COHERENT AND INCOHERENT INTERACTIONS OF HIGH-ENERGY HADRONS WITH HEAVY NUCLEI

G. Cocconi

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The description that will be given here of the coherent and incoherent interactions of hadrons with heavy nuclei at high energies is eminently qualitative and intends only to focus attention on the role played, in obtaining the final results, by the general properties of the nuclei and by the properties of the hadron-nucleon interactions.

This kind of approach finds justification in its simplicity and in the fact that at present a good fraction of the experimental evidence for coherent interactions in nuclei, though unmistakably clear, is still statistically too poor to warrant a detailed and rigorous analysis.

A somewhat formal review of coherent phenomena produced in nuclei by high-energy particles has been presented by A. and M. Goldhaber\(^1\), and several of the conclusions reached in the present paper are mentioned there. Coherent and incoherent processes of elastic scattering in light and heavy nuclei have been treated in detail by Glauber\(^2\), and we shall freely refer to this paper to justify some of our heuristic assumptions.

1. GENERAL CONSIDERATIONS

When speaking of hadron interactions with nuclei, the variety of processes is so large that often confusion arises because the same word is used to identify phenomena that are similar only superficially; to avoid this we should pay some attention to terminology. Of the various reactions that could be labelled coherent, because they involve several nucleons of the
same nucleus acting as a whole, we shall call coherent only those in which the nucleus, after interacting with the high-energy hadron, recoils remaining in its fundamental state (complete coherence). In this kind of process the final products of the interaction are, beside the recoiling nucleus, one or more high-energy secondaries, moving along directions close to the direction of the incoming hadron.

We shall instead call incoherent interactions the completely incoherent events in which the hadron has interacted with only one of the nucleons of the nucleus.

The disregard of other coherent phenomena, where an intermediate number of nucleons can be involved, is justified by the fact that whenever complete coherence occurs, the forward peak of the differential cross-section is anomalously high, while at larger angles there is always a characteristic region where complete incoherence dominates.

In the following we want to discuss completely incoherent and completely coherent interactions at the same time. The reason for this has its grounds in the nature of the experiments performed to detect these phenomena. In fact, in both cases the energy taken by the recoil, being a very small fraction of the hadron energy (some 10 MeV at maximum) is difficult to measure when heavy nuclei are involved both because the recoil itself is practically undetectable and because energy resolutions better than 20-50 MeV are hard to obtain when handling hadrons of 10 GeV or more. This means that the two kinds of events are experimentally indistinguishable, except for the fact that the angular regions where they are predominant are characteristically different. Their simultaneous study is then unavoidable.

The main simplifications that can be introduced in evaluating these phenomena derive from the fact that when high-energy hadrons, i.e., hadrons with more than, say, 5-10 GeV, interact with a free nucleon (we shall call these events elementary interactions), there are classes of reactions in which the target nucleon in the laboratory system simply recoils, otherwise undisturbed, while all secondaries are, so to speak, the result of modifications occurring in the high-energy hadron only. It is the existence of these classes of elementary interactions that makes possible coherent reactions of high-energy hadrons with nuclei.
One can picture the phenomenon of coherence as the occurrence of an elementary interaction between the incoming hadron and one of the nucleons bound in the nucleus in kinematic conditions such that the forces that keep together the nucleus transmit the recoil momentum of the nucleon to the rest of the nucleus, which consequently recoils coherently. Since many nucleons of both charge states can equally give rise to this kind of process in the same nucleus, it follows from a general principle of quantum mechanics that the final amplitude of the transition is the sum of the elementary amplitudes. The nuclear cross-section, proportional to the square of the final amplitude, is then considerably enhanced in the forward direction, where all elementary amplitudes are in phase, in comparison to the cross-section of the incoherent process, where the single nucleons act independently and out of phase. It also becomes clear that, in order to have complete coherence, the nucleus should remain in its fundamental state, because otherwise either the detection of the recoiling nucleon or, in case of excitation, that of the radiation emitted in the inverse transition could, in principle, be used to characterize the event, and all the nucleons could no longer be treated as equivalent.

2. ELEMENTARY INTERACTIONS

Following this line of approach, it is convenient to start by discussing the elementary interactions. We shall divide them into two classes: that of elastic interactions and that of production interactions.

