Affleck-Dine Baryogenesis
after
Thermal Inflation

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Abstract

We argue that an extension of the Minimal Supersymmetric Standard Model that gives rise to viable thermal inflation, and so does not suffer from a Polonyi/moduli problem, should contain right-handed neutrinos which acquire their masses due to the vacuum expectation value of the flaton that drives thermal inflation. This strongly disfavours SO(10) Grand Unified Theories. The $\mu$-term of the MSSM should also arise due to the vev of the flaton. With the extra assumption that $m_L^2 - m_{H_u}^2 < 0$, but of course $m_L^2 - m_{H_u}^2 + |\mu|^2 > 0$, we show that a complicated Affleck-Dine type of baryogenesis employing an $LH_u$ D-flat direction can naturally generate the baryon asymmetry of the Universe.
1 Introduction

Thermal inflation [1, 2, 3] provides the most compelling solution to the moduli (Polonyi) problem [4, 5, 6]. However, for a theory of the early Universe to be viable it must be capable of producing a baryon asymmetry [7]

\[ \frac{n_B}{s} \sim 3 \times 10^{-11} \]  

by the time of nucleosynthesis. Thermal inflation probably dilutes any pre-existing baryon asymmetry to negligible amounts, and the final reheat temperature after thermal inflation \( T_f \sim \text{few GeV} \) is probably too low even for electroweak baryogenesis. Thus if thermal inflation really is the solution of the moduli problem, then it is likely also to be responsible for baryogenesis.

In Section 2 we explain why the flaton that gives rise to thermal inflation probably also generates the masses of right-handed neutrinos as well as the \( \mu \)-term of the Minimal Supersymmetric Standard Model (MSSM). We also note the various ways in which a potential domain wall problem can be avoided. In Section 3 we describe how a somewhat complicated Affleck-Dine type mechanism can naturally generate the required baryon asymmetry after thermal inflation. In Section 4 we give our conclusions.

2 Thermal inflation, right-handed neutrinos, and the \( \mu \)-term

2.1 Thermal inflation and right-handed neutrinos

The superpotential of the MSSM is [8]

\[ W_{\text{MSSM}} = \lambda_1 QH_u t + \lambda_6 QH_d b + \lambda_\tau LH_d \tau + \mu H_u H_d \]  

Thermal inflation [1] requires that there is in addition at least one flaton \( \phi \) with vacuum expectation value \( |\phi| = M \) in the range \( 10^{10} \text{GeV} \lesssim M \lesssim 10^{12} \text{GeV} \), the lower bound coming from the requirement that thermal inflation sufficiently dilutes the moduli, and the upper bound from the requirement that the final reheat temperature after thermal inflation \( T_f \) be high enough to thermalise the lightest supersymmetric particles (LSP’s) produced in the flaton’s decay and so avoid an excess of LSP’s.

Also, in order for \( \phi \) to be held sufficiently strongly\(^2\) at \( \phi = 0 \) by the finite temperature during thermal inflation, \( \phi \) must have unsuppressed couplings to at

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\(^1\)Here, and throughout most of this paper, all indices (the usual gauge and generation indices, as well as any singlet indices) have been suppressed. We will for the most part be focusing on the third generation as is suggested by our notation.

\(^2\)Note the strong dependence on \( T_C \) in Eq. (35) of Ref. [1].
least one other field, say $\psi$, that is light when $\phi = 0$. We therefore either require a term $\lambda_\phi \phi \psi^2 / 2$ with $|\lambda_\phi| \sim 1$ in the superpotential, or require $\phi$ to spontaneously break a continuous gauge symmetry with gauge coupling $g_\phi \sim 1$ (with $\psi$ being the gauge field in this case). One reason to prefer the Yukawa coupling over the gauge coupling is that the renormalisation group [8] effect of the Yukawa coupling would be to drive the soft supersymmetry breaking mass squared of $\phi$ negative at low energies as is required for a flaton, while the gauge coupling would have the opposite effect. Another reason is that the gauge symmetry has no independent motivation, while, as we shall see, the Yukawa coupling is very well motivated. We will therefore focus on the case of the Yukawa coupling.

After $\phi$ acquires its vacuum expectation value $M$, $\psi$ will acquire a mass $|\lambda_\phi| M \sim 10^{10} \text{GeV}$ to $10^{12} \text{GeV}$ and so is not a MSSM field. In order for $\phi$ to be coupled to the thermal bath, which is presumably composed of MSSM fields, we therefore require $\psi$ to couple to the MSSM. We therefore require at least one of the terms $LH_u \psi$ or $\psi H_u H_d$ in the superpotential, as these are the only possible renormalisable couplings of a singlet to the MSSM.

The former is the standard coupling $\lambda_\nu LH_u \nu$ of a right-handed neutrino $\psi = \nu$ to the MSSM, and furthermore $\nu$ automatically acquires a mass $M_\nu = |\lambda_\phi| M \sim 10^{10} \text{GeV}$ to $10^{12} \text{GeV}$ in the right range for the seesaw mechanism [10] to generate a left-handed tau neutrino mass

$$m_{\nu_L} = \frac{m_D^2}{M_\nu} = \frac{|\lambda_\nu|^2 (174 \text{GeV})^2 \sin^2 \beta}{|\lambda_\phi| M}$$  \hspace{1cm} (3)

$$\sim 5 \text{eV} \left( \frac{3 \times 10^{12} \text{GeV}}{M/|\lambda_\nu|^2} \right) \left( \frac{1}{|\lambda_\phi|} \right) \left( \frac{\sin^2 \beta}{0.5} \right)$$  \hspace{1cm} (4)

suitable for the mixed dark matter scenario for the formation of the large scale structure of the Universe [11].

