Probing Lepton Flavor Violation at Future Colliders *

Nima Arkani-Hamed, Hsin-Chia Cheng, Jonathan L. Feng †, and Lawrence J. Hall

Theoretical Physics Group,
Lawrence Berkeley National Laboratory

and

Department of Physics
University of California, Berkeley, California 94720

Abstract

Supersymmetric theories with significant lepton flavor violation have $\tilde{e}$ and $\tilde{\mu}$ nearly degenerate. In this case, pair production of $\tilde{e}^+\tilde{e}^-$ and $\tilde{\mu}^+\tilde{\mu}^-$ at LEPII and at the Next Linear Collider leads to the phenomenon of slepton oscillations, which is analogous to neutrino oscillations. The reach in $\Delta m^2$ and $\sin^2 2\theta$ gives a probe of lepton flavor violation which is significantly more powerful than the current bounds from rare processes, such as $\mu \rightarrow e\gamma$. Polarizable $e^-$ beams and the $e^-e^-$ mode at the NLC are found to be promising options.
Two fundamental questions of particle physics are the origin of electroweak symmetry breaking and the pattern of the quark and lepton mass matrices, known as the flavor problem. Extensions of the standard model with weak scale supersymmetry have been widely studied: the enhanced spacetime symmetry offers an understanding of the hierarchy of the weak and Planck scales, electroweak symmetry breaking is triggered by the dynamics of a heavy top quark, and the unification prediction for the weak mixing angle is highly successful if there are superpartners at the weak scale. The experimental discovery of superpartners would represent enormous progress in understanding electroweak symmetry breaking, but would it allow progress on the flavor problem?

It is probable that all the quarks and leptons have been discovered. Although further light will be shed on the flavor problem by measuring the 13 fermion masses and mixings to greater accuracy, within the minimal three generation standard model there are no new flavor parameters to measure. In any supersymmetric extension of the standard model, the superpartners of the quarks and leptons must be given masses. There are 15 new flavor parameters in the scalar mass eigenvalues, and seven new flavor mixing matrices [1]

$$W_a = U_a^\dagger V_a , \quad a = u_{L,R}, d_{L,R}, e_{L,R}, \nu_L,$$

arising as relative rotations between the matrices $U_a$ and $V_a$ that diagonalize the scalar and fermion mass matrices, respectively. At the neutral gaugino vertex of species $a$, the $i$-th generation scalar is converted to the $j$-th generation fermion with amplitude $W_{aij}$. If supersymmetry is correct, it will furnish a large new arena for studying the problem of flavor.

The last twenty years have seen a succession of discoveries of heavy quarks and leptons. After the initial discovery and mass measurement, the focus has turned to measurement of the flavor violations via the parameters of the Cabibbo-Kobayashi-Maskawa matrix $V_{CKM} = U_u^\dagger V_d$. If supersymmetry is discovered, we envisage a similar progression: after measurements of superpartner masses, the focus will shift to a study of the new flavor mixing matrices at the gaugino vertices. Rare flavor changing processes, such as $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, $\tau \rightarrow e\gamma$, $b \rightarrow s\gamma$, and neutral meson mixing, provide important constraints on the $W_a$ mixing matrices via the virtual effects of superpartners. In this paper, however, we show that, once superpartners are discovered, it will be possible to probe these matrices much more powerfully by directly observing the change in flavor occurring at the superpartner production and decay vertices. We consider lepton flavor violation, and find that, if sleptons are made at LEP II or the NLC, the 21, 32, and 31 elements of $W_{e_L,e_R}$ will be probed considerably beyond the most stringent present limits, which result from $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, and $\tau \rightarrow e\gamma$, respectively.

There are many theoretical ideas for the origin of the scalar and fermion masses in supersymmetry and the symmetries that govern them [2]. In this letter we do not discuss these theories; we concentrate on the question of how well $W_{e_L,e_R}$ can be probed at future electron colliders in a model independent way, assuming only that sleptons are directly produced. Nevertheless, an important question is which experimental signature will provide the best probe of this physics, and hence is most likely to produce a signal. To evaluate this, we find the reach of NLC for $W_{e_{L,R}ij}$, $i \neq j$, and compare it to the corresponding CKM matrix element, $V_{CKM_{ij}}$. We find that only in the case $ij = 12$ can the NLC probe mixing angles as small as the CKM case, and in this case the probe can be very far beneath the CKM case. Hence in this paper we limit ourselves to an analysis of the lightest two
generations. Further details of this case, and the reach for flavor violation involving the tau, will be given in a subsequent paper [3].

