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1. Introduction

Interferometry in radio astronomy produces high resolution observations of distant sources by coherently connecting widely spaced antennae. Constructing a high fidelity image from interferometric data requires compensating for atmospheric and instrumental errors, and interpolating from the measured visibilities to the unsampled regions of the visibility plane. The constraints of positivity and compactness for real astrophysical sources allow the existing image reconstruction procedures to produce images with dynamic ranges up to 10^5 . Nonetheless, the image reconstructions are not unique and the dynamic range and reliability decline rapidly with sparse visibility data. The processing techniques are highly nonlinear, and frequently heuristic, so the systematic errors in the details of the reconstruction are poorly understood (Cornwell & Braun 1989, Perley 1989, Cornwell & Fomalont 1989).

Gravitational lenses provide a unique opportunity to evaluate reconstruction methods in real observations because multiple imaging adds the constraint that different parts of the image must be distorted images of a common source. The exact nature of the lens is not known *a priori*, but by extending the image reconstruction methods both the intrinsic source structure and the lens structure can be determined simultaneously. The application is somewhat specialized, but the principle is identical to that of self-calibration. We are concerned with small systematic errors because lens reconstruction appears to be a powerful tool in probing the structure of galaxies (e.g. Kochanek 1995, Chen, Kochanek, & Hewitt 1995) and determining H_0 through a measurement of the time delay (e.g. Lehár et al. 1992; Schild 1990; Vanderriest et al. 1989; Press, Rybicki, and Hewitt 1992a,b; Schild and Thomson 1995; Haarsma *et al.* 1996), but the conclusions rely heavily on the accuracy of error estimates. With the additional constraints from gravitational lensing, we examine the systematic errors in the reconstructions of real astrophysical sources, and their effects on

ABSTRACT

Multiple imaging by a gravitational lens strongly correlates widely separated regions of the image, allowing reconstruction of both the lens potential and the unlensed source structure. Thus lens inversion is analogous to self-calibration, because both techniques construct a model of the distorting medium constrained by consistency requirements in the data. The lens inversion algorithm, LensClean, has assumed that the image reconstruction techniques used in radio astronomy do not introduce errors which affect the lens modeling. We extend LensClean to work directly from visibility data, with the option of adding self-calibration steps, and investigate the level of systematic errors present in reconstructions of a point source system, MG 0414+0534, and an extended emission system, MG 1654+1346. We find that CLEAN-reconstructed radio maps contain significant deconvolution errors that degrade both the accuracy of reconstructed lens images and broaden the uncertainty in physical parameters of the lens models. Lens model reconstructions, even with a less than perfect lens model, are generally a better fit to the visibility data than a standard CLEAN map. Rigorous analyses of radio observations should be performed directly on the visibility data.

Subject headings: gravitational lensing - galaxies; radio continuum - deconvolution methods

$V^m(\vec{u}) = V(\vec{u})S(\vec{u})$, gives the true image convolved with the instrumental response,

$$I^d(\vec{x}) = I(\vec{x}) \otimes B(\vec{x}) \quad (3)$$

where $\otimes$ denotes the convolution operator and the dirty beam $B(\vec{x})$ is the Fourier transform of the sampling function $S(\vec{u})$. Convolution with the dirty beam introduces large spatial correlations, making interpretation of the dirty image, $I^d(\vec{x})$, difficult. Deconvolution techniques are used to remove the effects of the dirty beam and reconstruct the true image of the source using the constraints of positivity, compactness, and smoothness to estimate the missing visibilities. These constraints do not automatically satisfy the additional constraints required for a multiply-imaged object, so the interpolated visibilities can increase the error in the lens inversion and bias the solutions.

A further complication arises from atmospheric and instrumental effects manifesting themselves as phase and amplitude errors in the measured visibilities. Most of these errors can be characterized by time variable complex gains for each antenna, so that the true visibility is the product of the antenna gains and the measured visibility,

$$V(\vec{u}_{ij}) = g_i g_j^* V^m(\vec{u}_{ij}) \quad (4)$$

where g_i is the gain for the i^{th} antenna and $*$ denotes complex conjugation. Self-calibration (Pearson & Readhead 1984, Cornwell & Fomalont 1989) uses the knowledge that the source is positive and compact to estimate the g_i factors in an iterative scheme. At each step, the visibility data are Fourier transformed and the dirty image is deconvolved to yield a model of the true image. Self-calibration uses this model to solve for the complex gain factors. This procedure converges when a consistent solution for the source, atmosphere, and instrumental effects is found. If the deconvolution procedure introduces systematic biases which are inconsistent with lensing, then self-calibration will reinforce these errors by modifying the visibilities via the gain factors according to the biased model. The inconsistencies introduced in the data will propagate into errors in the lens model.

the estimated properties of the distorting lens.

In §2, we review the image reconstruction techniques employed in radio astronomy. In §3, we examine the extension of these techniques to gravitational lens systems and describe a modification to the LensClean algorithm which more accurately reconstructs the image and lens. In §4, we apply these techniques to two morphologically different gravitational lensing systems, MG 0414+0534 and MG 1654+1346, to examine the sources of error in the analysis and the reliability of the image reconstructions. In §5, we summarize our results.

2. Image Reconstruction in Radio Astronomy

An extensive presentation of interferometry theory and techniques can be found in Thompson, Moran and Swenson (1986), and we give only a brief review here. An interferometer measures the incident radiation field at many separated antennae, and correlates the signals from each antenna pair to measure the complex visibility function. For a sufficiently small source, the visibility function $V(\vec{u})$ is the two-dimensional Fourier transform of the source brightness distribution,

$$V(\vec{u}) = \int_S I(\vec{x}) e^{-2\pi i \vec{u} \cdot \vec{x}} d\vec{x} \quad (1)$$

where $\vec{u}$ is the baseline vector projected onto a plane perpendicular to the line of sight to the source. In principle, the problem of recovering the source brightness distribution can be solved by Fourier inversion of $V(\vec{u})$. In practice, $V(\vec{u})$ is sampled only at discrete points in the visibility plane determined by the antenna locations. The measured visibilities are the product of $V(\vec{u})$ and a sampling function,

$$S(\vec{u}) = \sum_{i,j}^N w_{ij} \delta(\vec{u} - \vec{u}_{ij}) \quad (2)$$

where w_{ij} is a weighting factor and $\vec{u}_{ij}$ is the baseline vector corresponding to the i th and j th antenna in an array of N elements. The direct Fourier inversion of the measured visibility,

and it is characteristic of CLEAN reconstructions of compact sources. Another well known problem affects extended sources, where CLEAN tends to subtract in periodic “stripes” with a period equal to the distance between the dirty beam peak and the first sidelobe (e.g. Cornwell 1983).

