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Jørgen Christensen-Dalsgaard

300 9613

Inst. of Physics and Astronomy
University of Aarhus
DK-8000 Aarhus C
Denmark

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Helioseismology, Solar Models and Solar Neutrinos

Jørgen Christensen-Dalsgaard^a

^aTeoretisk Astrofysik Center, and Institut for Fysik og Astronomi,
Aarhus Universitet, DK-8000 Aarhus C, Denmark

Very accurate measurements of oscillation frequencies of the Sun are providing detailed information about the structure of the solar interior. Somewhat surprisingly, the results are in excellent agreement with so-called ‘standard solar models’, provided up-to-date physics is used and settling of helium and heavy elements is taken into account. This suggests that an ‘astrophysical solution’ to the solar neutrino problem may be unlikely.

1. INTRODUCTION

The early attempts to explain the so-called solar neutrino problem, *i.e.*, the discrepancy between the measured flux of neutrinos and the predictions of solar models, generally concentrated on what might be called *astrophysical solutions*: modifications of some aspect of the model calculation aimed at reducing the central temperature of the model and hence the predicted neutrino production rate. Several such models were in fact constructed, although only with rather drastic assumptions. Furthermore, independent verification of these assumptions and of the resulting models was in general not possible. Indeed, although the evidence for nuclear reactions in the solar core, obtained from the neutrinos, is of fundamental importance, the measured neutrino counting rates are rather crude probes of conditions in the solar centre. Also, the combined evidence from the independent neutrino detectors now operating makes an astrophysical solution less likely (see [1]; Bahcall these proceedings).

Independently of the neutrino measurements we now have far more extensive and detailed information about the structure of the solar interior, in the form of frequencies of the solar five-minute oscillations. When first detected around 1960 (*e.g.* [2]) these oscillations were generally assumed to be local phenomena in the solar atmosphere. However, detailed observations have shown them to result from a superpositions of up to about 10^6 individual modes, each with a sharply defined frequency, in the period range be-

tween about 15 and 3 minutes. The frequencies depend on the structure of the entire solar interior, as characterized by the distribution of pressure, density and sound speed. Through analysis of the observed frequencies it is possible to obtain precise information about these quantities as a function of position, in most of the solar interior. Reviews of such helioseismic investigations were given in, for example [3,4]; see also [5] for a discussion of the relation between solar structure and oscillation frequencies.

Helioseismic investigations do not directly provide information about the temperature and composition of the solar interior. However, they place tight constraints on the structure of the solar core which are likely to lead to fairly strict bounds on the neutrino production rates, if the physics of the solar interior, particularly the opacity and nuclear reaction rates, can be assumed to be known with reasonable accuracy. In that case, the analysis of solar neutrinos can be reversed: rather than using the neutrinos to probe the solar interior, one may use the Sun as a well-calibrated source of neutrinos; then the measurements of the neutrino rates on Earth provide information about the physics of the neutrinos.

However, moving beyond neutrino physics, the significance of helioseismology lies principally in obtaining a detailed test of stellar evolution theory, including the properties of matter under stellar conditions. In this way, one may investigate the physics of hot and dense matter and establish a firm foundation for the application of stellar models in other areas of astrophysics.

2. THE ‘STANDARD’ SOLAR MODELS

Computations of solar models are based on a number of simplifying assumptions. Evolution is started from a chemically homogeneous model in hydrostatic equilibrium, either during the pre-main-sequence contraction phase or on the zero-age main sequence. Rotation and other departures from spherical symmetry are typically neglected, as are mixing processes outside the convectively unstable regions. Also, convection is treated using simple a prescription, such as mixing-length theory, with a free parameter characterizing the efficacy of convection.