In elastic interactions one has:

\[ N_i + n \rightarrow N_i + m; \text{ differential cross-section } \frac{d\sigma}{d\omega} \text{ }_{el} \]  \( (1) \)

where \( N_i \) is the hadron and \( m \) the target nucleon. Later on, we shall use the same symbols for the respective masses.
In production interactions:

\[ M_1 + m \rightarrow M_2 + m; \text{ differential cross-section } \left( \frac{d\sigma}{d\omega} \right)_{\text{prod}} \]  

(2)

Depending on whether what is produced is a single particle or several particles, \( M_2 \) stands for the single particle and its mass, or for the centre of motion of the particles produced and its mass. In both these reactions the nucleon remains unchanged, except for the gain of recoil momentum.

At high energies a common empirical property of both kinds of reaction is that the dependence of the differential cross-sections on the incoming hadron momentum, \( p \), and on the laboratory angle of emission of \( M_1 \) or \( M_2 \), \( \theta \), has the form

\[ \frac{d\sigma}{d\omega} = B \ p^2 \ e^{-(p\theta/a)^2} \]  

(3)

Since the parameter \( a \) has values between \( \frac{1}{4} \) and \( \frac{1}{2} \) GeV/c, practically all secondaries are contained within a forward cone of semi-aperture

\[ \alpha \simeq \frac{2a}{p} \simeq \frac{1 \text{ GeV/c}}{p} \]  

(4)

The value of the parameter \( n \) characterizes the energy dependence of the total cross-section of the reaction; the integration of Eq. (3) gives in fact:

\[ \sigma \propto p^{-n} \]  

(5)

Energy independent cross-sections (\( n \approx 0 \)) are observed for elastic interactions and for those of the production interactions in which, as in the elastic ones, the isotopic spin and the intrinsic parity of \( M_1 \) and \( M_2 \) are equal; otherwise, \( n \) assumes values around 1 and 2 [see Morrison\(^3\)].
When the elementary interactions described above take place between the hadron and a nucleon bound in a nucleus, four categories of processes can occur: the elastic coherent, the elastic incoherent, production coherent, and the production incoherent interactions. Of these, only the first, the elastic coherent, is, as far as the nucleus is concerned, really elastic, as this is the only process where the final products of the reaction are equal to the initial ones. The other three processes are inelastic. This is also illustrated in the following diagram:

\[
N_1 + A_1 \rightarrow \begin{cases} 
N_1 + A_2 & \text{elastic incoherent} \\
N_2 + A_1 & \text{production coherent} \\
N_2 + A_2 & \text{production incoherent}
\end{cases}
\]

where \(A_1\) stands for the nucleus in its fundamental state and \(A_2\) for the excited nucleus.

3. THE CYLINDRICAL NUCLEUS

Coming now to the interactions with a system composed of many nucleons, let us consider first a sphere of radius \(r\), large in comparison with the usual nuclear radii, in which are contained \(A\) free nucleons (which eventually we shall condense in a nucleus of mass number \(A\)). If \(r\) is large enough, the interaction of the hadrons with this gas of nucleons can be resolved in its elementary interactions, because the single nucleons are free and so far apart that the effects of multiple collisions and of the shadowing of one nucleon by another are negligibly small.
When the radius of the sphere is reduced to nuclear dimensions

\[ R = r_0 A^{1/3} \quad \text{with} \quad r_0 \approx 1.2 \times 10^{-13} \text{ cm}, \]

the multiple collisions and shadow effects become very important because the high-energy hadron-nucleon total interaction cross-sections \( \sigma \) have, typically, values of 20–40 mb, and consequently the hadron interaction mean free path in nucleon matter

\[ \lambda_{\text{int}} = \frac{4}{3} \pi r_0^3 \frac{1}{\sigma} \approx 2 \times 10^{-13} \text{ cm} \]

(7)
is considerably smaller than the nuclear radius, for heavy nuclei.

Before coming to the case of real nuclei, it is instructive to consider the case in which a parallel monoenergetic beam of high-energy hadrons collides with the \( A \) nucleons condensed to nuclear densities within a hypothetical nucleus having the shape of a cylinder of radius \( r \) and length \( \ell \), oriented with its axis parallel to the direction of the incoming hadrons (see Fig. 1).

\[ \begin{array}{c}
\hline \\
\rightarrow \\
\end{array} \quad P \quad \begin{array}{c}
\hline \\
\rightarrow \\
\end{array} \quad \begin{array}{c}
\hline \\
\rightarrow \\
\end{array} \quad \begin{array}{c}
\hline \\
\rightarrow \\
\end{array}
\]