Briggs (1994) has adapted the Non-Negative Least-Squares (NNLS) algorithm (Lawson & Hanson 1974) to the deconvolution problem. NNLS generalizes CLEAN to use a least-squares fit to the visibility data with a finite number of non-negative clean components. The algorithm constructs a model for the source by minimizing the mean-squared difference between the dirty image and the convolution of the source model with the dirty beam,

$$\varphi = \int d^2\vec{x} \left(B \otimes I^{model}(\vec{x}) - I^d(\vec{x}) \right)^2 \quad (5)$$

where $I^{model}(\vec{x})$ is the model of the true source distribution. At each iteration, the algorithm adds another component to the model, located at the pixel which produces the largest reduction in φ . The fluxes for all the components in the model, including the newly chosen component, are simultaneously optimized by minimizing φ in a least-squares sense with the criterion that all fluxes remain non-negative. The algorithm converges when adding a component does not reduce φ or when all the pixels in the image have been included. As with CLEAN, the final image is produced by convolving the clean components with the restoring beam and adding the residual image. The NNLS algorithm is “global” in the sense that the correlations between clean components are accounted for in the flux optimization step. NNLS also enforces positivity in the reconstructed source by constraining each pixel to be non-negative. The NNLS algorithm performs far better than CLEAN on partially resolved sources (Briggs et al. 1994), but NNLS is both a memory and time intensive calculation. It requires storage and inversion of a matrix with $N_d N_s$ entries, where N_d is the number of pixels in the dirty image and N_s is the number of source pixels in the reconstructed image. Since the matrix inversion scales in time approximately as $\mathcal{O}(N_d N_s^2)$,

Three deconvolution algorithms are commonly in use in radio astronomy: CLEAN, NNLS, and MEM. The most frequently used method is the CLEAN algorithm (Högbom 1974, Clark 1980, Schwab 1984), which is based on the assumption that sources can be decomposed into point sources. The normal implementation divides the procedure into minor and major cycles. During the minor cycle, the algorithm finds the position and flux of the peak in the dirty image. At this position, the dirty beam is subtracted from the dirty image, scaled to a fraction (called the loop-gain) of the peak flux and a point source with the same flux is added to the clean map. The procedure is repeated until a stopping criterion is satisfied, typically when all the pixels are less than a fixed fraction of the dirty image peak at the start of the minor cycle. In the major cycle, the accumulated clean components collected in the previous minor cycle are subtracted from the ungridded complex visibilities. When a specified number of clean components are subtracted or the maximum residual in the image is below a fixed value, the algorithm is terminated. Since spatial frequencies much larger than those sampled by the longest baseline are not well constrained, these frequencies are removed through convolution. The convolving function (referred to as the clean or restoring beam) is a gaussian which is fit to the central lobe of the dirty beam. The clean (final) image is created by convolving the final reconstructed image—the collection of clean components—with the clean beam. The residual image is added to the final image to include any flux which was not reconstructed in the deconvolution process.

A number of problems are intrinsic to this method of deconvolution. CLEAN is an inherently “local” algorithm in that each new clean component cannot alter any previously subtracted component. This independent nature leads to errors in both compact and extended structures. CLEAN subtracts from the peak of the dirty image, but due to noise and beam sidelobes, the peak position is not the actual location of the underlying compact source. CLEAN compensates for the initial error with many nearby low level components. This behavior adds spurious “skirts” around bright compact components,

are all present in the measured data. Reconstructing the lens should solve all these problems simultaneously. Lens modeling algorithms (e.g. Kochanek & Narayan 1992, Wallington, Narayan, & Kochanek 1994) have hitherto assumed that the reconstruction and self-calibration procedures do not affect the lens reconstruction, and we now examine the validity of this assumption.

3.1. The LensClean Algorithm

Kochanek & Narayan (1992) developed the LensClean algorithm as a general tool for inverting images of gravitational lens systems in the presence of finite resolution and noise. LensClean determines both the lens model and source structure simultaneously using an optimization procedure. LensClean deconvolves the image assuming a lens model to determine the residual error in the fluxes due to inconsistencies between the data and the lens model. The error is minimized by iteratively adjusting the lens model parameters. The LensClean algorithm uses the CLEAN algorithm, modified to include the distorting effects of the lens galaxy. If a source plane clean component $\mathcal{S}$ lies in the multiply-imaged region, the image plane has n distorted copies of $\mathcal{S}$ at positions $\vec{x}_k$ with magnifications M_k . LensClean chooses each source plane clean component to maximally reduce the error in the resulting image,

$$\epsilon^2 = \int d^2\vec{x} \left[I^d(\vec{x}) - \mathcal{F} \sum_{k=1}^n M_k B(\vec{x} - \vec{x}_k) \right]^2 \quad (6)$$

where $\mathcal{F}$ is the source plane flux of $\mathcal{S}$. In the absence of a lens, the optimum component is the peak of the dirty image, and LensClean reduces to the normal CLEAN algorithm.

The LensClean algorithm treats each clean component as a real feature in the source brightness distribution by requiring that these components are mapped into different regions in the multiply imaged region. The standard CLEAN algorithm allows negative clean components; however, we have found that the inclusion of these components can bias

this algorithm is only beginning to become computationally practical.

Unlike CLEAN and NNLS, the Maximum Entropy Method (MEM) (e.g. Narayan & Nityananda 1986) does not represent the source as a collection of point sources. MEM attempts to reconstruct the image under the constraints that the model is positive, fits the visibility data, and maximizes the image “entropy”. MEM performs well for sources with diffuse emission or simple structure, but poorly reconstructs compact sources embedded in extended emission (Cornwell & Braun 1989). The nature of the entropy function leads to reconstructions with edges only as sharp as the data requires. This “smoothness” constraint will create spurious extensions in the reconstructed images of compact sources. Wallington, Narayan, & Kochanek (1994; see also Wallington, Kochanek & Narayan 1995) have developed a lens inversion method based on the MEM deconvolution algorithm. The current implementation of MEM for lenses operates only on either CLEAN or NNLS images instead of directly on the visibilities, so this implementation of MEM for lenses will be susceptible to the same systematic errors explored in this paper. While future implementations of MEM for lenses will need to test the direct use of the visibilities, we study only the CLEAN-type algorithms here.