If \( W_{e_L,e_R} \) are comparable to the Cabibbo angle, then the rate for \( \mu \rightarrow e\gamma \) is typically four orders of magnitude above the experimental bound; this is part of the well-known supersymmetric flavor changing problem [4]. It is solved by having considerable degeneracy between the superpartners \( \tilde{e} \) and \( \tilde{\mu} \), leading to a superGIM cancellation in the amplitude for \( \mu \rightarrow e\gamma \). The near degeneracy of \( \tilde{e} \) and \( \tilde{\mu} \), together with their mass mixing, which results in non-zero \( W_{e_L,e_R} \), implies that the direct production of \( \tilde{e} \) and \( \tilde{\mu} \) results in lepton flavor oscillations, analogous to strangeness oscillations and neutrino oscillations. Unlike the neutrino case, however, \( \tilde{e} \) and \( \tilde{\mu} \) decay very quickly, and hence the relevant signal is the time integrated one. Nevertheless, the reach of an experiment is best described by plotting event rate contours in the \( (\sin 2\theta, \Delta m^2) \) plane [5].

The gauge eigenstate scalars \( |\tilde{e}\rangle, |\tilde{\mu}\rangle \) are related to the mass eigenstate scalars \( |1\rangle, |2\rangle \) via

\[
|\tilde{e}\rangle = + \cos \theta |1\rangle + \sin \theta |2\rangle \\
|\tilde{\mu}\rangle = - \sin \theta |1\rangle + \cos \theta |2\rangle ,
\]

where \( \sin \theta = W_{12} \). Suppose that at time \( t = 0 \) we produce a gauge eigenstate selectron in an \( e^-e^- \) collision: \( |\psi(0)\rangle = |\tilde{e}\rangle \). The state at time \( t \) is

\[
|\psi(t)\rangle = \cos \theta e^{-\frac{\Gamma}{2}t-\text{i}m_1t} |1\rangle + \sin \theta e^{-\frac{\Gamma}{2}t-\text{i}m_2t} |2\rangle \\
= (\cos^2 \theta e^{-\frac{\Gamma}{2}t-\text{i}m_1t} + \sin^2 \theta e^{-\frac{\Gamma}{2}t-\text{i}m_2t}) |\tilde{e}\rangle \\
- \sin \theta \cos \theta (e^{-\frac{\Gamma}{2}t-\text{i}m_1t} - e^{-\frac{\Gamma}{2}t-\text{i}m_2t}) |\tilde{\mu}\rangle ,
\]

where we have neglected the difference between the widths of the two mass eigenstates and set them equal to \( \Gamma \). The probability \( P(\tilde{e} \rightarrow f_\mu) \) that the gauge eigenstate selectron decays into the final state containing a muon, \( f_\mu \), is

\[
P(\tilde{e} \rightarrow f_\mu) = \int_0^\infty dt |\langle \tilde{\mu}|\psi(t)\rangle|^2 \times B(\tilde{\mu} \rightarrow f_\mu) \\
= 2 \sin^2 \theta \cos^2 \theta \frac{(\Delta m^2)^2}{4\tilde{m}^2\Gamma^2 + (\Delta m^2)^2} \times B(\tilde{\mu} \rightarrow f_\mu) ,
\]

