THE INELASTIC CONTRIBUTION IN THE SCATTERING OF 20 GeV PROTONS BY NUCLEIG. Matthiae[†] - CERN

The differential cross sections for scattering of 20 GeV protons by nuclei of Li^6 , Li^7 , Be^9 , C^{12} , Al^{27} , $\text{Cu}^{63,6}$, $\text{Pb}^{207,2}$ and $\text{U}^{238,0}$ have been reported by Bellettini et al¹⁾. In that work the angular distributions obtained for the nuclei of Cu, Pb and U were analysed in terms of a simple optical model, i.e., an absorbing sphere (only imaginary potential) with an appropriate charge distribution. Such a model was adequate for describing the general behaviour of the cross sections but failed in explaining the filling of the diffraction minima. When the Coulomb interaction was also taken into account, it was found that it could be responsible only for a partial filling of the first minimum.

Excellent fits to the experimental data were obtained, however, by Frahn and Wiechers²⁾ and by Dar and Varma³⁾, who used a more sophisticated optical model (the "strong absorption model"⁴⁾) which included the diffuseness of the nuclear surface and a real potential.

The main conclusions of these latter analyses were that the proton-nucleus scattering amplitude has a considerable real part, whose sign corresponds to a predominantly repulsive nucleon-nucleus interaction, and that it is this real part that is mostly responsible for the filling of the minima in the diffraction patterns. The Authors quoted above^{2,3)} considered significant the fact that the real part of the proton-nucleus scattering amplitudes derived by means of the strong absorption model is consistent with the sign and the values obtained for the real scattering amplitude of proton-proton⁵⁾ and proton-neutron⁶⁾ scattering at the same energy, using the small angle Coulomb interference technique.

This consistency has been particularly stressed by Frahn⁷⁾. Furthermore, Sakurai⁸⁾, using the data of refs. 5 and 6, and adopting for the real part of the central nucleon-nucleon potential an expression suggested by the vector meson exchange model, has calculated for the depth of the real nuclear potential for the Cu nucleus a value very close to that derived by Frahn and Wiochers.

An important consequence of this way of interpreting the filling of the nuclear diffraction minima is that it might become possible to deduce the value of the real part of the particle-nucleon amplitude from the measurement of the shape of the diffraction pattern in particle-nucleus scattering, thus avoiding the difficulties of the Coulomb interference experiments. For this reason, it seems to us worthwhile to examine in some details the applicability of the optical model method to the data of ref. 1.

In the experiment of Bellettini et al. scattered protons were analysed with a momentum resolution (± 50 MeV/c) sufficient for rejecting those events in which one or more pions were produced but not sufficient for rejecting the contribution of those inelastic events in which the nucleus was left in an excited state or in which a single nucleon was knocked out. As it was also noted in ref. 1, this finite momentum resolution could have been partially responsible for the filling of the diffraction minima.

The purpose of this note is to discuss this point in some detail and to present an evaluation of the contribution of the inelastic scattering for the nuclei of Cu, Pb and U.

A clear inelastic contribution to the differential cross sections measured by Bellettini et al.¹⁾ is given by those events which show an angular distribution independent of the nucleus and equal to that of free nucleon-nucleon scattering.

For the light nuclei, from Li to Al, this contribution, called incoherent in ref. 1, is so important that the smoothing of the diffraction pattern could well be due to this kind of quasi-free scattering. No reliable information can thus be obtained about the real part of the scattering amplitude from the study of the diffraction pattern of light nuclei.

For the heavier nuclei (Cu, Pb and U) the incoherent scattering is small compared to the coherent one, so that the strong absorption model is expected to apply well if inelastic scattering with excitation of discrete states of the nucleus turns out to be negligible. It will be seen, however, that in general this is not the case.

Scattering with excitation of the target nucleus has been extensively studied in low-energy nuclear physics in order to get information on nuclear levels. Particularly important is the excitation of the low-lying collective levels which is often treated using the adiabatic method developed by Blair^{9,10}.