3. Applications to Gravitational Lenses

The gravitational lens inversion problem is analogous to the self-calibration problem. The lens corrupts the true image of the source and introduces correlations between separated regions in the image plane through multiple imaging. As in self-calibration, the properties of the distorting medium are not known initially. The form of the distortions is significantly more complicated than the multiplicative gain factors from atmospheric and instrumental effects. The lensing-corrupted image is further distorted by these same discrete sampling and gain errors, so the lensing, convolution, and calibration errors

compact source. CLEAN compensates for the initial error with many nearby low level components. This behavior adds spurious “skirts” around bright compact components,

conservative scheme is used to determine the confidence levels on the parameters. First, to offset any overall systematic bias, the χ^2 surface is rescaled such that the minimum corresponds to the number of degrees of freedom, given by the number of beams which fit within the tangential critical line minus the number of parameters required by the lens model (Kochanek 1995),

$$N_{dof} = \frac{A_{tan}}{2\pi\sigma_b^2} - M. \quad (8)$$

Second, the confidence level on a parameter is defined as the largest variation of that parameter from the best fit value with $N_{dof}(\chi^2 - \chi_{min}^2)/\chi_{min}^2 \leq 15.1$, normally corresponding to the 99.99 percent confidence level. However, Kochanek (1995) suggests that the parameter limits should be interpreted as 95 percent confidence levels to compensate for the unknown systematic errors in the inversion procedure.

The CLC algorithm forces the lens model to fit not only the gain corrected measured visibilities, but also the entire reconstructed visibility plane. Since standard deconvolution methods are “local” in the sense that they ignore correlations caused by the lens, images of a common source need not be consistent with each other in the clean map used in the CLC analysis. Chen, Kochanek, & Hewitt (1995) find that for lens model reconstructions of the Einstein ring, MG 1131+0456, differences in the CLEAN reconstructions affect the CLC results. In particular, they examine the sensitivity of their results to changes in the loop-gain parameter. They find the differences in the lens model solutions are consistent with the conservative confidence levels adopted. However, variations in the loop-gain change the final rms residual by up to 50 percent. Since the tests were limited to CLEAN images, they could not fully explore the systematic errors, but they showed that the LensClean results are affected by the systematic errors in the reconstructed images.

the lens model solutions. Magnification gradients allow unresolved sets of components (i.e. components separated by much less than a beam width) to have zero flux at one image and positive net flux at others by distributing a pattern of positive and negative clean components. These nonphysical solutions can fit “better” than the physical solutions, but they require negative flux pixels in the source brightness distribution. Therefore, for the analysis presented in this paper we restrict the LensClean algorithm to subtract only positive clean components.

The LensClean algorithm is designed to account for both deconvolution and gravitational lensing effects. The method has so far been applied only to the clean map and beam; we refer to this as the Clean map LensClean (CLC) algorithm. This approach is computationally simpler because it manipulates the final images instead of the visibility data, thereby eliminating problems such as gridding and assigning weights to the visibilities. The gaussian restoring beam is significantly smaller than the dirty beam, reducing the computation time of the minor cycle subtractions. CLC does, however, assume that the systematic errors in the reconstructed clean maps do not impact the lens modeling results.

The goodness-of-fit of the CLC lens model is measured by comparing the rms residual in the multiply-imaged region to that of noise multiplied by the size of the multiply-imaged region in beam areas (Kochanek 1995),

$$\chi_{clc}^2 = N_{mul} \frac{\Delta x^2}{2\pi\sigma_b^2} \left(\frac{\sigma_{mul}}{\sigma_o} \right)^2 \quad (7)$$

where N_{mul} and σ_{mul} are the number of pixels of linear size Δx and the rms residual in the multiply-imaged region, respectively. The area of a beam is $2\pi\sigma_b^2$ and $\sigma_b = \text{FWHM}/\sqrt{8\ln 2}$. The noise in the data σ_o is estimated by the rms value in empty regions of the clean image. The error estimate should be limited to the multiply-imaged region, since the singly-imaged region contains no extra degrees of freedom (Wallington & Kochanek 1995, Kochanek 1995). Since the χ^2 surface near the minimum may be dominated by systematic errors, a

correct form for the χ^2 statistic. We, therefore, determine the lens model parameters by minimizing χ_{vlc}^2 .

To evaluate the goodness of fit, we need to determine the number of degrees of freedom in the system. If we fit N_{src} clean components in the source model and M parameters in the lens model, then the number of degrees of freedom is,

$$N_{dof} = 2N_{vis} - 3N_{src} - M. \quad (10)$$

Each complex visibility has two degrees of freedom and we lose three degrees of freedom in specifying the position and amplitude of each clean component. Because the lens model completely determines the relationship between image plane components and source plane components, we count the clean components in the source plane. A possible ambiguity in Equation 10 is whether multiple clean components can be subtracted from the same position. We believe N_{src} is the number of independent model components, or the number of nonzero pixels in the source plane model. Clean components at the same location correspond to multiple sinusoids with the same spatial frequency, and even though the clean components are chosen independently, fitting these identical functions to the measured data should not add degrees of freedom. We, therefore, conclude that clean components subtracted at the same location are not independent and should be treated as a single component.

We expect that the VLC results are less affected by systematic errors, and the formal errors more closely resemble the true errors. We use the same procedure for determining the parameter confidence levels as in CLC except we use the formal $\Delta\chi^2 \leq 4$ limit for calculating the 95 percent confidence intervals. We still compensate for any remaining systematic errors by rescaling the χ^2 such that the minimum χ^2 is equal to the expected number of degrees of freedom for the system. It is likely that this is a correct statistical interpretation, but it can only be confirmed by extensive Monte Carlo simulations.