The latter coupling $\psi H_u H_d$ does not have any good independent motivation, apart from the fact that it is possible. We will therefore focus on the former case. Later though, we shall make use of this possibility for coupling to the MSSM in a different context.

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3 Assuming $\psi$ is not charged with respect to the MSSM continuous gauge symmetries as this would, in general, destroy susy GUT gauge coupling unification. However, if $\psi$ is a complete SU(5) multiplet the unification of the gauge couplings will be unaffected. For example, one could have $\lambda_\phi \phi \psi^2 = \lambda_\phi \phi 5\bar{5}$. Alternatively, appropriate choices of representations could shift the unification scale to the string scale [9].

4 If we have both then we need two different $\psi$'s, one charged and one neutral under R-parity.
2.2 The final reheat temperature after thermal inflation and the $\mu$-term

Now that we have a more precise picture of the couplings of the flaton,

$$W_{\text{so far}} = W_{\text{MSSM}} + \lambda_\nu LH_\nu + \frac{1}{2} \lambda_\phi \phi \nu^2$$

(5)

we can hope to make a more precise estimate of the decay rate of the flaton and so the final reheat temperature after thermal inflation $T_f$.

We first note that the effective superpotential coupling

$$W_{\text{seesaw}} = - \frac{(\lambda_\nu LH_\nu)^2}{2\lambda_\phi}$$

(6)

obtained by integrating out $\nu$, i.e. eliminating $\nu$ via the constraint $\partial W/\partial \nu = 0$, will give a decay rate of order $\Gamma \sim m^5/M^4$ which is negligible.

The $\phi$ dependence of the low energy renormalised coupling constants will give larger decay rates. To estimate these we first need to know the contributions of the right-handed neutrinos to the renormalisation group equations. They are [12]

$$16\pi^2 \frac{d}{dt} \lambda_t = |\nu_t|^2 \lambda_t + \ldots$$

(7)

$$16\pi^2 \frac{d}{dt} \lambda_\tau = |\nu_\tau|^2 \lambda_\tau + \ldots$$

(8)

$$16\pi^2 \frac{d}{dt} \mu_H = |\nu_\mu|^2 \mu_H + \ldots$$

(9)

$$16\pi^2 \frac{d}{dt} m_L^2 = 2 |\nu_\nu|^2 \left(m^2_t + m^2_\nu + m^2_H + |A_{LH_\nu}|^2\right) + \ldots$$

(10)

$$16\pi^2 \frac{d}{dt} m^2_{H_u} = 2 |\nu_\nu|^2 \left(m^2_t + m^2_\nu + m^2_H + |A_{LH_\nu}|^2\right) + \ldots$$

(11)

$$16\pi^2 \frac{d}{dt} A_{QH_u t} = 2 |\nu_\nu|^2 A_{LH_\nu} + \ldots$$

(12)

$$16\pi^2 \frac{d}{dt} A_{LH_d \tau} = 2 |\nu_\nu|^2 A_{LH_\nu} + \ldots$$

(13)

$$16\pi^2 \frac{d}{dt} A_{\mu_H H_u H_d} = 2 |\nu_\nu|^2 A_{LH_\nu} + \ldots$$

(14)

where $t$ is the logarithm of the renormalistion scale, the $m$’s are the soft supersymmetry breaking masses, the $A$’s are the soft supersymmetry breaking parameters in the terms in the scalar potential of the form $AW + \text{c.c.}$, the $m$’s and the magnitudes of the $A$’s are of the order of the soft supersymmetry breaking scale $m_s \sim 10^2$ to $10^3$ GeV, and the ... stand for other terms independent of the right-handed neutrinos. $|\phi|$ sets the threshold for the right-handed neutrinos, and so writing $|\phi| = M + \delta \phi_r/\sqrt{2}$ we get the effective couplings

$$W_{\text{eff}} = \frac{|\lambda_\nu|^2}{16\sqrt{2} \pi^2 M} (\lambda_t QH_u t + \lambda_\tau LH_d \tau + \mu_H H_u H_d) \delta \phi_r$$

(15)
and

\[ V_{\text{soft eff}} = \frac{|V|}{8\sqrt{2}\pi^2 M} \left( m_L^2 + m_\nu^2 + m_{H_u}^2 + |A_{LH_u}\nu|^2 \right) \left( |L|^2 + |H_u|^2 \right) \delta \phi_r \]

\[ + \frac{|V|^2}{8\sqrt{2}\pi^2 M} A_{LH_u} (\lambda_t Q H_u t + \lambda_\nu L H_d \tau + \mu H_u H_d) \delta \phi_r \]  

(16)

From these couplings we estimate the total decay rate to be

\[ \Gamma \sim \frac{|V|^4 m_s^3}{10^4 M^2} \]  

(17)