Fig. 1

For \( A \) sufficiently large the dimensions of the cylinder can satisfy the following conditions:

\[ r \gg r_0, \quad \ell \gg \lambda_{\text{int}}. \]

(8)

The interaction of the hadrons with this cylindrical nucleus is peculiar because whenever the hadron hits the cylinder, it is completely absorbed and practically no product of the elementary interactions (except
the leptonic ones) can emerge unaffected by other collisions. In fact, the two conditions (8) assure that not only all primary hadrons with impact parameter smaller than \( r \), but also practically all hadronic secondaries, interact within the cylinder. Remembering relations (4) and (7), from the side walls of the cylinder can emerge only a small fraction of the secondaries produced within a depth

\[
\Delta r \ll 5 \alpha \lambda_{\text{int}} \approx \frac{10^{-12}}{p} \text{ cm},
\]

and \( \Delta r < r_0 \) when \( p \gg 10 \text{ GeV}/c \). (Here, as in the following numerical examples, \( p \) is measured in GeV/c.) Most of the energy of the hadron is thus degraded within the cylindrical nucleus which acts as a calorimeter.

It then follows that the ideal cylindrical nucleus has zero cross-section for all inelastic events classified in \( \omega_Q \) (6). The only hadron-nucleus interaction allowed in this case is the coherent elastic one, which, optically speaking, is generated by the disturbance introduced in the hadron plane wave by the cutting out of the circular disc\(^*)\). Following the optical model, the elastic cross-section is given by the equation \( (k = p/\hbar) \):

\[
\left( \frac{d\sigma}{d\omega} \right)_{\text{el. coh.}} = \left| \frac{1}{2ik} \sum_{0}^{\infty} (2\xi + 1)^2 P_{\xi}(\cos \vartheta)(1 - \eta_{\xi}) \right|^2
\]

\[
(9)
\]

\(^*)\) Here, as well as in the rest of this paper, we shall neglect the Coulomb interaction between charged hadrons and the nucleus because it influences the differential cross-sections noticeably, only at angles much smaller than those characteristic of coherent and incoherent phenomena. The expressions we are going to derive for the cross-sections are thus not valid for the angles where the effect of the Coulomb interaction becomes important, i.e., whenever

\[
\sigma^4 \frac{d\sigma}{d\omega} \approx \frac{4Z^2e^4}{p^2e^2}
\]
with the conditions
\[ \begin{align*}
\eta_\ell &= 0 \quad \text{for } \ell < kr \\
\eta_\ell &= 1 \quad \text{for } \ell > kr.
\end{align*} \]

At high energies $kr >> 1$ and the integration of Eq. (9) gives the familiar expression:
\[ \left( \frac{d\sigma}{d\omega} \right)_{\text{el.coh.}} = \left( \frac{k\sigma_{\text{tot}}}{4\pi} \right)^2 \left( \frac{2J_1(kr\sin \vartheta)}{kr\sin \vartheta} \right)^2 \frac{1}{\vartheta} \frac{d\vartheta}{\vartheta} \left( \frac{k\sigma_{\text{tot}}}{4\pi} \right)^2 e^{-\left(kr\vartheta/2\right)^2} \quad (10) \]

where $\sigma_{\text{tot}}$ is the total cross-section hadron-cylinder, and is equal to twice the geometrical cross-section of the cylinder;
\[ \sigma_{\text{tot}} = 2\pi r^2. \]

Twice, because it is the sum of two parts; the absorption cross-section and the elastic cross-section, both equal to $\pi r^2$. The small angle approximation is a Gaussian with r.m.s. angle
\[ \vartheta = \frac{1}{kr}. \quad (11) \]

This example makes it clear that in order to observe inelastic phenomena (elastic incoherent, production coherent, production incoherent) in heavy nuclei, the nucleus must have some transparency for the incoming hadron and for the outgoing secondaries. We shall see that this condition can be fulfilled in heavy nuclei only at the rim of the nucleus, a fact that makes the cross-sections of these phenomena very sensitive to the properties of the nuclear surface.