3.2. Visibility LensClean

We have developed an improved LensClean algorithm, which we name Visibility LensClean (VLC), to avoid the systematic errors of the CLEAN reconstruction by operating directly on the complex visibility data. We implement VLC in the National Radio Astronomy Observatory ⁴ (NRAO) Software Development Environment. The algorithm is a modification of the Cotton-Schwab CLEAN algorithm (Schwab 1984, Cornwell & Braun 1989). As in the CLC algorithm, the minor cycle cleans the image under the constraints of the lens model. During the major cycle, the accumulated clean components from the previous minor cycle are subtracted from the ungridded visibility data, which are then Fourier transformed to produce the new residual dirty image. The major cycle ends when the peak-to-peak residual does not drop by more than one percent of its value from the previous major cycle. We use the VLC algorithm to test the images produced by the standard image reconstruction techniques and to evaluate the conservative method of determining the confidence intervals on lens model parameters.

Since VLC operates on the ungridded visibility data, the definition of the χ^2 statistic is straightforward. Let N_{vis} be the number of measured visibility points. Each visibility has complex value V_i and noise σ_i . We assume that the noise shows sufficiently little variability over the V_i to allow us to replace σ_i by an average noise per visibility point σ_V . If the model visibilities are V_i^{model} then the χ^2 statistic for the fit of the model to the data is,

$$\chi_{vlc}^2 = \sum_{i=1}^{N_{vis}} \frac{|V_i - V_i^{model}|^2}{\sigma_V^2}. \quad (9)$$

There is no ambiguity in a visibility-based clean as to the number of data being fit and the

⁴The National Radio Astronomy Observatory is operated by Associated Universities, Inc. under co-operative agreement with the National Science Foundation.

4.1. MG 0414+0534

The MG0414 system was discovered in a radio survey for gravitational lenses (Hewitt et al. 1992). Figure 5.a shows the 15 GHz Very Large Array⁵ (VLA) image of MG0414 (Katz, Hewitt & Moore 1996). The beam full-width at half-maximum is 0."12 by 0."11. At this resolution, the system is clearly resolved into four compact components that are lensed images of a redshift $z_s = 2.63$ (Lawrence et al. 1994) radio quasar. The peak fluxes of A1, A2, B, and C are 158 mJy/Beam, 141 mJy/Beam, 56.5 mJy/Beam, and 24.2 mJy/Beam, respectively. Using empty regions of the image (which were not CLEANed), we estimate the noise level at 180 μ Jy/Beam, and find that the peak-to-peak error—the maximum value minus the minimum value in the empty region—is 1.39 mJy/Beam. The data set includes 120390 visibility points with a theoretical noise per point of 36.8 mJy for a 10 second integration period (Crane & Napier 1989). Ellithorpe (1995) has found that the system is fit reasonably well by an isothermal sphere plus the lowest two multipole moments arising from an external mass distribution. The potential for this lens model is given by,

$$\phi(\vec{r}) = b\sqrt{r^2 + s^2} + \frac{1}{2}\gamma r^2 \cos 2(\theta - \theta_\gamma) + \beta r^3 \cos 3(\theta - \theta_\beta) \quad (12)$$

where b is the critical radius, γ is the shear strength, θ_γ is the shear position angle, and β and θ_β control the $\cos 3\theta$ moment. Because there is little radial structure in the MG0414 system, the radial shape of the lens only weakly affects the fit. Therefore, we fix the core radius at 0."13.

When we model MG0414 using the CLC algorithm, we find strong residuals near all the components. Figure 5.a shows the residual image after subtraction of the clean components using the best fit lens parameters. The peak residual always lies near the A1 component,

⁵The Very Large Array is a facility of the National Radio Astronomy Observatory.

3.3. Self-Calibration with a Lensing Consistent Model

If the reconstruction of the image is inconsistent with the object being a gravitational lens, then self-calibration can reinforce the errors by adjusting the antenna gains to maximize the agreement with the incorrect reconstruction. Such a systematic bias in the self-calibration process will increase the estimated noise in the image, since the converged solution is not fully consistent with the data. We can test if errors are introduced by performing a self-calibration of the visibilities using the converged VLC model of the image. The decrease in χ^2_{vlc} gives a measure of the errors in the original self-calibration. This approach is not fully self-consistent because the VLC reconstruction was found using the visibility data produced by the standard self-calibration methods. To correct fully for the systematic biases, the self-calibration and lens modeling need to be combined so the complex gain factors and lens parameters are optimized simultaneously. The number of degrees of freedom after self-calibration is

$$N_{dof} = 2N_{vis} - 3N_{src} - M - \begin{cases} N_{gain} & \text{Phase only} \\ 2N_{gain} & \text{Amplitude and Phase} \end{cases} \quad (11)$$

where we have subtracted the number of gain parameters used.

4. Results from Two Gravitational Lens Systems

We have chosen two gravitational lensing systems, shown in Figure 5., as test cases: MG 0414+0534 and MG 1654+1346. The four images in MG0414 test the reconstruction of compact sources, and the diffuse emission in MG1654 tests the reconstruction of extended low-level flux. For simplicity, we examine these systems with only a lens parameterization known to fit each system reasonably well. The MG0414 results are summarized in Table 1 and the MG1654 results are summarized in Table 2.

a theoretical noise of 21.2 mJy. First, we apply VLC with no re-optimization of the lens parameters found by the CLC method. The residuals, shown in Figure 5.b, have significantly more structure. Since the VLC algorithm subtracts using the dirty beam rather than the compact clean beam, errors in the lens model are introduced throughout the residual image via the sidelobes of the dirty beam. Consequently, we see the rms residual in the multiply-imaged region increases to 625 μ Jy/Beam. However, the peak-to-peak error drops by 19 percent to 14.9 mJy/Beam, suggesting significant errors in the clean map reconstructions. The χ^2_{vlc} is 110000; since the system has 64700 degrees of freedom, the χ^2_{vlc}/N_{dof} is 1.70, which is 126σ from the expected best fit. This value is much smaller than that estimated by the CLC algorithm, although the model remains a poor fit to the data.