Assuming the nucleus to be completely absorbing, Blair⁹ has derived the following compact expressions for the differential cross section for inelastic scattering by an even-even nucleus with excitation from the 0^+ ground state to the 2^+ level (quadrupole excitation), to the 3^- level (octupole excitation) and to the 4^+ level, respectively.

$$\begin{aligned}\frac{d\sigma}{d\Omega}(0 \rightarrow 2) &= \frac{(K R^2)^2}{4} \frac{\beta_2^2}{4\pi} [J_0^2(KR\vartheta) + 3 J_2^2(KR\vartheta)] \\ \frac{d\sigma}{d\Omega}(0 \rightarrow 3) &= \frac{(K R^2)^2}{4} \frac{\beta_3^2}{8\pi} [3 J_1^2(KR\vartheta) + 5 J_3^2(KR\vartheta)] \\ \frac{d\sigma}{d\Omega}(0 \rightarrow 4) &= \frac{(K R^2)^2}{4} \frac{\beta_4^2}{64\pi} [9 J_0^2(KR\vartheta) + 20 J_2^2(KR\vartheta) + 35 J_4^2(KR\vartheta)]\end{aligned}$$

In these expressions β_ℓ is the deformation parameter of the collective model which may be derived from the measured inelastic cross section. It must be noted that the values of β_ℓ obtained with different particles (p, d, α) and at various energies are in general in good agreement¹⁰.

From the properties of the Bessel functions it is clear that the oscillations of the inelastic cross sections for even ℓ are out of phase with respect to the oscillations of the elastic cross section, which is proportional to $[J_1(KR\vartheta)/KR\vartheta]^2$. Therefore, if the magnitude of the inelastic scattering turns out to be substantial, the contribution to the filling of the minima coming from the quadrupole excitation could be important.

Blair's model is appropriate when the excited levels may be regarded as degenerate in energy with the ground state, for strongly absorbed nuclear projectiles and small scattering angles. These conditions are well satisfied in the experiment of ref. 1, for Cu, Pb and U target. However, in the original model of Blair⁹⁾ the diffuseness of the nuclear surface is not taken into account.

The effect of the diffuseness, as discussed by Blair et al.,¹¹⁾ is very similar on the inelastic cross sections as on the elastic one. It gives a reduction of the cross section which increases with the scattering angle.

Recently, Potgieter and Frahn¹²⁾ extended Blair's model including the diffuseness. They found that the differential cross sections, as given by Blair, must be multiplied by the square of a "form factor" $F(Kd\theta) = \pi Kd\theta / \sinh(\pi Kd\theta)$ where d is the nuclear diffuseness. This same form factor is used in the strong absorption model for elastic scattering⁴⁾.

In the following, inelastic contributions to the data of ref. 1 will be computed by means of Blair's model, taking into account the diffuseness according to Potgieter and Frahn¹²⁾ and using the values of d obtained in the fits of Frahn and Wiechers²⁾. The values of the deformation parameters derived in low-energy inelastic scattering and the radii given in ref. 1 will be used.

Let us first consider the case of Cu. Natural copper is composed of Cu⁶³ (69.1%) and Cu⁶⁵ (30.9%). These two isotopes have been widely investigated through the scattering of low-energy particles¹⁰⁾ mainly in order to check the weak particle-core coupling model. Perey et al.¹³⁾ using protons of 11 MeV have studied the excitation of the 2⁺ state of Ni⁶² and the corresponding multiplet of 4 levels of the neighbouring nucleus Cu⁶³. The inelastic scattering for the 2⁺ level of Ni⁶² shows the pattern characteristic of quadrupole excitation, as expected for a 0⁺ → 2⁺ transition. The four angular distributions for Cu⁶³ excitation are very similar to that of Ni⁶² and furthermore the total probability for exciting the four levels of the multiplet is essentially equal to that of exciting the 2⁺ level of Ni⁶². A similar comparison of both Cu⁶³ and Cu⁶⁵ with the neighbouring nuclei Ni⁶² and Ni⁶⁴, for quadrupole and octupole excitations