4. ELASTIC COHERENT AND INCOHERENT EVENTS IN REAL NUCLEI

When A nucleons are condensed into something that resembles more closely a real nucleus, i.e., a sphere of radius
\[ R = r_0 A^{1/3} \quad \text{with} \quad r_0 = 1.2 \times 10^{-13} \text{ cm}, \quad (12) \]
the elastic coherent cross-section is still given with good approximation 
by Eq. (10) in which

\[ \sigma_{\text{tot}} = \sigma_{\text{abs}} + \sigma_{\text{el,coh.}} = 2\pi R^2 = 2\pi r_0^2 A^{2/3} \]  

This is because for heavy nuclei \( A \geq 50 \), the thickness \( \ell \) of nuclear matter, 
along the path of the incoming hadron, that meets the nucleus with impact 
parameter \( b \), is

\[ \ell = 2(R^2 - b^2)^{1/2} \]  

(14)

Remembering Eq. (7), \( \ell \) is larger than \( \lambda_{\text{int}} \) whenever \( b < R(1 - 1/2A^{2/3}) \), i.e. up to impact parameters that differ from \( R \) by less than a few per cent (see Fig. 2).

\[ \begin{array}{c}
\hat{P} \\
\wedge \\
\vee \\
\end{array} \begin{array}{c}
b \\
\ell \\
2R \\
\end{array} \]

Fig. 2

The forward elastic coherent cross-section is thus

\[ \left( \frac{d\sigma}{d\omega} \right)_{\text{el,coh.}} (\vartheta = 0) \approx \left( \frac{k\sigma_{\text{tot}}}{4\pi} \right)^2 = 0.13 \, p^2 A^{4/3} \times 10^{-24} \, \text{cm}^2/\text{sr} \]  

(15)

while the r.m.s. value of its angular distribution is [ see Eq. (11)]

\[ \vartheta_{\text{el,coh.}} = \frac{1}{kr} \approx \frac{170}{pA^{1/3}} \, \text{mrad} \]  

(16)

The momentum of the recoiling nucleus being

\[ P_{\text{rec}} \approx p\vartheta \]  

(17)
its recoil energy when \( \theta \ll \theta_{\text{el.,coh.}} \) is

\[
T_{\text{rec}} = \frac{1}{2} \frac{p_{\text{rec}}^2}{A} \approx \frac{15 \text{ MeV}}{A^{1/3}},
\]

(18)

a fraction of MeV for heavy nuclei.

In the spherical nucleus the elastic incoherent events are not suppressed because whenever the elementary interaction takes place near the rim of the nucleus, i.e., whenever \( b \approx R \), then \( \ell \ll \lambda_{\text{int}} \) and the scattered hadron has a good chance of escaping from the nucleus without further interactions. The complete incoherence, however, can be obtained only when the recoil nucleon breaks the nuclear binding forces that, for a surface nucleon, are typically of the order of, say, 10 MeV. Since the recoil momentum in the elementary event is still given, to a good approximation, by expression (17), the recoil energy is larger than 10 MeV whenever

\[
\frac{p_{\text{rec}}^2}{2m} > 10 \text{ MeV}, \quad \text{i.e.,} \quad \theta > \frac{140}{p} \text{ mrad.}
\]

(19)

For heavy nuclei this angle is substantially greater than the angle [Eq. (16)] characteristic of the elastic coherent events, and the two phenomena can be observed separately in the laboratory by detecting the elastically scattered hadron only, even when the energy resolution is not good enough to distinguish between an energy loss of a fraction of MeV (elastic coherent events) and that of some 10 MeV (elastic incoherent events).

At this point we can also justify, for the elastic events, two statements made in the introduction. One is that at zero angle the coherent phenomenon is predominant; and in fact no elastic incoherent scattering can take place at zero angle as the recoil momentum would be too small to modify the nuclear structure. The second statement is that, where incoherent events begin to occur, the main contribution comes from interactions of the hadron with a single nucleon in the nucleus. The reason is that, at high energies, the partial cross-section of the elastic scattering in
hadron-nucleon collisions is only 10-15% of the total cross-section, and consequently the probability of two or more elastic events taking place inside the nucleus, and nothing else, cannot be substantial in the angular region $140 < \theta < 500$ mrad GeV/c.