Next, we optimized the lens model using VLC, and Figure 5.c shows the residual image. The best fit solution has $\sigma_{mul} = 548 \mu$ Jy/Beam and a peak-to-peak error of 13.3 mJy/Beam, down 27 percent from the CLC best fit solution. The drop in both the rms and peak-to-peak error suggests systematic errors in the clean map are affecting the CLC algorithm. Although the residual image statistics show a strong decrease, the χ^2_{vlc} drops by only 3 percent to 107000, slightly better than the previous model. The optimized solution used the same number of independent components (N_{src}) as the non-optimized solution. The χ^2_{vlc}/N_{dof} is 1.65. The lens parameters were found to be consistent within the confidence limits of the CLC solution.

During the self-calibration stage, the deconvolution methods did not use any lensing information. As our final experiment, we use the VLC clean component model to self-calibrate the visibility data set. This procedure tests the consistency of the gain corrected visibility data with gravitational lensing. Because the visibility amplitudes were not adjusted in the original self-calibration, we perform a phase only self-calibration with a 30 second solution interval. Figure 5.d shows the best fit residual image after optimizing the lens model in VLC on the re-self-calibrated data. The rms residual increases slightly, and

the peak in the image, and we see alternating positive and negative stripes. The best fit model has $\sigma_{mul} = 552 \mu\text{Jy/Beam}$ and a peak-to-peak error of 18.3 mJy/Beam . We find a large χ^2_{clc} value of 6620 for 315 degrees of freedom. If we assume normally distributed errors, we find the fit is 251 standard deviations from an optimal fit (i.e. $\chi^2/N_{dof} = 1$).

We believe some of the residual features are the result of the low flux level “skirts” that CLEAN constructs about the primary clean components. The A1/A2 images are highly magnified so the skirts of B/C images are lensed into arcs at the A1/A2 images, leading to the observed residual pattern. CLEAN will always create these features because of its “local” nature, raising the question whether a “global” scheme might solve the problem. To test this hypothesis, we replace the CLEAN reconstructed images with NNLS reconstructed images. The rms noise in the NNLS image agrees with that from the CLEAN image, and although the differences of less than 0.25 mJy/Beam between the images are significantly smaller than the features seen in the CLC residuals they are peaked about the component positions and indicate changes in the point source reconstructions. However, when we apply the CLC method to the NNLS image, we detected no significant change in the residual strength, pattern, or the lens model parameters. This test is not completely independent of the CLEAN algorithm because CLEAN was used to deconvolve the image during self-calibration. As another test, we repeat the entire self-calibration using the NNLS method. As before, the CLC residuals show the strong arcs and striping features, with no significant change in the results. The NNLS method appears to suffer from systematic effects similar to those in the “local” CLEAN algorithm.

For the VLC lens modeling, we use the NNLS self-calibrated data set, since both the NNLS and CLEAN self-calibrated data sets perform identically in the CLC analysis. VLC requires subtraction of each clean component from each visibility data point; therefore, we averaged the data set from 10 second intervals to 30 second intervals to increase computational efficiency. The number of visibility points drops to 32575, each with

results with his models; however, the data set used by Kochanek (1995) was unavailable so we use one differing slightly in its editing. Note that these data do not show the bridge across the center of the ring seen in Langston et al. (1990). Figure 5.b shows the 8 GHz VLA image of the core and lensed radio lobe with an angular resolution of 0."21 by 0."19. In empty regions of the image, the peak-to-peak value is 500 $\mu\text{Jy}/\text{Beam}$ and the estimated noise is 59 $\mu\text{Jy}/\text{Beam}$. The data set contains 18956 visibility points, each with a theoretical noise of 6.35 mJy (Crane & Napier 1989). The peak in the image is 5 mJy/Beam, although most of the flux in the system is contained in the fainter ring emission. Kochanek (1995) finds the lens in MG1654 is best fit by the first-order approximation to the ellipsoidal mass distribution with surface density,

$$\Sigma(\vec{r}) = \frac{1}{2}\Sigma_c b^{2-\alpha} \left(1 + \frac{r^2}{s^2}\right)^{\alpha/2-1} \left[1 - \frac{2-\alpha}{2} \frac{r^2}{r^2 + s^2} \epsilon \cos 2(\theta - \theta_\epsilon)\right] \quad (13)$$

where $\Sigma_c = c^2 D_{OS} D_{OL} / 4\pi G D_{LS}$ is the critical surface mass density for lensing. The monopole shape is controlled by the exponent α and the core radius s . The ellipticity ϵ and the position angle θ_ϵ governs the quadrupole shape.

Since the MG1654 system has already been well fit by Kochanek (1995), we expect to detect more clearly small systematic errors in the image reconstructions. Our data set is not identical to the one used by Kochanek (1995), so we first found the best fit CLC model for our data set (second CLC entry in Table 2). The residuals, shown in Figure 5.a, have a peak-to-peak error of 727 $\mu\text{Jy}/\text{Beam}$ and $\sigma_{mul} = 73.8 \mu\text{Jy}/\text{Beam}$, leading to a χ^2_{clc} of 310, significantly larger than those found by Kochanek (1995) (first CLC entry in Table 2). This difference is due to the positive clean component requirement and, to a lesser extent, the differences in the data sets. The best fit lens model was consistent in all parameters except for the monopole exponent α . The residual image shows strong arcs with alternating positive and negative flux. The largest residuals are clustered about the emission regions of the map, and they are caused by striping in the reconstruction of the ring. The two sides of

the peak-to-peak error does not change significantly. The χ_{vlc}^2 is 104000 and the number of degrees of freedom has dropped to 61700 through the inclusion of 2900 gain factors in the phase only self-calibration. The lens model parameters do vary slightly; however, the lens model does not fit significantly better.

The MG0414 tests show that the standard image reconstructions of compact sources are significantly affected by systematic errors and that these errors are detectable in lens models. These effects corrupt primarily the χ^2 , making interpretation of the absolute goodness of fit difficult. We find the best fit VLC model fit is a significant improvement over the CLC model fit. The decline of the peak error was even more dramatic, and the final peak residual is only 30 percent of the CLC peak residual. Even though we found that the χ_{clc}^2 value is strongly corrupted by the systematic errors, the VLC lens model parameters are consistent with the CLC model within the conservative error bars defined in §3.1. The VLC lens model solutions justify the conservative procedure used to estimate the confidence levels in CLC, since the error bars encompass the effects of deconvolution errors on the parameter estimates and more optimistic error estimation would not. Even though the VLC method significantly reduces the systematic errors, the coherent structures in the VLC residuals and the χ_{vlc}^2 indicate that the resulting lens model solution is still not a satisfactory fit to the system. In fact, the errors are dominated by the incorrect fit of the lens model, and further modeling of this system is being explored.

