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A Note on Monte Carlo Imitation of Bose - Einstein Interference Effects¹

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Abstract

Methods of implementing Bose - Einstein interference effects used in Monte Carlo codes are shortly discussed. The method used in JETSET is analyzed in detail; some of its drawbacks are indicated, and possible improvements are considered. General limitations of this approach are shown suggesting the necessity of other investigations.

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1 Introduction

In the last years there was a renewed interest in the Bose - Einstein interference effects in multiparticle production. It was triggered by the experimental observation [1,2] that the strong positive correlations observed for hadrons close in momentum space (giving rise to so-called "intermittency signals" [3,4]) occur only for pairs of identical particles. Thus the intermittency signals seem to be related to the Bose - Einstein interference effects (let us note in passing that this was suggested [5] soon after the original observation of first intermittency signals [3], but the experimental difficulties with e^+e^- pairs from γ conversion delayed this discovery). This does not make the intermittency signals trivial or uninteresting, as the observed distributions suggest non-trivial source structure [6].

More detailed investigations of the space-time development of multiple production processes are very difficult. To compare data with models we should be able to calculate how the assumptions influence the interference effects. Only the models using Monte Carlo generators allow us to produce arbitrary distributions and to investigate the effects of event selections or kinematical cuts reflecting the real experimental conditions. On the other side, however, these models use probabilities and not amplitudes. Thus incorporating quantum interference effects in them is not an easy task.

In this note we list shortly the existing approaches to the implementation of Bose - Einstein interference effects into Monte Carlo models. Then we discuss in more detail the procedure employed by the JETSET program [7], used most commonly by experimental groups. We indicate the drawbacks of the existing procedure, discuss its possible improvements and conclude with a few remarks on future possibilities. For simplicity, we restrict our discussion to the two-body distributions.

2 Implementation of interference effects in Monte Carlo models

The traditional way of introducing Bose-Einstein interference effects into the description of multiparticle production is by symmetrizing the production amplitudes [8]. To this aim one needs the space-time profile of the source, which is usually convoluted with plane waves (for charged particles, corrections for final state electromagnetic interactions are sometimes taken into account). To get simple results, rather unphysical assumptions are often adopted (spherically symmetric static Gaussian sources; no dependence on momenta of produced hadrons etc.).

One of the possible ways of incorporating Bose-Einstein interference into Monte Carlo models is therefore to use the original Monte Carlo program as a model of classical source [9]. This may be reasonable for the case of heavy ion collisions, where the source size exceeds significantly typical wavelengths of produced hadrons and the assumption of incoherence is plausible. However, for hadro- or leptonproduction such a separation of collision and production processes seems unnatural.

More promising is the application of Wigner functions' formalism [10]. As it was recently outlined, one may use it to justify the introduction of interference effects as weights to be attributed to the events depending on their momentum structure [11].

However, for the Monte Carlo programs containing many fitted parameters such weights could necessitate a long procedure of refitting to the huge collection of data.

Therefore at present the only working method to introduce interference effects into Monte Carlo generators for hadroproduction is that of shifting final state momenta to reproduce the assumed distribution of some variable. This is the method employed in new versions of JETSET [7], which will be described in the next section.

3 Procedure LUBOEI - imitation of interference effects

The FRITIOF [12] and PYTHIA [13] models, describing correctly many features of hadroproduction, did not include any interference effects. To improve this situation Sjöstrand introduced into the JETSET code for string hadronization (used by both models) [7] a new procedure LUBOEI shifting the generated momenta of final state hadrons in a way enhancing the probability of low values of two- particle invariant mass M for identical particles. More precisely, the procedure uses an input shape of Bose-Einstein factor f_{BE} in the Gaussian or exponential form in variable Q related to M by

$$M^2 \equiv (p_1 + p_2)^2 = Q^2 + 4m^2. \quad (1)$$

Let us denote the two-particle density in Q in absence of interference effects (integrated over all the components of p_1 and p_2) by $\rho(Q)$. For identical pions and small values of Q it is well approximated in PYTHIA or FRITIOF by the uncorrelated background, i.e. the convolution of single particle distributions

$$\rho(Q) \cong \rho_0(Q) = \int d^3p_1 d^3p_2 \rho_1(p_1) \rho_1(p_2) \delta[Q - \sqrt{-(p_1 - p_2)^2}]. \quad (2)$$