has been done by Bruge et al.¹⁴⁾ The conclusion has been reached that, as far as the low-lying levels are concerned, the excitation of Cu^{63} and Cu^{65} is mainly a core excitation so that the values of the deformation parameters obtained for Ni^{62} and Ni^{64} can be used. Dickens et al.¹⁵⁾ have measured the quadrupole deformation parameter β_2 of Ni^{62} and Ni^{64} , produced by the scattering of 11 MeV protons, obtaining $\beta_2 = 0.281$ and $\beta_2 = 0.231$ respectively. For natural copper the effective value turns out to be $\beta_2 = 0.266$. Eccles et al.¹⁶⁾ analysing the inelastic scattering of 19 MeV protons by Ni^{62} have found for β_2 a value in agreement with that quoted in ref. 15 and for β_3 a value of about 0.22. In our calculation the values $\beta_2 = 0.266$ and $\beta_3 = 0.22$ have been used for evaluating the quadrupole and octupole excitation for copper target. The result is described by the curve of Fig. 1.

The next target to be considered is natural lead, which is essentially composed of Pb^{206} (23,6%), Pb^{207} (22,6%) and Pb^{208} (52,3%). As discussed by Blair¹⁰⁾ there is considerable experimental evidence for the validity of the weak coupling model for these nuclei. In fact the core excitations of the doubly-magic Pb^{208} are present in the neighbouring nuclei with almost the same strength. Several consistent determinations have been done of the deformations parameters for the lead isotopes by means of the inelastic scattering of 31 MeV protons¹⁷⁾, 40 MeV protons¹⁸⁾, 42 MeV α particles¹⁹⁾, and 8 MeV neutrons²⁰⁾ as well as by means of life-time measurements.^{21,22)} The values presently used, $\beta_2 = 0.06$ and $\beta_3 = 0.12$, are quite low, as expected for nuclei with closed shells. The calculated differential cross section for both quadrupole and octupole excitations of lead is shown in Figure 2.

The last target nucleus considered is U^{238} which is known to have rotational bands. The first level above the ground state, with spin and parity 2^+ , is expected to be strongly excited. Using the value of the deformation parameter $\beta = 0.268$ given by Davidson²³⁾, the inelastic differential cross section shown in Fig. 3 has been calculated. The contribution of inelastic scattering is seen to be much larger for the U than for the Pb target. This is in agreement with the experimental data, which show for the U target a smoother trend than for Pb target.

In Figs. 1, 2 and 3 the dashed lines represent the predictions of the strong absorption model for the elastic cross section without real part. These curves have been obtained using the same parameters as used for the fits in ref.2.

Clearly, for U the smoothness of the observed cross section in the angular region where the minima are expected can be easily attributed to the quadrupole excitation, except in the region of the first minimum where the contribution of the excitation and of the Coulomb interaction seem to be insufficient to fill completely the dip.

For Cu, the Coulomb interaction is too small to give an appreciable contribution to the filling of the first minimum but the quadrupole and octupole excitations are of such a magnitude that they may be responsible for a nearly complete filling of the minima.

The situation seems to be different for the lead target. In this case the stability against deformation of the nearly doubly magic isotopes makes the inelastic cross sections with excitation of the states considered above so small that this excitation cannot account for the filling of the minima.

It must, however, be pointed out that the present evaluation of the inelastic contributions is surely an underestimate, because only one or two excitations have been considered. Unfortunately little is known about the excitation of levels at higher energy with respect to the ground state.

In conclusion, it seems evident to us that the contribution of the inelastic scattering with excitation of the first collective states, which is of quite different magnitude for different nuclei, must be considered carefully in any detailed analysis of data of the type of ref. 1, aiming at extracting informations on the real part of the elastic scattering amplitude.

In fact, the results of the present calculation show that the conclusions reached in refs. 2 and 3, concerning the real part of the scattering amplitude are incorrect, at least for the Cu and U nuclei. Since the amount of the real part of the amplitude reported in refs. 2 and 3 is about the same for the three nuclei, Cu, Pb and U, it may be that also the conclusions obtained for Pb are to be revised.

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FIGURE CAPTIONS

The experimental data of reference 1, the strong absorption model prediction without real part (dashed lines) and the inelastic cross section as evaluated in the text (solid lines) are quoted in

Figure 1 for $\text{Cu}^{63.6}$, in
Figure 2 for $\text{Pb}^{207.2}$ and in
Figure 3 for $\text{U}^{238.0}$.

FIG. 1

FIG. 2

FIG.3

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