The problem of finding the expression for the elastic incoherent cross-section at angles that satisfy relation (19) is now reduced to that of finding the number of nucleons at the rim of the nucleus that can scatter the hadrons as if they were free. We call this number $N_{\text{el. inc.}}$. Its evaluation is not simple as it depends critically on the density of nuclear matter at the surface of the nucleus and, for example, the crude spherical model used in our discussion above is inadequate. Recently, $N_{\text{el. inc.}}$ has been calculated for protons by Glauber and Matthiae\textsuperscript{4}) using for the nucleus the Woods-Saxon density. However, the first determination of $N_{\text{el. inc.}}$ was experimental and was made at CERN\textsuperscript{5}) with protons of 19.3 GeV and various nuclei, ranging from Be to Pb. From these measurements it was concluded that a reasonable numerical expression for it is:

$$N_{\text{el. inc.}} = 1.6 A^{\frac{1}{3}} \quad \text{(for 19.3 GeV protons).} \quad (20)$$

The curve calculated by Glauber and Matthiae for the same case is in numerical agreement with the experimental data, but suggests a different relation:

$$N_{\text{el. inc.}} = 0.67 A^{0.57} \quad \text{(for 19.3 GeV protons).}$$

In the following discussion, however, we shall use the empirical expression (20).

It is reasonable to think that for protons of different energies expression (20) will continue to be valid, as the proton-nucleon total cross-section is energy independent, above 5 GeV. As for the other hadrons, $N_{\text{el. inc.}}$ will certainly change in correlation with their total cross-sections;
it can be assumed, however, that what is valid for protons will be approximately valid also for the other hadrons. In any case, the nuclear elastic incoherent cross-section is given by the following expression:

\[
\left( \frac{d\sigma}{d\omega} \right)_{\text{el.inc.}} = N(\text{el.inc.}) \left( \frac{d\sigma}{d\omega} \right)_{\text{el.}} ; \ \theta > \frac{140}{p} \text{ mrad},
\]

(21)

where \( \left( \frac{d\sigma}{d\omega} \right)_{\text{el.}} \) is the elementary cross-section for elastic hadron-nucleon scattering, Eq. (1). The shape of the differential cross-section of nuclear elastic incoherent events, Eq. (21), is thus equal to that of the elementary interaction, Eq. (3). In the case of protons, the experimental values give the numerical relation:

\[
\left( \frac{d\sigma}{d\omega} \right)_{\text{el.inc.}} = 0.040 \ A^{1/2} p^{2} e^{-10 p^{2} \theta^{2}} 10^{-24} \text{ cm}^{2}/\text{sr}.
\]

(22)

The r.m.s. of this angular distribution, which is typical for all hadrons, is

\[
\theta_{\text{el.inc.}} = \frac{160}{p} \text{ mrad},
\]

(23)

a factor of \( A^{1/2} \) larger than the corresponding quantity for the elastic coherent events given by Eq. (16). In an angular distribution plot, it is thus easy to distinguish at first sight the elastic coherent from the elastic incoherent events, as the first are concentrated at small angles and have a very steep angular dependence, while the others are distributed at larger angles and decrease much more gently as \( \theta \) increases.

From Eq. (21) it also follows that, disregarding the dip at the smallest angles, the total cross-section of the elastic incoherent events, \( \sigma_{\text{el.inc.}} \), is equal to \( N(\text{el.inc.}) \) times the cross-section of the elementary event:

\[
\sigma_{\text{el.inc.}} \approx N(\text{el.inc.}) \sigma_{\text{hadron-nucleon elastic}}
\]

(24)

Another interesting number is the ratio \( R_{0} \), between the cross-section at \( \theta = 0 \) of coherent events and the cross-section also extrapolated to
\[ \sigma = 0 \] (though there the phenomenon really does not occur) of the incoherent events. For protons, which again are typical, one obtains from Eqs. (15) and (22), at \( \sigma = 0 \),

\[ R_0(\text{el.}) = \frac{\left( \frac{d\sigma}{d\omega} \right)_{\text{el. coh.}}}{\left( \frac{d\sigma}{d\omega} \right)_{\text{el. inc.}}} \approx 3A. \]  \hspace{1cm} (25)

For heavy nuclei the elastic coherent peak dominates by a factor of more than 100 the shoulders due to the elastic incoherent events, and this is well borne out by the results of the CERN experiments\(^3\).

These high values of \( R_0 \) also imply that for the heaviest nuclei the angles at which the same ratio becomes substantially smaller than 1 and incoherent events predominate, is somewhat larger (for protons on Pb, more than three times) than that given by condition (19).

The detailed study of \( N(\text{el. inc.}) \) for particles whose elementary cross-section is well known, e.g. protons and pions, will give, as pointed out by Glauber\(^2\), direct information about the surface density of nuclei.