For each pair of identical pions (option with kaons is also available) the original value of Q is replaced by Q' such that

$$\int_0^Q \rho(x) dx = \int_0^{Q'} \rho(y) f_{BE}(y) dy. \quad (3)$$

As already noted, in LUBOEI

$$f_{BE} = 1 + \lambda \exp(-Q/Q_0) \quad \text{or} \quad f_{BE} = 1 + \lambda \exp[-(Q/Q_0)^2]. \quad (4)$$

Two free parameters, λ and Q_0 can be taken from dedicated experiments or fitted to data (default values are 1 and 0.2 GeV, respectively). To simplify calculations, $\rho(Q)$ is taken from the spherical phase space as

$$\rho(Q) = cQ^2(Q^2 + 4m^2)^{-1/2} \quad (5)$$

and equation (3) is solved approximately using this ansatz. It is easy to check that if the original distribution is given by (5), shifting each Q value to Q' defined by (3) gives indeed as the new distribution $\rho'(Q')$ the product of the old distribution and the f_{BE} factor

$$\rho'(Q') = \rho(Q') f_{BE}(Q'). \quad (6)$$

Thus the procedure may be regarded as an imitation of the interference effect reproducing the assumed shape of distribution in Q . Obviously, this does not guarantee that other features of the interference effect will be well described; in fact, it is known that the dependence on the event multiplicity is wrong [14]. Also the higher order effects are over-estimated [2]. Nevertheless, one may hope that modifying the prescription (i.e. replacing (4) by some functions corresponding to more complicated sources, introducing different source shapes and sizes for direct pions and resonance decay products etc.) one will improve the agreement with the data. This could be then regarded as an indirect evidence for such features of source structure.

We have to use this procedure to construct the modified full finite states. Thus LUBOEI contains further a detailed prescription for calculating all components of new momenta of two pions of the pair (by keeping unchanged the sum of three-momenta in CM frame). For a single pair this shift from p_1, p_2 to p'_1, p'_2 would result exactly in shifting $\sqrt{-(p_1 - p_2)^2}$ from Q to Q' . The global shift of momenta for all particles is actually performed only after all new values of Q for all pairs were calculated. Finally, all three-momenta in CM are rescaled by a common factor to restore the original value of global CM energy.

All this is done before the long-living resonances are allowed to decay. Therefore the pions from such decays are not correlated, which lowers the effective value of λ in the final distributions.

Unfortunately, the procedure applied to the events generated by PYTHIA does not give the ratio of new to original distribution in Q equal to the input factor f_{BE} already after the first step (before the rescaling of momenta). This is illustrated in Fig.1 for exponential f_{BE} with Q_0 equal 0.1 and 0.4 GeV for $p\bar{p}$ collisions at 630 GeV CM energy - the conditions of experiment of Ref. [1], where PYTHIA/JETSET results were extensively used for comparison with data.

Fig.1. The $\log(\rho'(Q)/\rho(Q) - 1)$ values from PYTHIA/JETSET/LUBOEI for $p\bar{p}$ collisions at 630 GeV (diamonds and crosses) and the input function $-Q/Q_0$ (solid and broken line) for $Q_0 = 0.1, 0.4$, respectively.

Whereas for small Q_0 the agreement is reasonable, for the value of 0.4 the ratio exceeds 2 at smallest Q and then decreases faster than f_{BE} . Unless otherwise noted, we use always λ equal one; 10^5 PYTHIA events were generated.