4.2. MG 1654+1346

In contrast to MG0414, MG1654 (Langston et al. 1989, 1990) provides an example of a lensing system dominated by extended emission. The redshift $z_s = 1.74$ quasar consists of a compact core and two radio lobes. The southern lobe has been lensed into a ring. Kochanek (1995) has extensively modeled this system using the CLC algorithm. We compare our

the number of degrees of freedom has dropped to 34000, accounting for the 600 gain factors used in the self-calibration. The χ^2_{vlc}/N_{dof} dropped to 1.14, only 19σ from an optimal fit of $\chi^2_{vlc}/N_{dof} = 1$, assuming normally distributed errors. Since the lens model solution has not changed significantly from the best fit VLC solution, we conclude that the better fit is due to the reduction of systematic errors introduced in the original self-calibration.

These MG1654 tests show that sources dominated by extended emission are affected by the errors in the standard deconvolution methods. VLC shows a more dramatic effect in MG1654 than in MG0414 for two reasons. First, our lens model does not fit the MG0414 system very well, so the modeling errors are larger than the systematic errors. Second, the MG1654 data set contains fewer measured points for a significantly more complex source structure.

The constraints imposed by gravitational lensing allow a more ambitious interpretation of the reconstructed image. Since each of the four images is differentially magnified, the effective resolution of these images is significantly higher than estimated by the restoring beam (Kochanek & Narayan 1992). Since we have a lens model which fits the system well, we should be able to probe the higher spatial frequency structure in the image by convolving the converged VLC clean component model with a more compact beam. Figure 5.a shows the best fit VLC reconstructed image using the self-calibrated data set. We convolved the clean component model of the image with a $0''.1$ restoring beam, an increase by a factor of two in resolution. The super-resolved image, shown in Figure 5.b, is free of “striping” artifacts and appears to be reconstructed well. To test the accuracy of the super-resolution, higher frequency VLA observations are necessary.

the ring are striped independently, and when the lens model in CLC overlaps these stripes, this results in the alternating residual pattern.

The VLC algorithm dramatically reduces the striping problem in CLC. The best fit VLC residual image is shown in Figure 5.b. The ring-shaped residuals are significantly reduced and there is no longer the alternating pattern, although some coherent structures about the ring remain. The rms residual is $\sigma_{mul} = 61 \mu\text{Jy}/\text{Beam}$, consistent with the noise level in the CLEAN image, and the peak-to-peak error is $491 \mu\text{Jy}/\text{Beam}$ which is 32% smaller than for CLC. The χ^2_{vlc} is 42600. With 34600 degrees of freedom, we find $\chi^2_{vlc}/N_{dof} = 1.23$, 38σ from the optimal fit. A normal CLEAN with no lens model gives a χ^2_{vlc} of 43100 with 27800 degrees of freedom, or $\chi^2_{vlc}/N_{dof} = 1.55$, worse than the VLC fit. The normal CLEAN both has larger residuals and uses more independent clean components to fit the data. In some senses, the VLC reconstruction is a more accurate representation of the image than the standard CLEAN component model. The VLC model solution is consistent with the CLC solution found in our analysis. We found that the VLC solution is also consistent with that found by Kochanek (1995), except that the monopole exponent α is marginally outside the conservative CLC confidence interval. The CLC algorithm used by Kochanek did not constrain the clean components to be positive, leading to small changes in the reconstructions and the converged model. The error in the lens position is due primarily to the uncertainty in location of the quasar core between data sets rather than differences in the lens models. Within our data set, the CLC and VLC best fit lens positions are consistent.

As in the MG0414 system, we self-calibrated the MG1654 data using the best fit VLC reconstruction as a model for the true image and redid a full optimization of the lensing parameters using VLC. Again, we perform a phase-only self-calibration since the data were not amplitude self-calibrated. The residuals are shown in Figure 5.c. We found the rms residuals and peak-to-peak error did not change. However, the χ^2_{vlc} dropped to 38900 and

errors by introducing systematic biases into the visibility data. When we redo the phase-only self-calibration using the best fit VLC reconstructed model, we found significant reductions in the residuals, leading to better fits for both lensing systems. Although we did not perform a fully self-consistent solution for the gain factors and the lens model, we found that a single self-calibration with the converged VLC model can reduce the systematic errors introduced by self-calibration. Our results on a self-consistent self-calibration of MG0414 are inconclusive, probably because the errors are dominated by the poorly fitting lens model.

It is clear that the conservative error analysis described in §3.1 (Kochanek & Narayan 1992, Kochanek 1995, Chen, Kochanek, & Hewitt 1995) was justified and that strict statistical interpretation of the residuals in CLC would have been misleading. The difficulty in comparing CLC results derived from two different datasets reflects the ambiguities in the measure of goodness-of-fit when standard CLEAN maps are used as input data. In the VLC implementation of LensClean, the measure of goodness-of-fit is straightforward. Also, by eliminating many of the sources of systematic errors, we may now be able to interpret the measure of fit more reliably and use less conservative error bars in the VLC lens model solutions. However, Monte Carlo simulations are required to calculate the true parameter confidence levels.

In agreement with Kochanek (1995), we find that the expanded elliptical mass distribution model is a good but not perfect fit to the MG1654 system. The fit has improved, and an important conclusion to be drawn from this work is that a centrally concentrated smooth potential provides a remarkably complete description of the lens galaxy in MG1654. Furthermore, with the increased precision of the VLC models, we find that the radial behavior of the potential deviates slightly from isothermal. We have not yet found an adequate fit to the MG0414 data; we are currently exploring other models of this system.

5. Discussion

We had two goals in this paper: The general goal of examining the systematic errors in radio maps, and the specific goal of examining and removing the effects of these errors on gravitational lens modeling. The additional constraints provided by the knowledge that the source is lensed allows this analysis in the same manner as closure amplitudes and phases allow self-calibration. The CLC lens modeling algorithm was known to be affected by systematic errors (e.g. Kochanek & Narayan 1992, Kochanek 1995), although the tests performed by Chen, Kochanek, & Hewitt (1995) showed that while the CLC error does change significantly with small changes in the image reconstruction, the final lens model solutions were consistent if one adopted conservative confidence intervals. Using the VLC implementation, we explored the level of the systematic errors in the image reconstructions of both compact and extended emission systems.