This would give a final reheat temperature after thermal inflation of

\[ T_f \simeq g^\frac{3}{4} \Gamma^\frac{1}{2} M_{\text{Pl}}^\frac{3}{2} \]

\[ \sim 10 \text{ MeV} \left( \frac{3 \times 10^{12} \text{ GeV}}{M/|V|^2} \right) \left( \frac{m_s}{300 \text{ GeV}} \right)^\frac{3}{2} \]  

(18)

(19)

where the first bracket is constrained to be of order one, or perhaps less, by Eq. (4). However, in order not to over-produce LSP’s we require [13]

\[ T_f > 1 \text{ GeV} \left( \frac{m_s}{300 \text{ GeV}} \right)^{1.5} \]  

(20)

Thus our model seems to be in trouble unless we can add some extra coupling that gives a stronger decay rate. The only possibility is to couple \( \phi \) to \( H_u H_d \) in the superpotential. A term \( \phi H_u H_d \) would require a very small coupling constant to avoid generating too large a \( \mu \)-term, but a term\(^5\)

\[ W_{\text{decay}} = \frac{\lambda_\mu \phi^2 H_u H_d}{M_{\text{Pl}}} \]  

(21)

is not only allowed but could naturally generate a \( \mu \)-term

\[ \mu_\phi = \frac{\lambda_\mu \langle \phi \rangle_{\text{vac}}}{M_{\text{Pl}}} \]  

(22)

of the required size [14]. For example, for \( M = 10^{11} \text{ GeV} \) and \( |\mu_\phi| = 0.1 \) we get \( |\mu_\phi| = 400 \text{ GeV} \). From now on we will assume that the \( \mu \)-term is generated in this way so that \( \mu_H = 0 \), or at least \( |\mu_H| \lesssim |\mu_\phi| \), and \( |\mu_\phi| \sim m_s \).

Writing \( \phi = \mathcal{M} + \delta \phi \), where \( |\mathcal{M}| = M \) and \( \mu_\phi = \lambda_\mu \mathcal{M}^2/M_{\text{Pl}} \), we get the relatively unsuppressed couplings

\[ \mathcal{L}_{\text{decay}} = 2\mu_\phi \bar{H}_u H_d \frac{\delta \phi}{\mathcal{M}} - 2 |\mu_\phi|^2 \left( |H_u|^2 + |H_d|^2 \right) \frac{\delta \phi}{\mathcal{M}} \]

\[ - 2\mu_\phi \left( \lambda_t Q t H_d + \lambda_\nu Q b H_u + \lambda_\tau L \tau H_u + A_\mu H_u H_d \right) \frac{\delta \phi}{\mathcal{M}} + \text{c.c.} \]  

(23)

\(^5\)A term \( \lambda_\mu \phi^3 H_u H_d/M_{\text{GUT}}^3 \) would be an alternative if \( M \gtrsim 3 \times 10^{11} \text{ GeV} \). We assume the displayed case for simplicity.
where a tilde denotes the fermionic component of the superfield, a bar denotes the hermitian conjugate, and $A_\mu$ is the soft supersymmetry breaking parameter in $V_{\text{soft}} = A_\mu \mu H_u H_d + \text{c.c.}$ and so $|A_\mu| \sim m_s$. The decay rate is then estimated to be

$$\Gamma \sim \frac{m_s^3}{10^2 M^2}$$

(24)

where we have roughly assumed $m_\phi \sim |\mu_\phi| \sim |A_\mu| \sim m_s$. We therefore get a reheat temperature

$$T_f \sim (1 \text{ to } 10 \text{ GeV}) \left( \frac{10^{11} \text{ GeV}}{M} \right) \left( \frac{m_s}{300 \text{ GeV}} \right)^{\frac{3}{2}}$$

(25)

which is sufficiently high.

We thus expect the flaton to have the following couplings to the MSSM 6

$$W_{\text{couplings}} = \lambda_\nu L H_u \nu + \frac{1}{2} \lambda_\phi \phi^2 + \frac{\lambda_\mu \phi H_u H_d}{M_{\text{Pl}}}$$

(26)

### 2.3 The flaton potential and domain walls

In this section we consider the self-couplings of the flaton. The flaton (or, in the case of a multi-component flaton, at least one component of the flaton) should have a negative soft supersymmetry breaking mass squared $-m_\phi^2 |\phi|^2$ to drive it away from $\phi = 0$ after thermal inflation. It will also need a term to stabilise its potential at its vacuum expectation value $|\phi| = M \sim 10^{10}$ to $10^{12}$ GeV. The simplest possibility is 7

$$W_{\text{vev}} = \frac{\lambda_M \phi^4}{4 M_{\text{Pl}}}$$

(27)

and this is what we shall assume. In the case of a multi-component flaton this could be interpreted as for example $\lambda_M \phi^4/4 M_{\text{Pl}}$, $\lambda_M \phi_1 \phi_2^2/M_{\text{Pl}}$ or a sum of such terms. See Ref. [15] for an explicit multi-component example. Here, for simplicity, we will focus on the case of a single component flaton, though it should be born in mind that a multi-component flaton might be preferable from the model building point of view.

We then get the following scalar potential

$$V(\phi) = V_0 - m_\phi^2 |\phi|^2 + \left( \frac{A_M \lambda_M \phi^4}{M_{\text{Pl}}} + \text{c.c.} \right) + \frac{|\lambda_M|^2 |\phi|^6}{M_{\text{Pl}}^2}$$

(28)

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6 As mentioned before, $\lambda_\mu \phi^3 H_u H_d/M_{\text{GUT}}^2$ is a possible alternative to $\lambda_\mu \phi^2 H_u H_d/M_{\text{Pl}}$.