4 Modifications of LUBOEI

We have found an obvious reason why LUBOEI does not reproduce the input factor (4). The experimental uncorrelated background ρ_0 (well reproduced by PYTHIA and close to the prediction for ρ from PYTHIA without LUBOEI) does not resemble at all the shape from spherical phase space (5), as shown in Fig.2. Instead of monotonical increase, it has a maximum at a few hundred MeV and then falls down power-like for large Q , as expected from Mueller-Regge picture. The difference becomes significant already at Q above 0.1 GeV.

Fig.2. The shape of uncorrelated background (fit to PYTHIA/JETSET, solid line) compared to spherical phase space (5) (broken line).

Therefore to have a consistent procedure reproducing the assumed input factor one should replace the function (5) in LUBOEI by the form approximating reasonably well the uncorrelated background (2). Unfortunately, the approximations used in LUBOEI fail for ρ_0 which does not increase indefinitely with Q and one cannot use them; equation (3) must be solved numerically.

Moreover, one must modify this equation itself: since the procedure shifts momenta without changing the number of particles, the integrals of ρ and ρ' over the full range of Q must be equal. This can be reconciled with (3) and (6) only if the range in Q' is narrower than in Q , since ρ and ρ' are positive and f_{BE} is bigger than one. For the monotonically increasing ρ this is irrelevant: for $Q \gg Q_0$ the difference between Q and Q' falls down as a power of Q . However, since true ρ falls down for large Q , prescription (3) would give the upper limit for Q' which depends significantly not only on available energy, but also on the values of parameters of the f_{BE} factor. In fact, for ρ decreasing fast enough Q' is

limited even for $Q \rightarrow \infty$, and the limiting value is fully determined by ρ and Q_0 ! This is certainly not a reasonable prescription, since our aim was to modify the shape of ρ only for small Q . Instead, we would generate events which do not contain at all the pairs of identical pions with large values of Q .

The solution is to multiply the RHS of equation (3) by the ratio of integrals of ρ and ρf_{BE} over the full Q range

$$R = \int_0^{Q_{\max}} \rho(x) dx / \int_0^{Q_{\max}} f_{BE}(x) \rho(x) dx. \quad (7)$$

At high energy we may replace $Q_{\max}$ by infinity if ρ decreases fast enough. The normalization of ρ' relative to ρ_0 depends anyway on the procedure of generating uncorrelated background, and multiplying the RHS of (3) by R guarantees that for large Q we have $Q' \cong Q$, as it should be.

Thus the internally consistent version of LUBOEI should start with fitting a reasonably simple function to the uncorrelated background (2). Then one should choose a proper f_{BE} , calculate R and find the prescription to calculate Q' for each Q by solving numerically the modified equation (3)

$$\int_0^Q \rho(x) dx = R \int_0^{Q'} \rho(y) f_{BE}(y) dy. \quad (8)$$

This guarantees that the resulting ρ' will be equal to ρf_{BE} up to a normalizing factor (and possible small corrections from energy rescaling) and that the range of Q will not be modified. Unfortunately, the shape of uncorrelated background (2) depends on energy and other kinematical conditions. Thus one cannot write a single version of this procedure; each user should perform first a suitable fitting parametrizing ρ_0 as measured in his data.

In Fig.3 we show the results obtained by such procedure for the same conditions as in Fig.1. Obviously, the ratio ρ'/ρ reproduces now the input shape of f_{BE} quite well.

Fig.3. The $\log(\rho'(Q)/\rho(Q)) - 1/R$ values from modified PYTHIA/JETSET/LUBOEI for $p\bar{p}$ collisions at 630 GeV (diamonds and crosses) and function $-Q/Q_0 - \log R$ (solid and broken line) for $Q_0 = 0.1, 0.4$, respectively.

Unfortunately, applying our modified procedure we discovered a more fundamental drawback of this method. We have found that the global effect of shifting all momenta to adjust them to modified values of Q does not reproduce the modified Q distribution for small Q . This shifting is done after modified values of Q for all pairs were adopted. Thus every momentum is shifted many times and the resulting Q distribution differs strongly from the one it should reproduce, as shown in Fig.4.