The CLC analyses showed deconvolution errors are present in the reconstructed images of both compact and extended emission sources. These errors severely corrupt our ability to measure the absolute fit of the model to the data. The deconvolution techniques are unreliable in the reconstructions of slightly resolved components, as in MG0414, and diffuse regions, as in MG1654. We found that the standard imaging constraints used by CLEAN and NNLS are not sufficient to satisfy the lensing constraints. Therefore, the resulting image reconstructions are not fully consistent with the gravitational lensing hypothesis. Modifying the LensClean algorithm to operate directly on the visibility data improves the model fits by reducing the deconvolution errors. Moreover, the lensing requirements reduce the range of allowable solutions, and give better fits to the data than a normal clean. With a reasonable lens model, the VLC algorithm appears to better reconstruct the true image and suppress the residual sidelobes in the empty regions of the map.

In the case of MG1654, we also found that self-calibration reinforces these deconvolution

Table 1. Results for the MG 0414+0534 system.

Method	b arcsec	(x, y) arcsec	γ	θ_γ degrees	βb	θ_β degrees	Min mJy/Beam	Max mJy/Beam	σ_{mul} mJy/Beam	χ^2	N_{dof}
CLC	1.180 ± 0.001	-0.473 ± 0.001 -1.267 ± 0.001	0.082 ± 0.002	-14.7 ± 1.4	0.003 ± 0.001	38.5 ± 3.2	-7.9	10	0.55	6620	315
VLC ^a	1.180 ± 0.001	-0.473 ± 0.001 -1.267 ± 0.001	0.082 ± 0.002	-14.7 ± 1.4	0.003 ± 0.001	38.5 ± 3.2	-8.4	6.4	0.62	110000	64700
VLC	1.181 ± 0.001	-0.475 ± 0.003 -1.267 ± 0.001	0.082 ± 0.001	-14.4 ± 3.2	0.004 ± 0.001	36.2 ± 2.1	-7.9	5.4	0.55	107000	64700
VLC/SC	1.180 ± 0.001	-0.480 ± 0.003 -1.267 ± 0.001	0.081 ± 0.001	-14.9 ± 3.2	0.004 ± 0.001	37.5 ± 2.1	-7.4	5.7	0.60	104000	61700

^aError statistics from applying the VLC method to the best fit CLC model, i.e. with no optimization of the parameters with VLC.

Table 1: VLC/SC indicates the results using the visibility data set after self-calibration with the converged VLC model. The lens position (x, y) is measured relative to component B. θ_γ and θ_β are measured north through east. The CLC confidence intervals are calculated using $\Delta\chi^2 \leq 15.1$. Since VLC has reduced the systematic errors, the VLC confidence intervals use $\Delta\chi^2 \leq 4$. For the CLC models, χ^2 and N_{dof} are calculated using Equation 7 and Equation 8, respectively. For the VLC models, χ^2 is calculated using Equation 9. N_{dof} is calculated using Equation 10 for the unself-calibrated data and Equation 11 for the self-calibrated data.

Our analysis clearly demonstrates that the standard deconvolution methods used in radio astronomy produce systematic errors due to poor interpolation in the visibility plane and that the errors are easily detected in VLA images of gravitational lenses. The level of these errors depends on the accuracy of the interpolation, and reconstructed images from sparser arrays, such as those from VLBI and MERLIN observations, or from data sets containing fewer visibility points will have more serious problems. The results found in this paper show that any rigorous analysis of deconvolved radio images is plagued by systematic errors. Applications which require quantitative interpretation of radio images, such as measuring the time variability in gravitationally lens sources (e.g. Lehár et al. 1992), measuring the kinematic properties of T-Tauri stars (e.g. Koerner & Sargent 1995), and “snapshot” monitoring of complex radio sources (e.g. Vasisht, Frail & Kulkarni 1995), would certainly be more accurate if the visibility data were analyzed directly using modified reconstruction algorithms like VLC.

We see that we can achieve a remarkable level of precision in fitting lens models to radio data. We should be able to further refine the lens models using observations with higher dynamic range and greater sampling than the data sets used in this study, which do not fully exploit the current capabilities of radio interferometry.

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REFERENCES

- Blandford, R.D. & Narayan, R., 1992, ARA&A, 30, 311
- Briggs, D.S., 1994, Ph.D. Thesis, New Mexico Institute of Mining and Technology, Socorro, New Mexico
- Briggs, D.S., Davis, R.J., Conway, J.E., & Walker, R.C., 1994, NRAO Memo #697
- Chen, G., Kochanek, C.S., and Hewitt, J.N., 1995, ApJ, 447, 62
- Clark, B.G., 1980, A&A, 89, 377
- Cornwell, T., 1983, A&A, 121, 281
- Cornwell, T., & Braun, R., Deconvolution, in Richard A. Perley, Frederic R. Schwab, and Alan H. Bridle, editors, Synthesis Imaging in Radio Astronomy, number 6 in Conference Series, pages 167–183. Astronomical Society of the Pacific, 1989.
- Cornwell, T., & Fomalont, E.B., Self-Calibration, in Richard A. Perley, Frederic R. Schwab, and Alan H. Bridle, editors, Synthesis Imaging in Radio Astronomy, number 6 in Conference Series, pages 185–197. Astronomical Society of the Pacific, 1989.
- Crane, P.C., & Napier, P.J., Sensitivity, in Richard A. Perley, Frederic R. Schwab, and Alan H. Bridle, editors, Synthesis Imaging in Radio Astronomy, number 6 in Conference Series, pages 139–165. Astronomical Society of the Pacific, 1989.
- Ellithorpe J.D., 1995, Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge
- Haarsma, D. B., Hewitt, J. N., Burke, B. F., and Lehár, J., 1996, in C. S. Kochanek and J. N. Hewitt, editors, Astrophysical Applications of Gravitational Lensing, Proceedings of IAU Symposium 173, Dordrecht: Kluwer Academic Publishers, in press.
- Hewitt, J.N., Turner, E.L., Lawrence, C.R., Schneider, D.P., & Brody, J.P., 1992, ApJ, 104, 968

Table 2. Results for the MG 1654+1346 system.