7 $W_{\text{vev}} = \lambda_M \phi^5/M_{\text{GUT}}^2$ would be an alternative if $M \gtrsim 3 \times 10^{11}$ GeV. Again we assume the displayed case for simplicity. One might even be able to use the renormalisation group running of $m_\phi$ to stabilise the potential but then one would not automatically get a value for $M$ in the correct range.
where $m_\phi \sim |A_M| \sim m_s$. This potential has four degenerate minima with $|\phi| = M$, where

$$M^2 = \frac{2m_\phi M_{Pl}}{3|\lambda_M|} \left( \frac{|A_M|}{m_\phi} + \sqrt{\frac{|A_M|^2}{m_\phi^2} + \frac{3}{4}} \right)$$

(29)

For example, for $m_\phi = |A_M| = 300$ GeV and $|\lambda_M| = 0.1$, we get $M = 10^{11}$ GeV. The eigenvalues of the mass squared matrix at the minima are

$$m^2_{\delta \phi} = \frac{16m_\phi |A_M|}{3} \left( \frac{|A_M|}{m_\phi} + \sqrt{\frac{|A_M|^2}{m_\phi^2} + \frac{3}{4}} \right)$$

(30)

and

$$m^2_{\delta \phi} = 4m_\phi^2 + m^2_{\delta \phi} = 4m_\phi^2 \left[ 1 + \frac{4|A_M|}{3m_\phi} \left( \frac{|A_M|}{m_\phi} + \sqrt{\frac{|A_M|^2}{m_\phi^2} + \frac{3}{4}} \right) \right]$$

(31)

and the flatino mass squared is

$$m^2_{\delta \phi} = 3m_\phi^2 + \frac{3}{2}m^2_{\delta \phi} = 4m_\phi^2 \left( \frac{|A_M|}{m_\phi} + \sqrt{\frac{|A_M|^2}{m_\phi^2} + \frac{3}{4}} \right)^2$$

(32)

Requiring zero cosmological constant at the minima gives

$$V_0 = \frac{2}{3} m_\phi^2 M^2 \left[ 1 + \frac{2|A_M|}{3m_\phi} \left( \frac{|A_M|}{m_\phi} + \sqrt{\frac{|A_M|^2}{m_\phi^2} + \frac{3}{4}} \right) \right]$$

(33)

With four degenerate minima we clearly have to worry about a potential domain wall problem. The simplest way to eliminate the domain walls is to add a small term which breaks the degeneracy of the vacua, the difference in pressure exerted on the walls causing the domains with greater vacuum energy to collapse. They collapse before the domain walls come to dominate the energy density if the difference in vacuum energies $\epsilon$ satisfies $\epsilon \gtrsim \sigma^2/M_{Pl}^2$ where $\sigma$ is the energy per unit area of the domain walls [16]. For flaton domain walls $\sigma \sim m_s M^2$ and so we require

$$\epsilon \gtrsim \frac{m_s^2 M^4}{M_{Pl}^2}$$

(34)

Therefore a term in the superpotential of the form

$$W_{walls} \gtrsim \frac{m_s M^{4-n} \phi^n}{M_{Pl}^2}$$

(35)

with $n$ odd would be sufficient to eliminate the domain walls. A term with $n = 2 \mod 4$, for example $W_{walls} \sim \phi^6/M_{Pl}^3$, would reduce the $Z_4$ domain walls to $Z_2$ domain walls. Note that $W_{walls}$ can be extremely small, and hence have
a negligible effect on the dynamics to be discussed in the next section, but still solve the domain wall problem.

Another way to avoid a domain wall problem is to gauge the discrete symmetry so that there is really only one vacuum. However, non-trivial anomaly cancellation conditions must be satisfied [17]. In the case of a single-component flaton with the superpotential of Eq. (36), and no extra light SU(3) multiplets, the mixed discrete-SU(3) anomaly cancellation condition requires \( \phi^2 \) to be neutral under any unbroken\(^8\) anomaly free discrete gauge symmetry. We therefore cannot use a discrete gauge symmetry to remove the \( Z_4 \) domain walls. However, the anomaly free \( Z_4 \) subgroup of \( U(1)_{B-L+4Y} \), under which \( \phi \) has charge 2, can be used to gauge away the \( Z_2 \) domain walls left by \( W_{\text{walls}} \sim \phi^6/M_{\text{Pl}}^3 \) above. Furthermore, this symmetry is broken down to the standard matter parity of the MSSM (which is equivalent to the R-parity of the MSSM) by the vacuum expectation value of \( \phi \).

In the case of a multi-component flaton the discrete symmetry can be extended to a continuous symmetry which may or may not be gauged. In the case of a continuous global symmetry, for example the Peccei-Quinn symmetry of Ref. [15], the Goldstone bosons may prove troublesome [18]. One also has more freedom to satisfy the anomaly cancellation conditions in the case of a multi-component flaton.

