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GENERATED TURBULENCE

Axel Brandenburg¹, Nordita, Blegdamsvej 17,
DK-2100 Copenhagen Ø, Denmark.

Åke Nordlund, TAC, Blegdamsvej 17, DK-2100 Copenhagen Ø,
Denmark and Copenhagen University Observatory.

Robert F. Stein, Dept. of Physics and Astronomy, Michigan
State University, East Lansing, MI 48824, USA.

Ulf Torkelsson², Sterrenkundig Instituut, Postbus
80000, 3508 TA Utrecht, the Netherlands.

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¹After February 1996: Dept. of Mathematics and Statistics,
Univ. of Newcastle upon Tyne, NE1 7RU, UK.

²After November 1995: Institute of Astronomy, Madingley Road,
Cambridge CB3 0HA, UK.

NORDITA · Nordisk Institut for Teoretisk Fysik

Blegdamsvej 17 DK-2100 København Ø Danmark

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Axel Brandenburg¹

Nordita, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Åke Nordlund

Theoretical Astrophysics Center, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark; and
Copenhagen University Observatory

Robert F. Stein

Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824

and

Ulf Torkelsson²

Sterrenkundig Instituut, Postbus 80000, 3508 TA Utrecht, The Netherlands

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ABSTRACT

Dynamo-generated turbulence is simulated in a modified shearing box approximation that removes scale invariance and allows finite accretion rates for a given distance from the central object. The effective Shakura-Sunyaev viscosity parameter, α_{SS} , is estimated in three different ways using the resulting accretion rate, the heating rate, and the horizontal components of the Maxwell and Reynolds stress tensors. The results are still resolution dependent: doubling the resolution leads to 1.4-1.6 times larger values for the viscosity parameter. For $63 \times 127 \times 64$ meshpoints the result is $\alpha_{\text{SS}} = 0.007$.

Subject headings: accretion: accretion disks – hydrodynamics – MHD – turbulence

¹after Feb 1996: Department of Mathematics and Statistics, University of Newcastle upon Tyne, NE1 7RU, UK

²after Nov 1995: Institute of Astronomy, Madingley Road, Cambridge CB3 0HA, UK

1. Introduction

Over the last few years it has become clear that a strong candidate for the origin of turbulence in accretion disks is the magnetic shear instability, often called the Balbus-Hawley (1991) instability. In its original form, the instability arises in the presence of a vertical magnetic field, but a purely toroidal field also leads to instability (Chandrasekhar 1961, Foglizzo & Tagger 1995). The instability operates even in the fully nonlinear regime where the magnetic field is turbulent and mostly in the toroidal direction (Hawley, Gammie, & Balbus 1995a; Matsumoto & Tajima 1995). These instabilities may differ in the nature of the dominant restoring force (magnetic tension, pressure gradients), but the main mechanism leading to the instability is always the same: when two fluid parcels in a Keplerian orbit are pulled together, they are actually torn apart. (This is why docking maneuvers in space are difficult.) The generic role of magnetic fields is to provide a coupling between otherwise independent fluid parcels.

Important outcomes of accretion simulations are the resulting Maxwell and Reynolds stresses, which are expected to be proportional to the gas pressure times a dimensionless number, the Shakura-Sunyaev viscosity parameter. The viscosity parameter can depend on (i) the magnetic field strength, (ii) the pressure, and (iii) the distance from the central object. The dependence on the field strength has been investigated by Hawley, Gammie, & Balbus (1995a). It is, however, possible that the magnetic field results from dynamo action (Brandenburg *et al.* 1995, hereafter referred to as Paper I; Hawley, Gammie, & Balbus 1995b; Stone *et al.* 1995). The magnetic field strength is then determined by the dynamics and is no longer an independent control parameter. The dependence on the pressure is removed by allowing for vertical stratification (Paper I; Stone *et al.* 1995). In unstratified simulations the vertical extent of the box, L_z , takes the role of the vertical (gaussian) pressure scale height, H , in that both determine the largest length scale of magnetic structures. The

models presented by Torkelsson *et al.* (1995) show that for $L_z \approx 1.4H$ the unstratified simulations give results for α_{SS} similar to the stratified simulations. In this *Letter* we focus on point (iii), the dependence on the radius. The main result is that for a finite radius a nonvanishing accretion rate is possible.