Method	α arcsec	s arcsec	b arcsec	(x, y) arcsec	ϵ	θ_ϵ degrees	Min mJy/ Beam	Max mJy/ Beam	σ_{mul} mJy/ Beam	χ^2	N_{dof}
CLC ^a	$1.10^{+0.01}_{-0.16}$	$0.005^{+0.048}_{-0.005}$	1.119 ± 0.106	2.263 ± 0.008 -2.078 ± 0.016	0.286 ± 0.086	11.5 ± 1.1	-0.19	0.14	0.025	133	76
CLC	1.17 ± 0.03	0.010 ± 0.003	1.234 ± 0.035	2.213 ± 0.029 -2.028 ± 0.030	0.236 ± 0.009	11.8 ± 4.0	-0.41	0.32	0.074	310	76
VLC	1.16 ± 0.02	0.016 ± 0.007	1.206 ± 0.023	2.210 ± 0.014 -2.032 ± 0.015	0.226 ± 0.002	11.1 ± 0.1	-0.24	0.25	0.061	42600	34600
VLC/SC	1.15 ± 0.01	0.016 ± 0.001	1.203 ± 0.003	2.223 ± 0.003 -2.036 ± 0.003	0.226 ± 0.002	11.1 ± 0.1	-0.20	0.29	0.061	38900	34000

^aResults from Kochanek (1995) using a different data set.

Table 2: VLC/SC indicates the results using the visibility data set after self-calibration with the converged VLC model. The lens position (x, y) is measured relative to the quasar core. θ_ϵ is measured north through east. The CLC confidence intervals are calculated using $\Delta\chi^2 \leq 15.1$. Since VLC has reduced the systematic errors, the VLC confidence intervals use $\Delta\chi^2 \leq 4$. For the CLC models, χ^2 and N_{dof} are calculated using Equation 7 and Equation 8, respectively. For the VLC models, χ^2 is calculated using Equation 9. N_{dof} is calculated using Equation 10 for the unself-calibrated data and Equation 11 for the self-calibrated data.

- Thompson, A. R., Moran, J. M., and Swenson, G. W., 1986, *Interferometry and Synthesis in Radio Astronomy*, John Wiley & Sons, New York, NY.
- Vanderriest, C., Schneider, J., Herpe, G., Chevreton, M., Moles, M., & Wlérick, G., 1989, A&A, 215, 1
- Vasisht, G., Frail, D.A., & Kulkarni, S.R., 1995, ApJ, 440, L65
- Wallington, S., & Kochanek, C.S., 1995, ApJ, 441, 58
- Wallington, S., Kochanek, C. S., & Narayan, R. 1995, ApJin press.
- Wallington, S., Narayan, R., & Kochanek, C.S., 1994, ApJ, 426, 60

- Högbom, J., 1974, ApJS, 15, 417-426
- Katz, C.A., Hewitt, J.N., & Moore, C.B., 1996, *in preparation*
- Kochanek, C.S., and Narayan, 1992, ApJ, 401, 461
- Kochanek, C.S., 1995, ApJ, 445, 672
- Koerner, D.W., & Sargent, A.I., 1995, AJ, 109, 2138
- Langston, G.I., Schneider, D.P., Conner, S., Carilli, C.L., Lehár, J., Burke, B.F., Turner, E.L., Gunn, J.E. Hewitt, J.N., & Schmidt, M., 1989, AJ, 97, 1283
- Langston, G.I., Conner, S.R., Lehár, J., Burke, B.F., & Weiler, K.W., 1990, Nature, 344, 43
- Lawrence, C.R., Elston, R., Jannuzi, B.T., & Turner, E.L., 1995, AJ, *in press*
- Lawson, C.L. & Hanson, R.J., 1974, Solving Least Squares Problems, Prentice-Hall, Englewood Cliffs, NJ.
- Lehár, J., Hewitt, J.N., Roberts, D.H., & Burke, B.F., 1992, ApJ, 384, 453
- Narayan, R., & Nityananda, R., 1986, ARA&A, 24, 127
- Pearson, T.J., & Readhead, A.C.S., 1984, ARA&A, 22, 97
- Perley, R., High Dynamic Range Imaging, in Richard A. Perley, Frederic R. Schwab, and Alan H. Bridle, editors, Synthesis Imaging in Radio Astronomy, number 6 in Conference Series, pages 287–313. Astronomical Society of the Pacific, 1989
- Press, W. H., Rybicki, G. B., and Hewitt, J. N. 1992a, ApJ, 385, 404
- Press, W. H., Rybicki, G. B., and Hewitt, J. N. 1992b, ApJ, 385, 416
- Schild, R.E., 1990, AJ, 100, 1771
- Schild, R., and Thomson, D. J. 1995, AJ, 109, 1970
- Schwab, F. R., 1984, AJ, 89, 1076

Fig. 2.— The MG 0414+0534 residual images after LensClean. The absolute contour levels are the same as in Figure 1a. Panel (a) shows the best fit residuals after Clean Map LensClean. Panel (b) shows the residuals after Visibility LensClean using the best fit Clean Map LensClean lens model. Panel (c) shows the best fit residuals after Visibility LensClean. Panel (d) shows the best fit residuals after Visibility LensClean of the self-calibrated data set. See Table 1 for a summary of the properties of these maps and models.

Fig. 3.— The MG 1654+1346 residual images after LensClean. The absolute contour levels are the same as in Figure 1b. Panel (a) shows the best fit residuals after Clean Map LensClean. Panel (b) shows the best fit residuals after Visibility LensClean. Panel (c) shows the best fit residuals after Visibility LensClean of the self-calibrated data set. See Table 2 for a summary of the properties of these maps and models.

Fig. 4.— The reconstructed Visibility LensClean images of MG 1654+1346. The absolute contour levels are the same as in Figure 1b. Panel (a) shows the reconstructed image at its natural resolution. The size of the clean beam is 207 mas by 188 mas. The image peak is 4.75 mJy/Beam. Panel (b) shows the super-resolved image using a 100 mas beam. The peak in the image is 2.00 mJy/Beam.