2.4 Summary

An extension of the MSSM that gives rise to viable thermal inflation, and so does not suffer from a moduli problem, should have the following terms in its superpotential

\[
W_{ti} = \lambda_t Q H_u t + \lambda_b Q H_d b + \lambda_t L H_d \tau + \lambda_u L H_u \nu + \frac{1}{2} \lambda_\phi \phi \nu^2 + \frac{\lambda_\mu \phi^2 H_u H_d}{M_{\text{Pl}}} + \frac{\lambda_M \phi^4}{4M_{\text{Pl}}} \tag{36}
\]

3 \( LH_u \) Affleck-Dine baryogenesis after thermal inflation

To orientate the reader we will first sketch the basic idea we have in mind before plunging into the details.

The \( D \)-flat direction parametrised by \( LH_u \) provides an ideal Affleck-Dine field [19, 20, 5]. In order for it to behave as an Affleck-Dine field we must first get it away from zero. We therefore require its mass squared at the end of thermal inflation, \( (m^2_L - m^2_{H_u} + |\mu_H|^2) / 2 \), to be negative. It is simplest to assume that it rolls away from zero before \( \phi \) does. However, when \( \phi = 0 \) the right-handed

\(^8\)R-symmetries are broken down to \( Z_2 \) by hidden sector supersymmetry breaking.
neutrinos are light, and so $LH_u$ is not $F$-flat (it has a quartic term in its potential coming from the superpotential). $LH_u$ will thus be stabilised at a modest value.

Next $\phi$ will roll away from zero. The right-handed neutrinos become heavy and so can be integrated out leaving the effective seesaw coupling $W_{\text{seesaw}}$ given in Eq. (6). This is now the term that stabilises the $LH_u$ direction and we see that it gets smaller, and so the $LH_u$ direction gets flatter, as $\phi$ gets larger. Thus, as $\phi$ rolls away from zero, $LH_u$ will roll further away from zero. Furthermore, the soft supersymmetry breaking term derived from $W_{\text{seesaw}}$ will correlate the phases of $\phi$ and $LH_u$.

When $\phi$ becomes sufficiently large, it will start to feel the basin of attraction of one of the minima of its potential, and so will start curving in towards that minimum, i.e. its phase will be roughly determined modulo $\pi/2$. The phase of $LH_u$ will then be roughly determined modulo $\pi/4$ by the soft supersymmetry breaking term derived from $W_{\text{seesaw}}$.

When $|\phi|$ becomes of order $M$, a cross term from the supersymmetric part of the potential becomes significant and changes the correlation between the phases of $\phi$ and $LH_u$, and so gives the phase of $LH_u$ a kick. The direction of the kick is determined by the parameters in the lagrangian (this is our $CP$ violation) and so gives a non-zero net contribution when averaged over different spatial locations, unlike the rest of the angular momentum that is flying around. Furthermore, as this is happening $W_{\text{decay}}$ (see Eq. (21)) starts to give a significant contribution to the mass of $H_u$ and hence $LH_u$. For $m_1^2 - m_2^2 + |\mu_H + \mu_\phi|^2 > 0$ this gives the $LH_u$ direction an overall positive mass squared (as it must because $LH_u$ has a positive mass squared in the true vacuum), and so sends $LH_u$ spiralling back in towards zero.

The effective friction on the motion of $\phi$ and $LH_u$ coming from the Hubble expansion is negligible. However, the effective mass squareds of both $\phi$ and $LH_u$ have been changing sign during the above dynamics and so one would expect them both to decay via broad parametric resonance [21]. This will lead to approximately critical damping, and so it seems reasonable to expect that both $\phi$ and $LH_u$ will be trapped near their vacuum expectation values essentially immediately after the dynamics described above has occurred. Once they are trapped, parametric resonance becomes less efficient because the mass squareds are now always positive. $LH_u$'s potential near $LH_u = 0$ conserves angular momentum, or in other words lepton number, and so $LH_u$'s newly acquired lepton number is conserved.

The dynamics outlined above is illustrated in Figure 1.

The decay of the $LH_u$ Affleck-Dine condensate will generate enough partial reheating to restore the electroweak symmetry, and so its lepton number can be converted to baryon number by the usual electroweak effects [22]. Note that the energy density is still dominated by the flaton and the reheating in the Affleck-Dine sector has a negligible effect on the now decoupled flaton. Finally, after the temperature has dropped to a few GeV, the flaton decay will complete, releasing
substantial entropy.

3.1 Estimating the baryon asymmetry

Our basic model is

$$W_{ti} = \lambda t Q H_u t + \lambda_b Q H_d b + \lambda_t L H_d \tau + \lambda_u L H_u \nu + \frac{1}{2} \lambda_\phi \nu^2 + \frac{\lambda_\mu \phi^2 H_u H_d}{M_{Pl}} + \frac{\lambda_M \phi^4}{4 M_{Pl}}$$  \hspace{1cm} (37)$$