where \( \Delta m^2 = m_1^2 - m_2^2 \), \( \tilde{m} = (m_1 + m_2)/2 \), and \( B(\tilde{\mu} \rightarrow f_\mu) \) is the branching fraction for \( \tilde{\mu} \rightarrow f_\mu \). The term depending on \( \Delta m^2 \) is the quantum interference factor neglected in [5]. Note that when \( \Delta m^2 \gg \tilde{m}\Gamma \), this factor becomes 1 and the interference effect can be ignored. However, when \( \Delta m^2 \ll \tilde{m}\Gamma \), the factor goes to zero and the interference effect cannot be neglected. \( \Gamma \) is typically much smaller than \( \tilde{m} \); for instance, if the only decay mode for a gauge eigenstate right-handed selectron is \( \tilde{e} \rightarrow e\tilde{\chi}_0 \), where \( \tilde{\chi}_0 \) the lightest neutralino, \( \Gamma/\tilde{m} = \frac{\alpha}{2\cos^2 \theta_W} \left(1 - \frac{\tilde{m}_\tilde{\chi}_0^2}{\tilde{m}^2}\right)^2 \sim 0.01 \). Thus, as long as \( \Delta m^2/\tilde{m}^2 > 0.01 \), there is no interference suppression of the flavor changing process. However, \( B(\mu \rightarrow e\gamma) \) constrains the product \( \sin \theta \cos \theta \Delta m^2/\tilde{m}^2 \) to be (roughly) less than \( \sim 0.01 \), so there is competition between these different probes of flavor violation.

We have calculated the cross sections for the flavor-violating processes \( e^\pm e^- \rightarrow e^\pm \mu^\mp \chi_0^0 \chi_0^0 \) and \( e^-e^- \rightarrow e^-\mu^-\chi_0^0 \chi_0^0 \). In calculating these cross sections, we have correctly treated the
different interferences in different channels. In the $e^-e^-$ case, the amplitude comes from $t$-channel neutralino exchange producing gauge eigenstate selectrons, while in the $e^+e^-$ case there are additional contributions from $s$-channel annihilation into $\gamma/Z$ producing gauge eigenstate selectrons and smuons. In both cases the sleptons produced oscillate and decay into leptons and lighter superpartners. Consider the $e^-e^-$ case with both beams right polarized. The cross section for $e_R e_R \rightarrow e^- \mu^- \tilde{\chi}^0 \tilde{\chi}^0$ depends on the right-handed mixing angle in the combination $\sin 2\theta_R$, on the mass difference of the right-handed scalars $\Delta m_R^2 / \bar{m}_R^2$ (via the interference effect), on the average mass of the right-handed selectrons and smuons $\bar{m}_R^2$, and finally, assuming that the lightest superpartner (LSP) is pure Bino, on the Bino mass $M_1$. Fixing $\bar{m}$ and $M_1$, we will give contour plots below for the cross section in the $(\sin 2\theta_R, \Delta m_R^2 / \bar{m}_R^2)$ plane. For comparison, we will also include contours of $B(\mu \rightarrow e\gamma)$ in our plots. The $\theta_R$-dependent amplitude for $\mu \rightarrow e\gamma$ contains two pieces: one depending on the same parameters just discussed and the other depending further on the left-right scalar mass mixing parameter $A + \mu \tan \beta$ and the left-handed scalar masses.

Having discussed the flavor-violating cross sections, we now examine the possibility of detecting flavor-violating signals at future colliders. We first consider the sensitivity of the LEP II $e^+e^-$ collider, with a center of mass energy $\sqrt{s} = 190$ GeV and an integrated luminosity of roughly 500 pb$^{-1}$. We then turn to the NLC, with design energy $\sqrt{s} = 500$ GeV and luminosity 50 fb$^{-1}$/yr in $e^+e^-$ mode. The $e^-e^-$ luminosity is currently being studied [6], and may be degraded somewhat from the $e^+e^-$ luminosity. We will, however, assume an event sample of 50 fb$^{-1}$ in both modes. A 5$\sigma$ discovery signal then requires $S \geq 7.1 \sqrt{B / 0.5 / Y}$ for LEP II and $S \geq 0.71 \sqrt{B / 50 / Y}$ for the NLC, where $S$ and $B$ are the signal and background cross sections after cuts (in pb), and the total integrated luminosity is $Y$ fb$^{-1}$.