2. Making the Model Nonlocal

In the local approximation, cylindrical polar coordinates (r, ϕ, z) are replaced by cartesian coordinates (x, y, z) , that are valid near a given point (R, ϕ_0, z) with $x = r - R$ and $y = r(\phi - \phi_0)$. Terms of order $1/R$ are neglected in comparison with $\partial/\partial x$. This is, however, not valid with respect to volume averaged quantities. For example, in the present case, where we assume sliding periodic and periodic boundary conditions respectively in x and y , the term $\langle B_x B_y \rangle / R \neq 0$ is of course *not* small compared with $\langle \partial B_x B_y / \partial x \rangle = 0$. (In fact, all averages over x and y derivatives vanish.)

It is straightforward to restore all terms of order $1/R$. The new set of model equations is formally equivalent to the equations used in Paper I,

$$\frac{\mathcal{D}\mathbf{A}}{\mathcal{D}t} = \mathbf{u} \times \mathbf{B} + \frac{3}{2}\Omega_0 A_y \hat{\mathbf{x}} - \eta\mu_0 \mathbf{J}, \quad (1)$$

$$\frac{\mathcal{D}\mathbf{u}}{\mathcal{D}t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \mathbf{g} + \mathbf{F} + \frac{1}{\rho}\mathbf{J} \times \mathbf{B} + \frac{1}{\rho}\nabla \cdot (2\nu\rho\mathbf{S}), \quad (2)$$

$$\frac{\mathcal{D}e}{\mathcal{D}t} = -(\mathbf{u} \cdot \nabla)e - \frac{p}{\rho}\nabla \cdot \mathbf{u} + \frac{1}{\rho}\nabla \cdot (\chi\rho\nabla e) + 2\nu\mathbf{S}^2 + \frac{\eta\mu_0}{\rho}\mathbf{J}^2 + Q, \quad (3)$$

$$\frac{\mathcal{D}\ln\rho}{\mathcal{D}t} = -(\mathbf{u} \cdot \nabla)\ln\rho - \nabla \cdot \mathbf{u}, \quad (4)$$

where $\mathcal{D}/\mathcal{D}t = \partial/\partial t + u_y^{(0)}\partial/\partial y$ is the time derivative that includes the transport by the linearized Keplerian shear flow, $u_y^{(0)}(x) = -\frac{3}{2}\Omega_0 x$. The full velocity consists thus of the rigid rotation $\Omega_0 R$, the linear shear flow $u_y^{(0)}$, and a three-dimensional (turbulent) component $\mathbf{u}$. The magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ is derived from the magnetic vector potential $\mathbf{A}$, the current

density is $\mathbf{J} = \nabla \times \mathbf{B}/\mu_0$, where μ_0 is the vacuum permeability. Gravity in the vertical direction is given by $\mathbf{g} = -z\Omega_0^2$, and $p = (\gamma - 1)\rho e$ is the equation of state for a perfect gas, which is used with $\gamma = 5/3$. The microscopic viscosity ν enters via the rate of shear tensor $\mathbf{S}$. In practice we use instead ‘numerical’ and shock viscosities, but their functional forms are similar. The same applies to the magnetic diffusion term $\eta\mathbf{J}$. For further details see Paper I. The difference lies in the term $\mathbf{F}$, which now consists of three components:

$$\mathbf{F} = \mathbf{f}_\Omega + k_1\mathbf{f}_1 + k_2\mathbf{f}_2, \quad (5)$$

where $\mathbf{f}_\Omega = (2u_y, -\frac{1}{2}u_x, 0)\Omega_0$ (as in Paper I), and the new terms of order $1/R$ are

$$\mathbf{f}_1 = R^{-1} \left(u_y^2 - B_y^2/\mu_0\rho, -u_xu_y + B_xB_y/\mu_0\rho, 0 \right), \quad (6)$$

$$\mathbf{f}_2 = R^{-1}u_y^{(0)} \left(u_y - \frac{1}{3}u_y^{(0)}, -u_x, 0 \right), \quad (7)$$