5. PRODUCTION PROCESSES

The substantial cross-section of elastic incoherent interactions assures that in real nuclei it is possible to have not-negligible cross-sections also for the other two inelastic processes listed in Eq. (5), the production coherent and the production incoherent processes. Actually these processes are the most interesting ones as far as high-energy hadron physics is concerned, because their cross-sections are sensitive to the properties of the interaction between the secondaries and the nucleons.

First we shall discuss incoherent production events. In the elementary production reaction (2)

\[ N_1 + m \rightarrow N_2 + n, \]

the momentum of the recoil nucleon

\[ p_{\text{rec.}} = \sqrt{p_{\|}^2 + p_{\perp}^2} \]
has parallel and transverse components that, in the high-energy approximation, are given by the expressions

$$p_n = \frac{H_n^2 - H_n^2}{2p} \quad ; \quad p_\perp = p^0,$$

(26)

where $p_n$ as usual, is the incoming hadron momentum (measured in GeV/c in the numerical examples). The transverse momentum recoil being equal to that of the elastic case, Eq. (17), incoherent events occur when condition (19) is satisfied and their differential cross-section is given in analogy with Eq. (21) by the expression:

$$\left(\frac{d\sigma}{d\omega}\right)_{\text{prod., inc.}} = N(\text{prod., inc.}) \left(\frac{d\sigma}{d\omega}\right)_{\text{prod.}} \quad ; \quad \theta > \frac{140}{p} \text{ mrad.}$$

(27)

The total incoherent cross-section, always disregarding a possible dip at the smallest angles, is then:

$$\sigma_{\text{prod., inc.}} \approx N(\text{prod., inc.}) \sigma_{\text{hadron-nucleon prod.}},$$

(28)

similar to Eq. (24).

$N(\text{prod., inc.})$ is the number of peripheral nucleons that act as free in the hadron-nucleus interaction. The value of $N(\text{prod., inc.})$ should depend rather critically on the nature of the primary and of the secondaries involved in the elementary interaction, but not on the incoming hadron momentum, $p$.

The detailed study of $N(\text{prod., inc.})$ promises to be very interesting, especially when short-lived particles are produced, because the measure of $N(\text{prod., inc.})$ can be related to the cross-section of these particles with free nucleons, a quantity that in certain cases cannot be measured with any other method. Note that the only condition for the production of these incoherent events is that the nucleon is left intact and no limitations come from spin and parity; consequently the class of reactions that can be studied is rather wide.
It is also worth pointing out that at high energies all known unstable particles will cross the nucleus with small probability of decaying within, because the largest width of the known baryon resonances are of the order of 100–200 MeV, and their decay mean free path

\[ \lambda_{\text{decay}} = \frac{\hbar e}{\tau} \frac{p}{\mu_c} \approx 10^{-13} \text{ p cm} \]

is considerably longer than the thickness of the rim of the nucleus.

So far, the only measurements of production incoherent interaction in nuclei that can be analysed in terms of Eq. (27) are those of Allard et al. \(^6\). These authors have observed in the heavy liquid of a bubble chamber (C\(_2\)F\(_5\)Cl) the production by 16 GeV \(\pi^-\) mesons of three-body final states consisting of \(\pi^- + \pi^+ + \pi^-\). The elementary interaction involved is

\[ \pi^- + m \to (\pi^- \pi^+ \pi^-) + m , \]  

(29)

where \(m\) is either a proton or a neutron. What has been previously called \(M_2\) is here the system \(\pi^- \pi^+ \pi^-\), that in most of the cases consisted of a \(\pi^-\) and a \(\rho^0\) meson. Allard et al. \(^6\) found that at small four-momentum transfers the differential cross-section has the steep angular dependence characteristic of coherent production (this will be discussed later), while at \(\theta \gtrsim 20\) mrad the angular dependence resembles closely that of the elementary reaction. The evaluation of \(N(\text{prod.inc.})\) from these results is made uncertain by two facts. One is that different nuclei are involved at the same time, and an "a priori" weight must be given to each of them. The second is that the cross-section of the elementary reaction Eq. (29) has so far only been measured up to 11 GeV, and its value at 16 GeV must be guessed by extrapolation \(^*)\). Assuming that \(N(\text{prod.inc.}) \propto A^{1/2}\), we find for the reaction in question

\[ N(\text{prod.inc.}) \approx 5, \text{ for } \langle A \rangle = 18 , \]  

(30)

a value, as expected, not very different from that valid for protons in elastic incoherent events [Eq. (20) would give 4.2].