To discuss the flavor violation discovery potential of LEP II, we first choose some representative values for the various SUSY parameters. (Some implications of deviations from these choices will be discussed below.) Sleptons with mass below 85–90 GeV are expected to be discovered at LEP II. We therefore consider the case where $m_{\tilde{e}_R} \approx m_{\tilde{\mu}_R} \approx 80$ GeV. The LSP must be lighter than this, and we assume that it is Bino-like with mass $M_1 = 50$ GeV. For simplicity, we also assume that the production of all other supersymmetric particles is suppressed, either kinematically, or, for example, in the case of neutralinos, through mixing angles. The sleptons, then, decay directly to the LSP, and the flavor-violating signal is $e^+e^- \rightarrow (\tilde{e}_R, \tilde{\mu}_R)(\tilde{e}_R, \tilde{\mu}_R) \rightarrow e^+\mu^\mp \tilde{\chi}^0 \tilde{\chi}^0$.

At LEP II energies, the dominant standard model background to the $e^\pm\mu^\mp$ final state is $W$ pair production, where both $W$ bosons decay to $e$ or $\mu$, either directly or through $\tau$ leptons. Including branching ratios, this cross section is 680 fb. The $WW$ background may be reduced with cuts, as has been discussed in a number of studies [7]. (Of course, if sleptons are significantly lighter than 80 GeV, one may run below $\sqrt{s} = 160$ GeV and eliminate $WW$ production altogether.) Depending on the LSP mass, the cuts may be optimized to reduce the background to $\sim 10$–100 fb, while retaining 40%–60% of the signal. Given an integrated luminosity of 500 pb$^{-1}$, then, the required cross section for a 5$\sigma$ effect is $\sim 40$–185 fb. The flavor-violating cross section is plotted in Fig. ??, along with the constraint from $B(\mu \rightarrow e\gamma)$ for various values of $\tilde{t} \equiv -(A + \mu \tan \beta) / \bar{m}_R$. (In the limit of large left-handed scalar mass, the $A + \mu \tan \beta$ contribution to $\mu \rightarrow e\gamma$ vanishes, and so this limit corresponds to the $\tilde{t} = 0$ contour.) The cross section contours extend to $\sin \theta_R \sim 0.15$, and for low values of $\tilde{t}$ extend
the reach in parameter space significantly.

We have assumed above that no other supersymmetric particles are produced. If this is not the case, there may be supersymmetric backgrounds. However, the supersymmetric backgrounds tend to be small relative to the WW background; in the case of stau pairs, for example, after branching ratios are included, this background is $O(10 \text{ fb})$. We have also assumed above that we are in the region where the lighter chargino and neutralinos are gaugino-like. If they are Higgsino-like, the slepton decay widths are greatly reduced, and the $\Delta m_R^2/(\bar{m}_R \Gamma)$ suppression takes effect only for smaller $\Delta m_R^2$. Thus, the $e^+\mu^\pm$ signal can probe regions of even smaller $\Delta m_R^2/\bar{m}_R^2$. However, for $e^+e^-$ machines, the cross section become smaller when $M_1$ is large. The reach in mixing angle may be slightly worse.

If sleptons are not produced at LEP II, they may be discovered at the NLC. To consider the potential for discovering slepton flavor violation there, we consider right-handed slepton masses $m_\tilde{e}_R, m_\tilde{\mu}_R \approx 200 \text{ GeV}$, and $M_1 = 100 \text{ GeV}$. Again, we assume that we are in the gaugino region, and that the production of other sparticles is suppressed.

There are many options at the NLC, as both highly polarized $e^-$ beams and $e^+e^-$ and $e^-e^-$ modes may be available. We consider first the $e^+e^-$ modes. At NLC energies, $W$ pair production is still a large background at 7 pb, but there are now others, including $e^+\nu W^\mp$, which, at 5 pb, is a large background even though the electron tends to disappear down the beampipe, and $(e^+e^-)W^+W^-$, which is only 200 fb, but is difficult to remove from the signal. Nevertheless, efficient cuts have been devised for (flavor-conserving) selectron and smuon pair production [8,9], and these also effectively isolate the flavor-violating signal. Applying the cuts of Ref. [8], we find that the standard model $e^+\mu^\mp$ background is reduced to 5.2 fb for unpolarized beams, while $\sim 30\%$ of the signal is retained. This may be improved further by using an $e_R$ beam, which doubles the signal and removes $W$ pair production, reducing the background to 2.6 (2.3) fb for 90% (95%) beam polarization. (Note also that the $e_R$ beam also highly suppresses the pair production of Wino-like charginos.) Given a year’s running at design luminosity, the required $5\sigma$ signal is 3.8 (3.6) fb. Cross sections for $e^+e^-\rightarrow e^+\mu^\mp\tilde{\chi}_0^0\tilde{\chi}_0^0$ at the NLC are given in Fig. ???. We see that the NLC in $e^+e^-$ mode is also a powerful probe of the flavor-violating parameter space, extending to sin $\theta_R = 0.06$ and probing parameter space for which $B(\mu \rightarrow e\gamma) = 10^{-14}$ ($10^{-11}$) for $\tilde{t} = 2$ (50). The extent of parameter space probed is seen to be insensitive to beam polarization.

