions of generated Cherenkov angles for tracks (2, 3) of Fig. 3, (c, d) the corresponding reconstructed Cherenkov angles.

Fig. 5. The Cherenkov angle width vs impact parameter \( x_e \) for a 1 Gev/c pion track 85 cm long in water. The contributions shown are \( \sigma_0(ms) \) from multiple scattering, \( \sigma_0(E) \) chromatic, \( \sigma_0(xyz) \) from pixel size, \( \sigma_0(sl) \) from energy loss, \( \sigma_0(z_e) \) from track length and \( \sigma_0(x_e) \) from impact parameter.

Fig. 6. The momentum resolution \( \sigma_p/p \) and photoelectron number \( n_{pe} (\equiv N) \) for a 1 Gev/c pion track of 85 cm in water vs \( x_e \). The loss of momentum resolution is due completely accounted for by the loss of \( n_{pe} \), in accord with Fig. 5.

Fig. 7. The momentum resolution \( \sigma_p/p \) vs \( p \) for \((\mu, \pi, K, P)\) in water and geometry of Fig. 1. The solid curves are from multiple scattering (Eq. 15) whereas the dot-dash curves are from the \( \beta \) measurement (Eq. 7) assuming mass is known.

Fig. 8. The mass resolution \( \sigma_m \) vs \( p \) for \((\mu, \pi, K, P)\) in water and geometry of Fig. 1. The solid curves (Eq. 16) are from combined measurements of multiple scattering and \( \beta \).

Fig. 9. The momentum resolution \( \sigma_p/p \) from Eq. 7 and mass resolution \( \sigma_m \) from Eq. 16 vs \( p \) for muons in \( C_2F_6 \) gas RICH and geometry of Fig. 1.
Fig. 10. A simulated image of a 1 GeV/c pion in water and geometry of Fig. 1. The pion is created on axis at \( z_v = 20.75 \) m in direction \( \theta_p = 0 \) and \( x_e = 0 \) with only emission point \( \sigma_\theta(z_v) \) errors, i.e. photons are emitted uniformly along the pion path (1 \( \lambda = 85 \) cm). The 40 cm wide bins roughly approximates the multiple scattering contribution.

Fig. 11. Same as for Fig. 10 but the pion is created on axis at \( z_v = 20.75 \) m with \( \theta_p = 300 \) mrad and \( x_e = 6.13 \) m (Eq. 21).

Fig. 12. A 5 GeV electron shower ring image simulated in the geometry of Fig. 1. The electron starts at \( z = 25.2 \) m in direction \( \theta_p = 0 \) and \( x_e = 0 \). Ten images are superposed but each with \( N/10 \) photons so the \( N \) value is correct but the distributions are smoothed.

Fig. 13. The radius distribution of the 5 GeV electron shower simulation of Fig. 12. Note that the width is still sharp i.e. \( \text{FWHM} = 200 \) cm or \( \sigma_\theta = 48 \) mrad but much larger and with more tails than a hadron image.

Fig. 14. The radius distribution of a simulated single 5 GeV shower (with \( N \) reduced by 10). Note that the width and tails agree with the smoothed distribution of Fig. 13.

Fig. 15. A view of the showering electrons from the 5 GeV electron shower simulation of Fig. 12.

Fig. 16. The depth distribution of the showering electrons (from the 5 GeV electron shower of Fig. 12) showing the \( Z \) origin of the imaged photons. The shower is largely spent in 5 m (13.9 \( X_0 \)) and completely spent in 6 m (16.7 \( X_0 \)).

Fig. 17. The momentum distribution \( p \) of pions produced by 12 GeV/c \( v_\mu \)'s interacting via charged and neutral currents. Solid curves are for all pions and the dotted curves are for those above water threshold.

Fig. 18. The angular distribution \( \theta_p \) of pions produced by 12 GeV/c \( v_\mu \)'s interacting via charged and neutral currents. Solid curves are for all pions and the dotted curves are for those above water threshold.