\(^*)\) We are grateful to Dr. E. Fiorini for supplying information about the elementary cross-section and unpublished data on the bubble chamber experiment of Allard et al.
Consider now the coherent production. The condition for its occurrence is that the single nucleon affected in the elementary act receives a recoil momentum so small as not to leave the nucleus in an excited state.

Kinematically, this condition means that for heavy nuclei, where the first excited levels are as little as a fraction of MeV above the fundamental level,

\[ T_{\text{rec}} = \frac{1}{2} \frac{p_{\text{rec}}^2}{m} < 1 \text{ MeV}. \]  

Now, in the case of production reactions, the recoil energy does not become negligible when \( \delta \to 0 \) because the change in mass from \( N_1 \) to \( N_2 \) implies a change in longitudinal momentum; as a consequence, even at \( \delta = 0 \) the coherent production is possible only when, using Eqs. (26) and (31),

\[ L_2 \leq \left( L_1^2 + \frac{p_0}{10} \right)^{1/2} \text{ GeV (p\text{e} and N in GeV)}. \]  

Implicit in our definition of coherence is also the condition that the angular momentum of the nucleus does not change, because a change of spin would also imply the excitation of some nuclear levels. This requirement, however, does not imply that \( N_2 \) must have the same spin and parity as \( N_1 \).

In fact, in the forward direction orbital angular momentum of the incoming hadron can be added to the spin of \( N_2 \) without violating the above-mentioned condition, and for each unit of angular momentum absorbed the parity of \( N_2 \) changes. What should not change is the intrinsic parity. For incident hadrons of spin zero this means that coherence can be observed provided

\[ \text{parity of } N_2 = \text{parity of } N_1 \times (-1)^{\Delta J}, \]  

where \( \Delta J \) is the spin of \( N_2 \). When the spin of the incident hadron is \( \frac{1}{2} \), more combinations are allowed.

As already pointed out by Berman and Drell\(^7\), a consequence of these selection rules is that when \( \Delta J \neq 0 \), the particles produced coherently
have their spins predominantly oriented in a direction perpendicular to that of their momentum, and the study of their anisotropic decay can be used for determining their spin and parity.

Phenomena of not complete coherence, where the nuclear spin changes and tidal waves are excited, have been observed especially with low-energy particles. However, in that case the order of coherence and the cross-section are much smaller than in the case we are contemplating.

Once conditions (32) and (33) are satisfied at \( \theta = 0 \), the contributions of the elementary transitions induced in the \( N(prod.,coh.) \) nucleons add coherently* and the nuclear differential cross-section is given by the expression:

\[
\left( \frac{d\sigma}{d\omega} \right)_{prod.,coh.} (\theta = 0) = \left[ N(prod.,coh.) \right]^2 \left( \frac{d\sigma}{d\omega} \right)_{prod.} (\theta = 0)
\]

(34)

Here also it can be expected that \( N(prod.,coh.) \) is practically independent of the hadron momentum, while \( (d\sigma/d\omega)_{prod.} \) follows the rules of Eq. (3). The angular distribution of this coherent production cross-section is different from that of the elastic case because here the contributing nucleons are distributed only along the rim of the nucleus. In the extreme case of a large nucleus, where the rim thickness is small in comparison with the radius, the diffraction figure is equal to that produced by a thin ring and is given, as long as condition (31) is satisfied, by the following equation:

\[
\left( \frac{d\sigma}{d\omega} \right)_{prod.,coh.} = \left( \frac{d\sigma}{d\omega} \right)_{prod.,coh.} (\theta = 0) \left[ J_0(k R \theta) \right]^2
\]

(35)

For a given nucleus, this angular distribution is narrower than that found for the elastic coherent cross-section, Eq. (16). Its r.m.s. angle is

\[
\theta_{prod.,coh.} = \frac{1}{\sqrt{2} k A} = \frac{120}{p A \sqrt{3}} \text{ mrad.}
\]

(36)

* Here we neglect the phase shifts created by the passage of the particles through the nucleus because the phenomenon takes place on the periphery and the amount of nuclear matter crossed is not large.
By integrating Eq. (35) up to $k_R \theta = 2$, a value where coherent and incoherent differential cross-sections are about equal, and with the aid of Eqs. (34) and (3), one obtains for the total coherent production cross-section

$$\sigma_{\text{prod. coh.}} \approx \frac{3}{2} \pi \left[ \frac{N(\text{prod. coh.})}{R} \right]^2 \left[ \frac{4 \pi}{k^2} \left( \frac{d\sigma}{d\omega} \right)_{\text{prod.}}(\theta = 0) \right] = \text{const.} \times p^{-n}. \quad (37)$$

The dependence of $\sigma_{\text{prod. coh.}}$ on the incident hadron momentum is thus equal to that of the corresponding elementary interaction.