which are turned off or on depending on whether k_1 and k_2 equal 0 or 1. These terms result basically from partially integrating terms of the form $(1/r)(\partial/\partial r)(rB_xB_y)$, etc., which come from the nonlinear terms such as $\mathbf{J} \times \mathbf{B}$ and $(\mathbf{u} \cdot \nabla)\mathbf{u}$. The importance of those terms has been stressed by Christodoulou, Contopoulos, & Kazanas (1995). Note that we have singled out the $u_y^{(0)}$ -terms. The contribution $\frac{1}{3}u_y^{(0)2}/R$ includes the second order contribution in the expansion of gravity in the radial direction. We still retain the shearing box approximation, i.e. $\mathbf{A}$, $\mathbf{u}$, e , and $\ln\rho$ are periodic in x , but with respect to a time dependent shift $\delta y = -\frac{3}{2}\Omega_0 t L_x$, where L_x is the extent of the box in the x -direction. The presence of $u_y^{(0)}$ in Eq. (7) breaks the pseudo-periodicity with respect to x (provided $k_2 = 1$), but in practice this contribution is weak and remains unnoticeable in the solution. Following Paper I, we include in some cases a cooling term of the form $Q = -\sigma_0(e - e_0)$, which removes the heat generated by viscous and resistive dissipation.

We solve equations (1)-(4) in a box $|x| < \frac{1}{2}L_x$, $|y| < \frac{1}{2}L_y$, and $0 \leq z \leq L_z$, imposing symmetry conditions on $z = 0$. On $z = L_z$ we impose stress-free, insulating boundary

conditions and assume that the magnetic field is vertical. For further details regarding initial and boundary conditions, nondimensionalization, as well as details of the code, see Paper I. No magnetic field is applied, so the field is entirely due to dynamo action. Initially, there is a seed magnetic field that is needed to get the turbulence going, but this field has zero flux. We should point out that a large-scale toroidal magnetic field develops that changes direction in a cyclic manner with a typical period $T_{\text{cyc}} \approx 30T_{\text{rot}}$, where $T_{\text{rot}} = 2\pi/\Omega_0$. We cannot exclude that this is a finite size effect which might go away in a truly global model, but it provides a convenient means of estimating the dependence of various transport properties on the mean magnetic field from their mutual correlation.

We performed simulations at two resolutions, comparing results for $31 \times 63 \times 32$ and $63 \times 127 \times 64$ meshpoints, using $R = 10$, $L_x = 1$, $L_y = 2\pi$, and $L_z = 2$. The initial scale height H_0 was chosen to be unity, which determines the initial temperature (or internal energy), $e_0 = \frac{1}{2}H_0^2\Omega_0^2/(\gamma - 1)$, and the density of the hydrostatic equilibrium $\rho = \rho_0 \exp(-z^2/H_0^2)$, where $\rho_0 = 1$.

3. Accretion Rate and Viscosity

We consider the instantaneous accretion rate $\dot{M}$, which is related to the radial mass flow $\langle \rho u_x \rangle$ via

$$\dot{M} = -(2\pi R)(2L_z)\langle \rho u_x \rangle, \quad (8)$$

and normalize it by $\dot{M}_c = 2\pi R\Sigma c_s$, where $\Sigma = 2L_z\langle \rho \rangle$ is the vertically integrated average density (which remains constant), and $c_s^2 = \frac{1}{2}H^2\Omega_0^2 = (\gamma - 1)\langle e \rangle$ is the squared isothermal sound speed. Here, angular brackets denote volume averages. Note that H may increase with time (depending on the cooling), and therefore we refer to the initial and final values, H_0 and H_{max} , respectively. The ratio $\dot{M}/\dot{M}_c$ corresponds to the Mach number of the accretion flow.

EDITOR: PLACE FIGURE 1 HERE.

We find that $\dot{M}$ undergoes epicyclic oscillations with the orbital period, $T_{\text{rot}} = 2\pi/\Omega_0$, where $\Omega_0 = R^{-3/2} = 0.032$ (Fig. 1a). Such oscillations occur also for $k_1 = k_2 = 0$, but there the range of $\dot{M}/\dot{M}_c$ never exceeds ± 0.02 . Most importantly, however, the running mean of $\dot{M}$ taken over one orbit is now positive during periods of high magnetic activity (Fig. 1b and c). Indeed, $\dot{M}$ and $\langle B_y \rangle^2$ are clearly correlated (Fig. 2). The results for a number of similar cases are summarized in Table 1.