2.1. Physics of the solar interior

Given these assumptions, the structure and evolution of the model is essentially determined by the microscopic properties of solar matter. These include thermodynamic relations such as the dependence of pressure on density, temperature and chemical composition, the opacity and the nuclear reaction parameters. Very substantial improvements have been made in recent years in the treatment of the physics of the solar interior. Two detailed descriptions of the equation of state have been implemented: the so-called MHD formulation (*e.g.* [6]) and the Livermore formulation [7]. Although conceptually very different, they give rather similar results to leading order (*e.g.* [8]); however, the details are subtly different, at a level which is significant in the helioseismic analysis (see also Section 5, below). The Livermore calculations [9] have resulted in significant revisions of the opacity in the solar radiative interior, while others have recomputed the low-temperature opacities relevant to the solar atmosphere [10,11]. Finally, nuclear reaction parameters and electron screening in nuclear reactions have received a great deal of attention, not least as a result of the solar neutrino problem (see Bahcall these proceedings; Turck-Chièze these proceedings).

Compared with other time scales of relevance to the chemical composition motion is so rapid in convectively unstable regions that they may be taken to be chemically homogeneous. In the absence of macroscopic motion, however, there is a

tendency for heavier elements to sink relative to lighter elements, while sharp composition gradients are smoothed by diffusion. The time scale for these processes in the radiative interior of the Sun is very long; as a result, until fairly recently they were generally neglected (for exceptions, see [12,13]), even though they, properly speaking, belong to the ‘standard’ description of solar evolution. Cox, Guzik & Kidman [14] carried out detailed calculations of solar models including settling and diffusion of a substantial number of elements and investigated the effects of these processes on the oscillation frequencies of the models. Christensen-Dalsgaard, Proffitt & Thompson [15] found that inclusion of helium settling reduced the difference in sound speed between solar models and the Sun (see also Fig. 4 below). Detailed investigations of the effects of diffusion and settling on the computed neutrino fluxes were carried out by Bahcall & Pinsonneault [16,17].

2.2. Constraints on solar models

The computed models must satisfy the overall observational constraints on the Sun. The solar mass, radius and luminosity are known with substantial accuracy. The age of the Sun can be estimated from ages of meteorites, combined with models of the formation of the solar system (*e.g.* [18]; Wasserburg in [17]). Some information about the composition of the solar photosphere can be obtained spectroscopically; in particular, the ratio between the current surface abundance by mass of elements heavier than helium and the hydrogen abundance has been found to be $Z_s/X_s = 0.0245 \pm 0.0015$ [19]. On the other hand, the surface helium abundance Y_s cannot be determined directly. As a result, the ‘standard’ computation of solar models contains two essentially free parameters: the initial helium abundance Y_0 and the parameter α_c characterizing convection. These two parameters are adjusted until the model has the correct luminosity and radius at the present age of the Sun. With this calibration, the ‘standard solar model’ is completely determined by the assumed physics and input parameters.

2.3. Complications

This standard picture is certainly an oversimplification. The mixing-length calculation of the convective flux is at best a parametrization with a modest physical basis but certainly with no claim to realism. Also, the mean dynamical effect of the convective motions on the structure, often described as “turbulent pressure”, is generally ignored. Attempts are being made to incorporate more realistic descriptions of convection into stellar evolution calculations (*e.g.* [20]). Furthermore, detailed time-dependent hydrodynamical simulations of parts of the convection zone are providing insight into the properties of turbulent convection under solar conditions, which may lead to improved parametrized formulations (*e.g.* [21,22]). These efforts offer hope for a realistic description of the upper part of the convection zone. In any case, the uncertainties in this region have little effect on the overall structure of the model, once the convection parameter has been adjusted to obtain the proper solar radius (*e.g.* [23,24]).

More serious uncertainties concern the radiative interior. It is likely that convective motion penetrates beyond the region of instability, changing the temperature gradient (*e.g.* [25]). Beyond the region of fairly vigorous motion weaker penetration, or induced gravity waves, might lead to significant mixing of material over the time scale of solar evolution (*e.g.* [26]). Additional mixing could be caused by rotationally induced instabilities, related to meridional circulation or the spin-down of the solar interior from the usually assumed state of initial rapid rotation [27,28]. Such processes would counteract the settling of helium and heavier elements and hence would modify the abundance profile below the convection zone. Mixing of the energy-generating core could change the evolution of the Sun and, by bringing additional hydrogen to the central regions, would decrease the central temperature of the Sun required to provide the solar luminosity. Indeed, fairly substantial mixing has been proposed as a solution of the deficit of ^{37}Cl neutrinos (*e.g.* [29]), although this possibility seems unlikely in the light of more recent helioseismic evidence [30,31].