Fig.4a. The ratio of Q distributions after and before the momenta shifting for $p\bar{p}$ collisions at 630 GeV for $Q_0 = 0.1, 0.2, 0.4$ (diamonds, crosses and squares, respectively) from PYTHIA/JETSET/LUBOEI.

Fig.4b. The ratio of Q distributions after and before the momenta shifting for $p\bar{p}$ collisions at 630 GeV for $Q_0 = 0.1, 0.2, 0.4$ (diamonds, crosses and squares, respectively) from modified PYTHIA/JETSET/LUBOEI.

Already for the original LUBOEI prescription the ratio of shapes of $\rho(Q)$ before and after the shifting deviates quite strongly from unity for $Q_0 \geq 0.2$ (Fig.4a), and for our

prescription this deviation grows even more rapidly with Q_0 (Fig.4b). Thus the final shape of ratio ρ/ρ_0 has little to do with original shape of assumed f_{BE} for Q_0 values above 0.1 (corresponding to source size below 2 fm). This is illustrated in Fig.5.

Fig.5a. The ratio of final Q distribution to the uncorrelated background for $p\bar{p}$ collisions at 630 GeV for $Q_0 = 0.1, 0.2, 0.4$ (diamonds, crosses and squares, respectively) from PYTHIA/JETSET/LUBOEI. Solid, broken and dotted lines represent functions (4) with corresponding values of Q_0 and reduced values of λ .

Fig.5b. The ratio of final Q distribution to the uncorrelated background for $p\bar{p}$ collisions at 630 GeV for $Q_0 = 0.1, 0.2, 0.4$ (diamonds, crosses and squares, respectively) from modified PYTHIA/JETSET/LUBOEI. Solid, broken and dotted lines represent functions (4) with corresponding values of Q_0 and reduced values of λ .

Here this ratio for both original and modified procedure is compared with input shapes corrected for normalization (taking into account resonance decay products and the R factor in eq. (8)).

In fact, this makes our modification pointless, as the distortions introduced by using wrong shape of ρ_0 in LUBOEI are negligible for sources large enough to neglect distortions due to global momentum shifting.

5 Conclusions and outlook

We have shown that the most commonly used implementation of Bose-Einstein interference effect in Monte Carlo programs does not reproduce the assumed input shape of distributions. We have explained the reason of this inconsistency and proposed a modified version which reproduces correctly the input shape.

Unfortunately, the shape of distribution is distorted even more violently by the global effect of the momenta shifting performed to adjust to new values of Q . Only for sources of size above 2 fm ($Q_0 \leq 0.1$ GeV) the method reproduces the assumed shape in the final distributions (and there our modification is negligible). Thus we conclude that imitating BE effect in JETSET by momenta shifting is reasonable only for large sources (heavy ion collisions?). For the sources of sizes typical for hadron and/or lepton collisions we obtain the final sample of events for which the ratio of $\rho(Q)/\rho_0(Q)$ does not reproduce the shape assumed for f_{BE} . Obviously, one may treat parameters of f_{BE} just as free parameters to be fitted to the data at the level of final distributions, but then there is no reason to attach to them any physical significance. In particular, replacing exponential shape by e.g. power-like function, or adding terms with different slopes can no longer be understood as modifying physical assumptions about the source structure.

Let us stress once more that the implementation of the interference effects discussed here is only an imitation, i.e. it does not explain the origin of shapes of distributions, but simply should reproduce input shapes (as very clearly stated in [7], but often forgotten by users). Thus even for large sources it can only serve as a tool to test some ideas about source structure and to check which of the observed effects are independent, and which are just kinematical reflections. Its inadequacy to describe smaller sources underlines the necessity of further development of other approaches (in particular, applications of Wigner functions formalism), since at present this implementation remains the only one extensively used to compare Monte Carlo results with data on short range correlations.

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