The squark fields have no linear terms in their potential and have positive mass squareds. They will therefore be held at zero apart from thermal fluctuations, and so can be ignored apart from their contribution to the finite temperature effective potential. The zero temperature potential for the other fields is

$$V = |\lambda_\tau L H_d|^2 + |\lambda_\nu L H_u + \lambda_\phi \nu|^2 + |\lambda_\tau H_d \tau + \lambda_u H_u \nu|^2 + \left| \lambda_\nu L \nu + \frac{\lambda_\mu \phi^2 H_d}{M_{Pl}} \right|^2 + \left| \lambda_\tau L \tau + \frac{\lambda_\mu \phi^2 H_u}{M_{Pl}} \right|^2 \hspace{1cm} + D\text{-terms}$$

$$+ \left( A_\tau \lambda_\tau L H_d \tau + A_\nu \lambda_\nu L H_u \nu + A_\phi \lambda_\phi \nu^2 + A_\mu \lambda_\mu \phi^2 H_u H_d + A_M \lambda_M \phi^4 + \text{c.c.} \right)$$

$$+ m_\tau^2 |\tau|^2 + m_\nu^2 |\nu|^2 + m_L^2 |L|^2 - m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 - m_\phi^2 |\phi|^2$$  \hspace{1cm} (38)$$

where the $m$’s and the magnitudes of the $A$’s are of order $m_s$. We assume

$$m_{LH_u}^2 \big|_{\phi=0} = \frac{1}{2} \left( m_L^2 - m_{H_u}^2 \right) < 0 \hspace{1cm} (39)$$

so that the $D$-flat direction parametrised by $LH_u$ is also unstable, in addition to the flaton $\phi$. Note that after $\phi$ acquires its vacuum expectation value $M \sim \sqrt{m_s M_{Pl} / |\lambda_M|}$, it will give an extra contribution $|\lambda_\mu|^2 M^4 / M_{Pl}^2$ to $H_u$’s mass squared. This will be of order $m_s^2$ if $|\lambda_\mu| \sim |\lambda_M|$. We assume

$$m_{LH_u}^2 \big|_{\phi=M} = \frac{1}{2} \left( m_L^2 - m_{H_u}^2 + \frac{|\lambda_\mu|^2 M^4}{M_{Pl}^2} \right) > 0 \hspace{1cm} (40)$$

so that the $LH_u$ direction is stable in the true vacuum.

A rigorous study of the dynamics of this model is beyond the scope of this paper. Instead we will make some simplifying assumptions in order to illustrate how the Affleck-Dine mechanism might be implemented after thermal inflation and to crudely estimate the resultant baryon asymmetry.

We assume that all fields are initially held at zero by the finite temperature during thermal inflation. We assume that the $LH_u$ direction rolls away from zero first. It will be quickly stabilised by the term $|\lambda_\nu L H_u|^2$ at a value $|LH_u| \sim$
Then the term governed by the zero temperature potential cause some early time because all its couplings would be small. The term \( A_\nu \nu \) say the right-handed electron sneutrino, which could plausibly have a small quartic coupling \( L_\nu \) writing.We assume the\( D \)-terms constrain to the minimum of its potential

\[
V = |\nu| \left[ \lambda_\nu L_H + /|\nu| \right]^2 + |\lambda_\nu H_u \nu| \left[ \nu_\mu \phi^2 H_u + \frac{\lambda_\mu \phi^2 H_u}{M_{Pl}} + \frac{\lambda_\mu \phi^2 H_u}{M_{Pl}} \frac{1}{2} \right] + \left[ A_\nu \lambda_\nu H_u \nu + A_\nu \lambda_\nu \phi^2 + A_\nu \lambda_\nu \phi^2 + c.c. \right] + m_\nu^2 |\nu|^2 + m_\nu^2 \left[ L \phi^2 - m_H^2 |H_u|^2 - m_\nu^2 |\phi|^2 \right] \tag{42}
\]

As \( |\phi| \) increases, \( \nu \) will quickly acquire a large mass \( \sim |\lambda_\phi \phi| \), and so will be constrained to the minimum of its potential

\[
\nu \simeq - \frac{\lambda_\nu L_H}{\lambda_\phi \phi} \tag{43}
\]

The effective potential then becomes

\[
V = \left( |\nu| \left[ \lambda_\nu H_u \right] + |\lambda_\nu H_u \left] \right] \left[ \lambda_\nu L_H \left] \right] \frac{2}{\lambda_\phi \phi} \right) + \left[ \lambda_\nu L_H \left\frac{2}{\lambda_\phi \phi} \right] + \left[ \lambda_\nu L_H \left\frac{2}{\lambda_\phi \phi} \right] + \frac{\lambda_\nu L_H \left\frac{2}{\lambda_\phi \phi} \right] + D\text{-terms} + \left[ A_\nu \lambda_\nu H_u \nu + A_\nu \lambda_\nu \phi^2 + A_\nu \lambda_\nu \phi^2 + c.c. \right] + m_\nu^2 |\nu|^2 + m_\nu^2 \left[ L \phi^2 - m_H^2 |H_u|^2 - m_\nu^2 |\phi|^2 \right] \tag{44}
\]

We assume the\( D\)-terms constrain \( L \) and \( H_u \) to the \( D\)-flat direction \( L_H \). Then writing \( L_H = \psi^2 / 2 \) we get

\[
V = \frac{\lambda_\nu L_H \psi^2}{2\phi} + \frac{1}{2} \left[ \frac{\lambda_\nu L_H \psi^2}{M_{Pl}} \right] + \left[ \frac{\lambda_\nu L_H \psi^2}{M_{Pl}} \right] - m_\phi^2 |\phi|^2 \tag{45}
\]