Fig. 19. The momentum distribution \( p \) of protons produced by 12 GeV/c \( v_\mu \)'s interacting via charged and neutral currents. Solid curves are for all protons and the dotted curves are for those above water threshold.
Fig. 20. The angular distribution $\theta_p$ of protons produced by 12 GeV/c $\nu_\mu$'s interacting via charged and neutral currents. Solid curves are for all protons and the dotted curves are for those above water threshold.
Fig. 2
Fig. 4
Fig. 6
Fig. 7
Fig. 8
Fig. 9
1 GeV pion theta=0 xe=0 vertex=(0,0,2075), emis. along 1 inter. length
theta = 300 mrad zv=2075 cm

RINGS FROM WATER RADIATOR

Fig. 11
10 events Normal incidence

Fig. 13
Fig. 14
Fig. 15

10 Events Normal Incidence
Fig. 16

10 Events Normal incidence

Z distribution of electrons above Ch. threshold (cm)
12 GeV muon neutrino - proton (3000 interactions)

charged current

neutral current

Fig. 17
12 GeV muon neutrino – proton (3000 interactions)

charged current

neutral current

polar angle (rad) for charged pions

Fig. 18
Fig. 19

12 GeV muon neutrino - proton (3000 interactions)

Charged current

Neutral current
12 GeV muon neutrino – proton (3000 interactions)

charged current

polar angle (rad) for protons

neutral current

polar angle (rad) for protons

Fig. 20
Résumé

Nous proposons d'utiliser au Gran Sasso une cible de 27 ktonnes d'eau exposée à un faisceau de neutrinos en provenance du CERN pour réaliser une expérience d'oscillations à grande distance. Les secondaires chargées provenant des interactions des neutrinos produisent des photons Čerenkov dans l'eau qui sont visualisés comme des anneaux par un miroir sphérique.

Les photons sont détectés par 14 400 photomultiplicateurs de 127 mm de diamètre, sensibles à un photon. Un signal de coïncidence de \(\approx 300\) PMs en temps avec l'éjection du faisceau par le SPS déclenche la lecture des PMs pendant 100 ns avec une précision de 1 ns. Ceci définit une granularité effective du détecteur de 1.44 Mpxels.

L'impulsion, la direction et la masse des hadrons et des muons sont déterminées respectivement par la largeur, le centre et le rayon des anneaux. L'impulsion est mesurée si la diffusion multiple est l'effet dominant sur la largeur de l'anneau, ce qui est le cas pour la plupart des particules intéressantes. Dans l'eau, les seuils des muons, des pions, des kaons et des protons sont respectivement 0.12, 0.16, 0.55 et 1.05 GeV/c.

On prévoit des résolutions de 1 à 10\% sur l'impulsion, de 5 à 50 MeV sur la masse et inférieures à 1 urad sur la direction. Les électrons et les photons peuvent être mesurés avec une résolution en énergie de \(\sigma_E/E \approx 7\%/\sqrt{E}\) (GeV) et avec une résolution angulaire d'environ 1 urad.

Le faisceau SPS est pulsé de telle sorte que le rejet du fond des rayons cosmiques est excellent et par conséquent, il est inutile d'enterrer le détecteur.

Abstract

A 27 kton water volume is considered as a target for a long base line neutrino beam from CERN to Gran Sasso. Charged secondaries from the neutrino interactions produce Čerenkov photons in water which are imaged as rings by a spherical mirror.

The photon detector elements are 14 400 photomultipliers (PMs) of 127 mm diameter with single photon sensitivity. A coincidence signal of \(\approx 300\) PMs in time with the SPS beam burst starts readout of the PMs in bins of 1 ns over a period of 100 ns. This defines the effective detector granularity to be 1.44 Mpxels, quite sufficient for the maximum expected event size of \(\approx 2 \times 10^4\) photon hit points.

Momentum, direction and velocity of hadrons and muons are determined from the width, center and radius of the rings, respectively. Momentum is measured if multiple scattering dominates the ring width, as is the case for most of the particles of interest. Thresholds in water for muons, pions, kaons and protons are 0.12, 0.16, 0.55 and 1.05 GeV/c, respectively.

Momentum resolutions of 1-10\%, mass resolutions of 5-50 MeV and direction resolutions of < 1 mrad, are achievable. Electrons and gammas can be measured with energy resolution \(\sigma_E/E \approx 7\%/\sqrt{E}\) (GeV) and with direction resolution \(\approx 1\) mrad.

The detector can be sited above ground because it is directional and the SPS beam is pulsed thus it has excellent rejection of cosmic ray background.