2.4. A set of solar models

Here I consider several models, differing in aspects of the physics or parameters; some details of the calculations were described in [32]. All use the Livermore opacities [9], nuclear parameters from [17] and the Böhm-Vitense [33] formulation of mixing-length theory. Evolution started at a chemically homogeneous initial model, and the age of the present Sun was taken to be 4.6 Gyr. The models were calibrated to a present luminosity and radius of $3.846 \times 10^{33} \text{ erg s}^{-1}$ and $6.9599 \times 10^{10} \text{ cm}$, respectively. Where settling and diffusion were included, the coefficients of Michaud & Proffitt [34] were used. Heavy-element diffusion was represented by the rates for ^{16}O .

The physics of the models, and some properties of them, are summarized in Table 1. The models are principally distinguished by the choice of equation of state or the inclusion of diffusion and settling. Comparison of models 4 and 5 show the effect of the current uncertainty in the observed ratio Z_s/X_s .

3. OBSERVATIONS OF SOLAR OSCILLATIONS

The solar oscillations are associated with the solar interior, but they may be observed in a number of different ways in the solar atmosphere. The most extensive data on modes probing the deep solar interior have been obtained through Doppler measurements of the line-of-sight velocity of the solar atmosphere. The basic principle of the techniques may be presented quite simply: light in the vicinity of a spectral line is passed through a filter which alternates between transmitting the red and the blue wing of the line. The difference $I_r - I_b$, between the resulting intensities I_r and I_b , changes as the line is shifted and may therefore be used as a measure of the line-of-sight velocity. The intensities are typically determined with an area detector, such as a CCD, and hence one obtains a velocity image, giving the velocity as a function of position on the disk of the Sun. In this way it is possible to determine the velocity with a resolution of perhaps 1000×1000 pixels, a cadence of more than once per minute, and an overall error of less than 1 m/s per velocity image.

Table 1

Properties of solar models. T_c and ρ_c give the central temperature and density, Φ^{Cl} and Φ^{Ga} the neutrino capture rates in the ^{37}Cl and ^{71}Ga experiments, while ^8B is the flux of ^8B neutrinos.

No.	Equation of state	Settling?	Z_s/X_s	T_c (10^6 K)	ρ_c (g cm^{-3})	Φ^{Cl} (SNU)	Φ^{Ga} (SNU)	^8B $10^6 \text{ cm}^{-2} \text{ s}^{-1}$
1	MHD	No	0.0245	15.444	150.15	6.67	123.4	4.67
2	MHD	He	0.0245	15.563	154.41	7.59	128.1	5.43
3	MHD	He, Z	0.0245	15.674	154.49	8.50	132.6	6.15
4	Livermore	He, Z	0.0245	15.668	154.24	8.20	131.9	5.87
5	Livermore	He, Z	0.0259	15.737	154.86	8.79	135.0	6.33

For a single mode of oscillation, the radial component of velocity is approximately given, as a function of co-latitude θ , longitude ϕ and time t , by

$$V_{nlm}(\theta, \phi, t) \simeq A_{nlm} \Re \left[Y_l^m(\theta, \phi) e^{-i(\omega_{nlm}t + \delta_{nlm})} \right], \quad (1)$$

where A_{nlm} is the amplitude and δ_{nlm} is the phase of the mode, and Y_l^m is a spherical harmonic. Thus the mode is characterized by three wave numbers: the degree l which measures total horizontal wave number $k_h = 2\pi/\lambda_h = \sqrt{l(l+1)}/r$ at distance r from the solar centre, λ_h being the horizontal wavelength; the azimuthal order m which measures number of nodes in longitude; and the radial order n , which characterizes the behaviour in the radial direction. As indicated, the angular frequency ω_{nlm} in general depends on n , l and m . In addition to ω_{nlm} the cyclic frequency $\nu_{nlm} = \omega_{nlm}/2\pi = 1/P_{nlm}$, P_{nlm} being the period, is commonly used.