In accretion disk theory the accretion rate is related to the turbulent viscosity ν_t via the relation

$$\nu_t \Sigma = \dot{M}/(3\pi) \quad (9)$$

(Frank, King, & Raine 1992). Expressing ν_t in terms of c_s and H , i.e. $\nu_t = \alpha_{\text{SS}} c_s H$, we have

$$\alpha_{\text{accr}} = \dot{M}/(3\pi \Sigma c_s H) = (2/3)(H/R)(\dot{M}/\dot{M}_c), \quad (10)$$

where the subscript “accr” refers to how the value of α_{SS} is obtained. Since α_{accr} is correlated with the magnetic activity level (Fig. 2), we write

$$\alpha_{\text{accr}} = \alpha_{\text{accr}}^{(0)} + \alpha_{\text{accr}}^{(B)} \langle B_y \rangle^2 / B_{\text{eq}}^2, \quad (11)$$

where $B_{\text{eq}} = (\mu_0 \rho c_s^2)^{1/2}$ is the equipartition value based on the thermal energy. The resulting values of $\alpha_{\text{accr}}^{(0)}$ and $\alpha_{\text{accr}}^{(B)}$ are given in Table 1 for various runs.

EDITOR: PLACE FIGURE 2 HERE.

EDITOR: PLACE TABLE 1 HERE.

The heating rate. In all cases the disk heats up somewhat initially due to viscous and Joule heating. In particular when $\sigma_0 = 0$ (Run A), the disk heats continuously without limit. The rate of heating depends on the turbulent viscosity,

$$\dot{E} = \nu_t \Sigma \left(r \frac{\partial \Omega}{\partial r} \right)^2, \quad (12)$$

where $\dot{E}$ is the average rate of heating, $d\langle \rho e \rangle / dt$, integrated vertically over the disk (i.e. multiplied by $2L_z$). Since $r\partial\Omega/\partial r = -\frac{3}{2}\Omega_0$, we obtain

$$\alpha_{\text{heat}} = \dot{E} / \left[\left(\frac{3}{2}\Omega_0 \right)^2 \Sigma c_s H \right]. \quad (13)$$

This equation is only valid for early times (less than 10–20 orbits), because H increases with time. The evolution of α_{heat} and $\langle B_y \rangle^2 / B_{\text{eq}}^2$ is shown in Fig. 3a. Note that there is a phase shift; $\langle B_y \rangle^2 / B_{\text{eq}}^2$ leads α_{heat} by $\Delta t \approx 1.5 T_{\text{rot}} \approx 0.05 T_{\text{cyc}}$. The correlation between $\alpha_{\text{heat}}(t + \Delta t)$ and $\langle B_y(t) \rangle^2 / B_{\text{eq}}^2$ is shown in Fig. 3b, and the coefficients $\alpha_{\text{heat}}^{(0)}$ and $\alpha_{\text{heat}}^{(B)}$, defined analogous to (11), are given in Table 1.

EDITOR: PLACE FIGURE 3 HERE.

The viscous stress. Finally, we estimate the turbulent viscosity parameter directly from the (x, y) -components of the Maxwell and Reynolds stress tensor,

$$\alpha_{\text{stress}} = \langle \rho u_x u_y - B_x B_y / \mu_0 \rangle / \left(\frac{3}{2}\Omega_0 \langle \rho \rangle c_s H \right). \quad (14)$$

In Fig. 4 we compare the results for runs at different resolution. Note that the run with higher resolution gives 1.4–1.6 times larger values of α_{SS} than in the case of low resolution (compare Runs B and C). Likewise, a larger toroidal extent of the box (e.g. $L_y = 4\pi$ in Run E) also gives rise to larger values of α_{SS} .

EDITOR: PLACE FIGURE 3 HERE.

Finally we wish to add two comments regarding the generated magnetic field. First, we confirm that the kurtosis of the total and toroidal magnetic fields, $\langle \mathbf{B}^4 \rangle / \langle \mathbf{B}^2 \rangle^2$ and $\langle B_y^4 \rangle / \langle B_y^2 \rangle^2$, respectively, is typically closer to 3 than to 6, indicating that the field is not strongly intermittent (Paper I). Second, the mean toroidal field lags the mean radial field by $\approx 3\pi/4$ (Fig. 1c). This corresponds to the $(\langle B_x \rangle, \langle B_y \rangle, 0)$ -vector rotating with the shear.

4. Conclusions

We have shown that the shearing box approximation can be extended to allow for nonvanishing net accretion flows. The radial mass flow is dominated by epicyclic oscillations, but its running mean has a positive net value. The turbulent viscosity parameter α_{SS} inferred from the average accretion flow is consistent with the value obtained from other methods. The value of α_{SS} is still somewhat dependent on the resolution and the toroidal extent, so it would be important to carry out such simulations on larger meshes and for larger boxes. The value of α_{SS} is approximately equal to $\langle \mathbf{B}^2 \rangle / \langle 2\mu_0 p \rangle$ times $\langle B_x B_y \rangle / \langle \mathbf{B}^2 \rangle$. The results in Table 1 show that higher resolution gives rise to an increase in $\langle B_x B_y \rangle / \langle \mathbf{B}^2 \rangle$ by 25%, while the increase in $\langle \mathbf{B}^2 \rangle / \langle 2\mu_0 p \rangle$ is only 10%. It is also noteworthy that the inclusion of mass accretion does not significantly affect the values of α_{stress} . The contribution from the Maxwell stress is 3 times larger than from the Reynolds stress, which is also approximately the ratio of magnetic to turbulent kinetic energy.