\footnote{One might imagine that our Affleck-Dine type mechanism could also be implemented using say the right-handed electron sneutrino, which could plausibly have a small quartic coupling \( \lambda_\phi \), instead of \( L_H \). However, unlike \( L_H \), if it was unstable it would roll away from zero at some early time because all its couplings would be small. The term \( A_\nu \lambda_\phi \nu^2 \phi + c.c. \) would then cause \( \phi \) to roll away from zero causing a premature end to thermal inflation.}
where \( m^2_\psi = (m^2_{H_u} - m^2_{L})/2 \). To make this potential more transparent, we make the following change of variables

\[
V = m^2_\psi M^2 \tilde{V}, \quad \phi = M\tilde{\phi}, \quad \psi = M\tilde{\psi}, \quad m_\phi = m_s a_\phi, \quad m_\psi = m_s a_\psi
\]  
(46)

\[
A_\nu - A_\phi = m_s \alpha_\nu, \quad A_M = m_s \alpha_M, \quad \lambda_\mu = \frac{m_s M_{Pl}}{M^2} \beta_\mu, \quad \lambda_\phi = \frac{M}{m_s} \gamma
\]  
(47)

where the \( a \)'s and the magnitudes of the \( \alpha \)'s and \( \beta \)'s are of order one and we assume \( |\gamma| \ll 1 \). We then get

\[
\tilde{V} = -a^2_\phi |\phi|^2 + |\beta_M|^2 |\tilde{\phi}|^6 + \left( -a^2_\psi + \frac{|\tilde{\psi}|^2}{2|\tilde{\phi}|} + \frac{1}{2} |\beta_\mu|^2 |\tilde{\phi}|^4 \right) |\tilde{\psi}|^2
\]

\[
+ \left[ \alpha_M \beta_M \tilde{\phi}^4 \left( \alpha_\nu - \alpha_M \tilde{\phi} \frac{\tilde{\phi}^4}{2 |\alpha_M \tilde{\phi}|^2} \right) \frac{\tilde{\psi}^4}{4|\tilde{\phi}|} + \text{c.c.} \right]
\]  
(48)

When \( \tilde{\phi} \ll 1 \), \( \tilde{\psi} \)'s potential is stabilised at

\[
|\tilde{\psi}|^2 \sim |\gamma| |\tilde{\phi}|
\]  
(49)

while its phase is coupled to that of \( \tilde{\phi} \) by the term\(^{10}\)

\[
- \left( \alpha_\nu - \alpha_M \tilde{\phi} \right) \frac{\tilde{\psi}^4}{2 |\alpha_M \tilde{\phi}|^2} \frac{\tilde{\phi}^4}{4|\tilde{\phi}|} + \text{c.c.}
\]  
(50)

the second term in the brackets being negligible at this stage.

When \( \tilde{\phi}^3 \gtrsim \gamma \), the potential for the phase of \( \tilde{\phi} \) will be dominated by the term

\[
\alpha_M \beta_M \tilde{\phi}^4 + \text{c.c.}
\]  
(51)

and so in some sense we can regard the phase of \( \tilde{\phi} \) as being determined modulo \( \pi/2 \). Put in a different way, \( \tilde{\phi} \) will be pulled towards one of the minima of its potential and so its phase will be strongly biased towards

\[
\alpha_M \beta_M \tilde{\phi}^4 = - |\alpha_M \beta_M \tilde{\phi}^4|
\]  
(52)

The phase of \( \tilde{\psi} \) is then determined modulo \( \pi/8 \) by the term in Eq. (50).

When \( \tilde{\phi} \) becomes of order one, two things happen. First, the second term in the brackets in Eq. (50) becomes of order one and gives the phase of \( \tilde{\psi} \) a kick in

\(^{10}\)The correlation induced by this term is different from that of Eq. (41) and so the phase of \( \psi \) will get a kick in the direction \( \sin(\arg A_\phi - \arg A_\nu) \) while the phase of \( \phi \) will get a kick in the opposite direction. This may contribute to the net lepton number generated, in addition to the similar effect to be described below.
the direction $\sin(\arg \alpha_N - \arg \alpha_M)$. Note that even before this $\tilde{\psi}$ will have had some angular momentum about $\tilde{\psi} = 0$, but it averages out to zero in the Universe as a whole, as is shown in Figure 2(a). This new contribution has a direction determined by the parameters of the lagrangian and so will give a non-zero net contribution, as is shown in Figure 2(b). Put another way, the difference in phase between $\alpha_N$ and $\alpha_M$ is our source of $CP$ violation. Secondly, the last term in the brackets in

$$
\left(-a^2_{\psi} + \frac{|\tilde{\psi}|^2}{2\gamma\phi} + \frac{1}{2} |\beta_\mu|^2 |\phi|^4 \right)|\tilde{\psi}|^2
$$

becomes of order one giving $\tilde{\psi}$ a net positive mass squared and so causing it to spiral back in to $\tilde{\psi} = 0$.

Assuming the expected broad parametric resonance [21] provides enough damping, both $\phi$ and $\psi$ should then become trapped near their vacuum expectation values, after which the parametric resonance becomes less efficient. $\psi$’s potential near $\psi = 0$ conserves angular momentum, or in other words lepton number, and so $\psi$’s lepton number is conserved.

The Affleck-Dine condensate $\psi$ will decay well before the Hubble expansion reduces its amplitude to the electroweak scale, and so will release enough thermal energy to restore the electroweak symmetry. The lepton asymmetry will then be converted into a baryon asymmetry

$$
\frac{n_B}{s} \sim \frac{1}{3} \left( \frac{n_L}{s} \right)
$$

by the usual electroweak effects [22, 23]. Finally, after the temperature has dropped to a few GeV, the flaton decay will complete, releasing substantial entropy.