The dependence of frequency on m arises from departures from spherical symmetry, for example induced by rotation. Here I shall be concerned with the spherically symmetric component of solar structure; this corresponds to frequencies ω_{nl} , and cyclic frequencies ν_{nl} , which do not depend on m .

In reality A_{nlm} and δ_{nlm} depend on time, as a result of the processes that excite and damp the oscillations. However, the coherence time of the modes is quite long, reaching several months for the lowest-frequency modes currently observed. Consequently, the frequencies can be determined with very high precision.

The observed signal consists of a superposition of contributions of the form given in eq. (1). Through a suitable spatial transform one may isolate time strings that each contains only a few l and m ; from these, the individual modes can be isolated through a temporal Fourier transform.

As an example of the quality of current observations Fig. 1 shows cyclic frequencies ν_{nl} as functions of degree l , from 6-month set of data obtained with the High Altitude Observatory's LOWL instrument on Mauna Loa [35]. Each of the ridges corresponds to a separate value of the radial order n . Note that the error bars represent 1000 standard deviations; the most accurately determined frequencies have a relative standard deviation below 5×10^{-6} , making them the most accurately known properties of the Sun.

4. ANALYSIS OF THE OBSERVED FREQUENCIES

4.1. Comparison between observations and models

An obvious first step in the analysis of the observed frequencies is to compare them with frequencies computed for a solar model. Given the structure of the model, it is straightforward to compute the frequencies, assuming that the oscillations can be treated as small perturbations around an equilibrium structure and that they behave adiabatically. The former assumption is amply justified: the largest amplitude of any individual mode corresponds to a velocity of around 20 cm s^{-1} , or a displacement of the solar photosphere of about 20 m. The assumption of adiabaticity breaks down only very near the surface,

Figure 1. Frequencies observed with the HAO LOWL instrument. The error bars indicate 1000 standard deviations. The circles mark the frequencies of the corresponding modes in model 4 of Table 1

where the thermal time scale of solar matter is comparable with the oscillation period. In this region the coupling between the oscillations and the turbulent convection introduces additional uncertainties in the description of the modes. However, as discussed below these problems, and the uncertainties in the description of the structure of the model in this region, can be eliminated in the analysis of the observed frequencies.

A rough idea about the diagnostic possibilities of the observed frequencies can be obtained from the overall properties of the modes. Most observed modes are predominantly of the nature of standing acoustic waves following ray paths which, when going from the solar surface towards the interior, curve away from the vertical because of the increasing sound speed with increasing depth. As a result waves corresponding to a mode of degree l and frequency ω_{nl} undergo total internal reflection at a *lower turning point* at a distance $r = r_t$ from the centre, determined by

$$\frac{c(r_t)}{r_t} = \frac{\omega_{nl}}{\sqrt{l(l+1)}}. \quad (2)$$

The frequency of such a mode is predominantly determined by the structure of that part of the

Sun which lies outside r_t . It follows from eq. (2) that modes of high degree are trapped in the outer parts of the Sun, whereas low-degree modes extend into the solar core; spherically symmetric modes, with $l = 0$, penetrate to the centre. It is this variation of penetration depth with degree and, to a smaller extent, frequency which allows the frequencies to be used to resolve solar structure, providing localized information about essentially all parts of the Sun.

In Fig. 1 the circles show adiabatic frequencies computed for model 4 of Table 1. It is evident that the agreement between observations and computations is excellent, when presented in this form, giving confidence in the identification of the oscillations, and some hope for the overall correctness of solar models.