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UNI•C in Copenhagen, Denmark.

Table 1: Comparison of various models. A comparison of the values reproduced in bold is given in the text. The values for the shorter runs (smaller values of $t_{\max}$) are uncertain because of insufficient statistics.

Run	O	A	B	C	D	E
Mesh	$31 \times 63 \times 32$	$31 \times 63 \times 32$	$31 \times 63 \times 32$	$63 \times 127 \times 64$	$31 \times 63 \times 32$	$31 \times 127 \times$
σ_0	$10\Omega_0$	0	Ω_0	Ω_0	Ω_0	Ω_0
k_1	0	1	1	1	1	1
k_2	0	0	0	0	1	1
L_y	2π	2π	2π	2π	2π	4π
$t_{\max}/T_{\text{rot}}$	164	52.9	26.4	11.6	8.2	19.2
$H_{\max}$	1.1	3.0	1.8	2.0	1.6	2.1
$\langle \alpha_{\text{accr}} \rangle$	–	0.004	0.004	0.008	0.002	0.005
$(\alpha_{\text{accr}})_{\max}$	–	0.025	0.010	0.018	0.020	0.018
$\alpha_{\text{accr}}^{(0)}$	–	-0.000	0.002	0.006	0.000	0.001
$\alpha_{\text{accr}}^{(B)}$	–	1.36	0.47	1.8	0.42	0.79
$\langle \alpha_{\text{heat}} \rangle$	–	0.005	–	–	–	–
$(\alpha_{\text{heat}})_{\max}$	–	0.023	–	–	–	–
$\alpha_{\text{heat}}^{(0)}$	–	0.000	–	–	–	–
$\alpha_{\text{heat}}^{(B)}$	–	1.51	–	–	–	–
$\langle \alpha_{\text{stress}} \rangle$	0.005	0.005	0.005	0.007	0.005	0.006
$(\alpha_{\text{stress}})_{\max}$	0.018	0.013	0.012	0.012	0.008	0.015
$\alpha_{\text{stress}}^{(0)}$	0.002	0.003	0.004	0.005	0.005	0.004
$\alpha_{\text{stress}}^{(B)}$	0.029	0.61	0.13	1.51	0.42	0.52
$\langle \alpha_{\text{Maxw}} \rangle / \langle \alpha_{\text{Reyn}} \rangle$	3.4	3.1	3.1	3.0	3.0	2.8
$\langle \mathbf{B}^2 \rangle / \langle \mu_0 \rho \mathbf{u}^2 \rangle$	6.2	3.4	3.9	3.1	4.4	3.7
$\langle \mathbf{B}^2 \rangle / \langle 2\mu_0 p \rangle$	0.066	0.049	0.037	0.054	0.052	0.058
$\langle B_x B_y \rangle / \langle \mathbf{B}^2 \rangle$	0.065	0.087	0.084	0.109	0.076	0.089
$\langle \mathbf{B}^4 \rangle / \langle \mathbf{B}^2 \rangle^2$	3.0	5.8	3.7	4.7	2.8	3.3
$\langle B_y^4 \rangle / \langle B_y^2 \rangle^2$	2.7	4.2	3.7	3.2	2.4	3.2

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Fig. 1.— (a) Evolution of instantaneous mass flux, (b) running mean of the mass flux, and (c) the mean magnetic field. (The dotted line refers to the radial field, multiplied by a factor of 30.) Run A.

Fig. 2.— Correlation of the viscosity parameter (as derived from the accretion rate) and the mean magnetic field. Run A.

Fig. 3.— (a) The evolution of α_{heat} (solid line), compared with $0.64\langle B_y \rangle^2/B_0^2$ (dotted line). Note the phase shift between the magnetic field and α_{heat} . (b) Correlation of α_{heat} and $\langle B_y \rangle^2/B_0^2$.

Fig. 4.— The evolution of α_{stress} for runs at different resolution.