Finally, the ratio between coherent and incoherent production, extrapolated to $\theta = 0$, is from Eqs. (28) and (34)

$$R_0(\text{prod.}) = \left( \frac{d\sigma}{d\omega} \right)_{\text{prod. coh.}} / \left( \frac{d\sigma}{d\omega} \right)_{\text{prod. inc.}} = \frac{[N(\text{prod. coh.})]^2}{N(\text{prod. inc.})}, \quad (38)$$

and its numerical value depends on that of the two $N$'s. When conditions (32) and (33) are met, one expects that $N(\text{prod. coh.}) = N(\text{prod. inc.})$ and that consequently

$$R_0(\text{prod.}) = N(\text{prod. inc.}). \quad (39)$$

As we have seen earlier, $N(\text{prod. inc.})$ is a number small in comparison with $A$ (typically around $1.5 A^{1/2}$) and consequently the ratio $R_0$ in the case of production events is much smaller than the corresponding ratio [(Eq. (25))] for the elastic events. For example, in the case of $A = 200$, $R_0(\text{el.}) = 600$ and $R_0(\text{prod.}) \approx 10$. Thus in the case of production events the coherent peak in the forward direction is much less prominent with respect to the incoherent shoulders than in the case of elastic events. The production coherent peak is, however, somewhat steeper than the elastic coherent one.

When conditions (32) and/or (33) are not satisfied, the production coherent reaction does not take place, and at $\theta = 0$ the differential cross-section does not rise to the level predicted by relation (38). In cases where the elementary process is depressed at $\theta = 0$, as in some
charge exchange reactions, in the forward direction one actually expects a depression also with nuclei.

The experimental information thus far available on production coherent events is not abundant, but seems to be in agreement with our conclusions.

In the already quoted experiment of Allard et al.\textsuperscript{6}), the three pions of reaction (29) can assume a configuration that satisfies conditions (32) and (33), and the differential cross-section of their production presents a peak in the forward direction that has the characteristics of the production coherent peak. The value of $R_0$ (prod.) that can be deduced from these experiments is $7.4 \pm 2$, in reasonable agreement with Eq. (39) when one takes into account the great uncertainties connected with the determination of $N$(prod. inc.) that led to the value quoted in Eq. (30). This result also shows that practically all the incoherent production due to reaction (29) takes place through a channel satisfying the condition (33) and with $\Lambda_2 \approx 1.3$ GeV.

The shape of the angular distribution at the smallest angles is a Gaussian with an r.m.s. angle that, interpreted as the diffraction figure of an opaque disk, gives a nuclear radius somewhat larger than that expected for the average nucleus of the bubble chamber liquid. The experimental errors are, however, still too large and the nuclei involved perhaps too light to test the validity of Eq. (36).

Information about the ratio $R_0$ for 15 GeV $\pi^-$ on Pb nuclei is given in a paper by Ratti and Vegni\textsuperscript{9}). The elementary interaction is again the three-pion production, Eq. (29), observed in a Pb sheet placed inside a heavy-liquid bubble chamber. Though only a few hundred events were observed in total, the coherent peak at small angles and the incoherent events at large momentum transfer are clearly recognizable. One obtains $R_0$(prod.) $\approx 10$ ($A = 208$), again in agreement with our expectations.

In conclusion, the description given here of coherent and incoherent events in heavy nuclei seems to be confirmed by the existing experimental material, and we feel justified in making some predictions about the behaviour of coherent and incoherent phenomena in heavy nuclei at energies much higher than those presently available. Coherent and incoherent elastic events will continue to be present with cross-sections practically equal to those presently observed. In production interactions, coherence in nuclei
will be observed for values of the mass, $M_2$, much larger than the present ones; according to Eq. (32), up to $M_2 \approx 6$ GeV when $p = 300$ GeV/c. However, for many of the possible processes the sharp energy dependence of the elementary cross-sections, expressed by Eq. (3), will make the detection hard. The production processes that will persist with practically constant cross-sections will be only those, elastic-like, in which the isotopic spin and the intrinsic parity of $M_2$ is equal to that of $M_1$.

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