Figure 2. Scaled differences between the observed frequencies shown in Fig. 1 and the frequencies of model 4, in the sense observations – model

4.2. Analysis of frequency differences

To utilize fully the precision of the observations a much more careful analysis is required, however. A first step is to consider differences between the observed and computed frequencies. Here some account must be taken of the uncertainties introduced by the near-surface region. Since the high-degree modes are trapped

near the surface they involve a smaller fraction of the solar mass and hence are more susceptible to the superficial effects. This dependence on degree can be eliminated by scaling the frequency differences by a quantity Q_{nl} which is essentially the ratio between the energy of a given mode and the energy of mode with $l = 0$ having the same frequency and photospheric amplitude [36,37]. It may be shown that the near-surface effects cause scaled frequency changes that are essentially slowly-varying functions of frequency alone. As shown in Fig. 2, the actual differences between the observed frequencies and frequencies of model 4 do indeed have this property, indicating that most of the problems with the model are in the surface region.

Assuming that the model is indeed a good approximation to the actual solar structure, the differences $\delta\omega_{nlm} = \omega_{nlm}^{(\text{obs})} - \omega_{nlm}^{(\text{mod})}$ between the observed and computed frequencies can be linearized around the model, resulting in (e.g. [38])

$$\frac{\delta\omega_{nl}}{\omega_{nl}} = \int_0^R K_{c^2, \rho}^{nl}(r) \frac{\delta c^2(r)}{c^2(r)} dr + \int_0^R K_{\rho, c^2}^{nl}(r) \frac{\delta \rho(r)}{\rho(r)} dr + \frac{F_{\text{surf}}(\omega_{nl})}{Q_{nl}} + \epsilon_{nl}, \quad (3)$$

where the integrals extend to the surface radius R of the Sun. Here δc^2 and $\delta \rho$ are differences between the Sun and the model in the squared sound speed c^2 and density ρ . The kernels $K_{c^2, \rho}^{nl}$ and K_{ρ, c^2}^{nl} are known functions, determined from the oscillation eigenfunctions, and $F_{\text{surf}}(\omega_{nl})$ results from the near-surface errors in the model. Finally, ϵ_{nl} is the error in the observations.

Other suitable pairs of variables may be used instead of the pair (c^2, ρ) in eq. (3). In particular, if the equation of state of solar matter and the heavy-element abundance are assumed to be known, the frequency differences may be expressed in terms of $\delta \rho$ and the correction δY to the helium abundance. The dependence on Y arises essentially through the variation of the adiabatic exponent $\Gamma_1 = (\partial \ln p / \partial \ln \rho)_a$ in the ionization zones of hydrogen and helium. Hence the frequency differences are largely determined by the difference in ρ ; the dependence on Y may in principle be used to infer the helium abundance

in the ionization zones, although the remaining uncertainty in the equation of state has a significant effect on the result (e.g. [39]).

4.3. Inverse analysis

Equation (3) forms the basis for most *inverse analyses* to infer the corrections required to the solar models to match the observations. Although several different techniques exist (e.g. [38]) the principles are most easily illustrated in terms of so-called optimally-localized averages. The idea is to form a linear combination of both sides of eq. (3),

$$\begin{aligned} \sum_{nl} c_{nl}(r_0) \frac{\delta\omega_{nl}}{\omega_{nl}} &= \int_0^R \sum_{nl} c_{nl}(r_0) K_{c^2, \rho}^{nl}(r) \frac{\delta c^2(r)}{c^2(r)} dr \\ &+ \int_0^R \sum_{nl} c_{nl}(r_0) K_{\rho, c^2}^{nl}(r) \frac{\delta \rho(r)}{\rho(r)} dr \\ &+ \sum_{nl} c_{nl}(r_0) \frac{F_{\text{surf}}(\omega_{nl})}{Q_{nl}} + \sum_{nl} c_{nl}(r_0) \epsilon_{nl}, \end{aligned} \quad (4)$$

choosing the coefficients $c_{nl}(r_0)$ such that

$$K_{c^2, \rho}(r_0, r) = \sum_{nl} c_{nl}(r_0) K_{c^2, \rho}^{nl}(r) \quad (5)$$

is localized near $r = r_0$, and the other terms are small. In particular, the effects of the near-surface errors, which dominate the frequency differences in Fig. 2, are suppressed. In this way the left-hand side of eq. (4), which can be determined from the observations, gives a measure of δc^2 at $r = r_0$. More precisely, one obtains an average of $\delta c^2 / c^2$ over a region determined by the *averaging kernel* $K_{c^2, \rho}$ defined in eq. (5). The last term on the right-hand side of eq. (4) gives an estimate of the error in the inferred value of $\delta c^2 / c^2$. Techniques for determining the coefficients c_{nl} were described in, for example, [40].