The baryon asymmetry generated in this way is roughly estimated to be

$$
\frac{n_B}{s} \sim \frac{T_m L}{m_\phi n_\phi} \sim \frac{\theta T_f n_\psi}{m_\phi n_\phi} \sim \frac{\theta|\lambda_\phi|T_f}{|\lambda_\nu|^2 M} \sim \frac{\theta(100 \text{ GeV})^2 T_f}{m_\nu L M^2}
$$

$$
\sim 10^{-10} \theta \left( \frac{10 \text{ eV}}{m_\nu L} \right) \left( \frac{T_f}{\text{GeV}} \right) \left( \frac{10^{11} \text{ GeV}}{M} \right)\quad (56)
$$

where we have used Eqs. (49) and (3). $\theta$ is defined by this equation and will depend on the phase difference between $\alpha_N$ and $\alpha_M$ as well as the detailed dynamics.

As discussed in Section 2, we expect $T_f \sim 1$ to 10 GeV and $M \sim 10^{10}$ to $10^{12}$ GeV. Neutrino phenomenology [11, 24] suggests $m_\nu L \sim 5 \text{ eV}$, $m_\nu L \sim 10^{-2}$ to $10^{-3}$ eV and $m_\nu L \lesssim m_\nu L$. Therefore, in order for Eq. (56) to give the baryon asymmetry of Eq. (1), we require $\theta$ to be roughly

$$
\theta_f \sim 10^{-4} \; \text{to} \; 10
$$

$^{11}$ $\theta \gtrsim 1$ corresponds to the scenario being unviable.
\[ \theta_e \gtrsim \theta_\mu \sim 10^{-7} \text{ to } 10^{-2} \] (58)
depending on which generations make up the Affleck-Dine \( LH_u \) direction. The eventual measurement of the Higgs and slepton masses should help to determine which of these ranges is the appropriate one (or rule out the whole scenario), and a measurement of \( m_{\nu_e L} \) would narrow the uncertainty in \( \theta_e \).

### 4 Conclusions

Right-handed neutrinos should acquire their masses due to the vacuum expectation value of the flaton that gives rise to thermal inflation, not some composite GUT operator. This will have important implications for GUT model building. In particular, SO(10) GUT’s are strongly disfavoured because the flaton would have to be in a \( \mathbf{126} \) representation which is difficult to derive from superstrings and one would have a flaton-125 splitting problem in addition to the usual doublet-triplet splitting problem.

The \( \mu \)-term of the MSSM should also be generated by the vev of the flaton.

Our Affleck-Dine type mechanism generates a baryon asymmetry which is roughly estimated to be

\[ \frac{n_B}{s} \sim 10^{-10} \theta \left( \frac{10 \text{ eV}}{m_{\nu_e}} \right) \left( \frac{T_f}{\text{GeV}} \right) \left( \frac{10^{11} \text{ GeV}}{M} \right)^2 \] (59)

where \( \theta \) is the lepton asymmetry per Affleck-Dine particle. \( \theta \) depends on the difference in phase between the soft supersymmetry breaking parameters of \( W_{\text{seesaw}} \) and \( W_{\text{vev}} \) (c.f. Eqs. (6) and (27)), as well as the detailed dynamics.

We also make the following prediction

\[ m_{\nu_e L}^2 < m_{H_u}^2 \] (60)

modulo renormalisation effects, where \( -m_{H_u}^2 \) is the soft supersymmetry breaking mass squared of \( H_u \), and \( m_{\nu_e L}^2 \) is the soft supersymmetry breaking mass squared of a lepton doublet.

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References


Figure captions

Figure 1

Numerical simulation to illustrate the dynamics of $\phi$ and $LH_u = \psi^2/2$ after thermal inflation. The potential of Eq. (48) was used with the parameters $a_\phi = a_\psi = \alpha_\nu = 1$, $|\alpha_M| = \beta_\mu = 2$, $\gamma = 10^{-3}$ and $\arg\alpha_M = 3.1$. The initial conditions were $|\phi| = |\psi| = 10^{-3}$, $\arg\phi = 1$ and $\arg\psi = 0.25$. A friction term $\Gamma \dot{\phi}$ with $\Gamma = 0.75$ was added to the equation of motion of $\phi$ to crudely simulate the effects of parametric resonance. A friction term was not added for $\psi$ because it would obscure the total lepton number generated which in reality is contained in both the homogeneous $\psi$ field and its decay products (such as the inhomogeneous $\psi$ modes produced by parametric resonance).

Figure 2

Numerical simulation to show the non-zero net lepton number generated. The same parameters as in Figure 1 were used except for the following. Motivated by Eq. (41), the initial phase of $\psi$ was taken to be random while the initial phase of $\phi$ was taken to be given by $\arg(\phi) = 4 \arg(\psi) + C$. The lepton number produced, as measured by $\arg\psi(t = 100) - \arg\psi(t = 0)$, is plotted against the initial phase of $\psi$. The dotted line gives the average value. The plots correspond to the following values of the parameters (a) $\arg\alpha_M = \pi$ (which is $CP$ conserving) and $C = 0$, (b) $\arg\alpha_M = 3.1$ and $C = 0$. 