Figure 3 shows examples of averaging kernels, obtained for the mode set illustrated in Fig. 1 and keeping the error in the inferred solution at a moderate level. The resolution is clearly highest in the outer parts of the Sun. However, it is remarkable that relatively well-localized information can be obtained even in the solar core.

Figure 3. Averaging kernels for inversion for sound speed, based on the LOWL data, at target radii $r_0 = 0.25R$ (dotted curve) $r_0 = 0.5R$ (dashed curve) and $r_0 = 0.75R$ (solid curve)

5. ANALYSIS OF SPECIFIC MODELS

We may now apply the inversion procedures discussed in the preceding section to the models listed in Table 1 (for further details, see [41]). In Fig. 4 differences in squared sound speed between the Sun and models 1 and 2 are compared. Here and in the following plots the vertical error bars show the estimated error in the solution, obtained from the quoted errors in the observed frequencies, while the horizontal error bars extend from the first to the third quartile point of the corresponding averaging kernels. It is evident that, as was also found in [15], the inclusion of helium settling reduces the difference between the Sun and the model very substantially.

The inclusion of heavy-element settling has only a modest effect on the sound speed (see also [42]). However, as illustrated in Fig. 5 the results are sensitive to the choice of equation of state or the heavy-element abundance. It is interesting that the Livermore equation of state appears to yield models in somewhat better accord with solar structure than does the MHD formulation. Also, the observational uncertainty in the determination of the surface heavy-element abundance clearly has a substantial effect on the sound

Figure 4. Relative difference in the squared sound speed between the Sun and two reference models, in the sense Sun - model, inferred from inversion of the LOWL frequencies. The symbols show results for model 1, neglecting settling; the vertical error bars show the standard deviation based on the errors in the observations, while the horizontal error bars measure resolution. The dashed curve is for model 2, which includes He settling

speed.

Results of density inversions, using the pair of variables (ρ, Y) , are shown in Fig. 6. Again, it is evident that the inclusion of settling improves the agreement with the Sun. Also, it appears that the increased Z_s brings the density in model 5 into close agreement with the Sun. Here, however, it must be recalled that this result is subject to any remaining errors in the equation of state.

6. HELIOSEISMOLOGY AND NEUTRINOS

As shown by Table 1, all models discussed in Section 5 have high neutrino counting rates. Also, it is interesting that inclusion of settling, which greatly improves the models as far as the helioseismic inferences are concerned, tends to increase the neutrino fluxes, although the effect is relatively modest. Thus the helioseismic results accentuate the neutrino problem, for standard solar models and within the traditional view of neu-

Figure 5. Relative difference in the squared sound speed between the Sun and three reference models, in the sense Sun - model, inferred from inversion of the LOWL frequencies. The symbols (see caption to Fig. 4) show results for model 3, the solid line results for model 4 and the dot-dashed line results for model 5

trino physics. Given this result, and the sensitivity of the inferred sound-speed and density differences to even fairly modest modifications in the physics, the chances of finding models with low neutrino fluxes which at the same time satisfy the helioseismic constraints might appear bleak. In fact, models that have been constructed specifically to reduce the neutrino flux through some modification to the physics are generally inconsistent with the observed frequencies (see, for example [30,43,31]). Such results argue against an “astrophysical solution” to the neutrino problem.

This conclusion may be premature, however. Adiabatic stellar oscillation is a purely dynamical process. As such, the frequencies are determined by the “mechanical” properties of the Sun, *i.e.*, the distribution of gravity $g(r)$, pressure $p(r)$ and density $\rho(r)$, as well as the relation between the pressure and density fluctuations induced by the oscillations, determined by the adiabatic exponent $\Gamma_1(r)$. This property was already employed implicitly in eq. (4) where the frequency differences were expressed in terms of differences in c^2 and ρ . Here $c^2 = \Gamma_1 p / \rho$, and $p(r)$ can be

Figure 6. Relative difference in density between the Sun and four reference models, in the sense Sun - model, inferred from inversion of the LOWL frequencies. The symbols (see caption to Fig. 4) show results for model 1, the dotted line results for model 3, the solid line results for model 4 and the dot-dashed line results for model 5

obtained from $\rho(r)$ with the assumption of hydrostatic equilibrium. Thus the frequencies are uniquely determined by $c^2(r)$ and $\rho(r)$.

In contrast, the neutrino fluxes depend crucially on temperature T and chemical composition. It is true that T and composition is related to p , ρ and Γ_1 through the equation of state; furthermore, in a solar model these variables satisfy the equations of stellar structure, further constraining them. However, these relations assume that the physics of the solar interior is known. If the neutrino problem is taken to imply basic errors in stellar modelling, it is not obviously permissible to use the equations of stellar structure to infer $T(r)$ from the helioseismically determined sound speed and density.

To provide a simple illustration of these problems, I note that for a fully ionized, ideal gas

$$c^2 \simeq \frac{\Gamma_1 k_B T}{\mu m_u} \quad (6)$$

where k_B is Boltzmann’s constant, μ is the mean molecular weight and m_u is the atomic mass unit. A determination of c^2 from helioseismology therefore constrains T/μ , but not T and μ separately.

Antia & Chitre [44] explored the possible thermal structure and neutrino fluxes consistent with the helioseismic results. They were able to obtain a model yielding the measured ^{37}Cl flux, although only by assuming also a reduction in the opacity by about 30 %, which appears unrealistic. Similar results have been obtained by Baturin & Roxburgh (see [45]). It remains to be seen whether such solutions can be found without demanding what might be considered unreasonable changes in the physics of the solar interior.

7. CONCLUSION

‘Standard solar models’ have been remarkably successful in reproducing solar structure as inferred from helioseismic analysis. As a typical example may be taken model 4 of Table 1: this was computed using as far as possible the best current physics and parameters, but with no *a priori* attempt to match the observed frequencies: yet, as shown in Figs 5 and 6, throughout most of the solar interior the squared sound speed and density of the model match the Sun to within less than about 0.3 and 1.6 per cent, respectively. Although we cannot exclude that this close agreement is a coincidence, hiding much more substantial differences in the thermal and compositional structure, it seems more plausible that solar models do indeed give a good representation of the properties of the solar interior. Thus it appears unlikely that the solar neutrino problem will find an astrophysical solution.

The results demonstrate the sensitivity of the oscillation frequencies to even fairly subtle changes in the physics of the model. Thus, while the differences in the neutrino fluxes between the models in Table 1 are barely significant, the models can be very clearly distinguished in the inversion results. In particular, it is interesting that the evidence for settling and diffusion is so unequivocal. Also, it should be noticed that the current data, even though restricted to acoustic modes, may be used to probe the solar core in some detail.

Major improvements in the helioseismic data will result from the GONG project which started full observation in October 1995 and the helioseis-

mic instruments on the SOHO satellite launched in December 1995. This will enable better resolution of the structure of the core, particularly if, as is hoped, it is possible to detect internal gravity modes. Also, the uncertain region at the base of the convection zone will be resolved in considerable detail; and studies of the upper parts of the convection zone will yield subtle information about the thermodynamics of ionizing gases and provide some insight into the properties of the strongly turbulent region very near the solar surface. The results of these experiments will help to place strong constraints on the neutrino production in the solar core. Given also the results of the remarkable new neutrino detectors now under construction, we may hope to get a deep understanding of both the properties of the solar interior and the physics of the neutrinos.

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