An intriguing feature of the NLC is its ability to run in $e^-e^-$ mode. This option allows one to polarize both beams, and has extremely low backgrounds. For example, WW production, previously our most troublesome background, and chargino production are both eliminated. However, as first noted in Ref. [10], slepton pair production is allowed, as SUSY theories naturally provide Majorana particles, the neutralinos, which violate fermion number. The flavor-violating signal is slepton pair production followed by decays to the final state $e^-\mu^-\tilde{\chi}_0^0\tilde{\chi}_0^0$. In fact, the backgrounds are so small in $e^-e^-$ mode that $\mu^-\mu^-$ final states may also be used [3].

Assuming excellent lepton charge identification and a hermetic detector, there are essentially no backgrounds for the RR beam polarization. For LR (LL), the dominant background is $e^-\nu W^-$ with cross section 43 (400) fb [11], where the electron is required to have rapidity $\eta < 3$ and the branching fraction of $W^- \rightarrow \mu^-\bar{\nu}_\mu$ has been included. Thus, without any additional cuts, if both beams are 90% (95%) right-polarized, the background is reduced to 12 (5.1) fb, and the required $5\sigma$ signal is 2.4 (1.6) fb. Cross sections for $e_R^-e_R^-\rightarrow e^-\mu^-\tilde{\chi}_0^0\tilde{\chi}_0^0$
are given in Fig. ???. This proves to be the most sensitive mode considered so far, probing mixing angles with \( \sin \theta_R = 0.02 \) and probing parameter space for which \( B(\mu \rightarrow e\gamma) = 10^{-15} \) \( (10^{-12}) \) for \( \bar{\ell} = 2 \) (50).

Finally, we note that once lepton flavor violation is detected, the next step will be to identify its sources and measure it precisely. For simplicity, we have chosen scenarios in which the flavor-violating signal results solely from \( W_{eR} \) mixing. This analysis may be applied to \( W_{eL} \) mixing with the analysis of left-handed sleptons. Of course, in more general settings, flavor-violating signals from both \( W_{eL} \) and \( W_{eR} \) mixing may be accessible. For example, if both left- and right-handed charged sleptons are available, the flavor-violating cross section in \( e^+e^- \) mode may depend on both mixings. However, in the \( e^-e^- \) mode, one can isolate the flavor violation to either \( W_{eL} \) or \( W_{eR} \) by polarizing both beams, a considerable aid in disentangling the flavor-violating matrices.

If sleptons are discovered at the NLC, the \( e\mu \) signal will provide the most powerful probe of flavor violation mixing between the two lightest generations. Many theories of flavor will be probed: for example, those that give \( W_{12} = \sqrt{m_e/m_\mu} \), in analogy to \( V_{us} = \sqrt{m_d/m_s} \). Although it is possible that lepton flavor is exactly conserved, many unified theories give \( W_{12} \) and \( \Delta m^2 \) large enough to be detected, as, for example, in Ref. [12]. Important tests are also possible in the third generation: \( \tau \rightarrow \mu\gamma \) \( (\tau \rightarrow e\gamma) \) do not give bounds on \( W_{32} \) \( (W_{31}) \), (although \( \mu \rightarrow e\gamma \) constrains their product,) but can be probed down to approximately 0.2 (0.05) at the NLC [3].

We are grateful to H. Murayama for many useful discussions. L.J.H. thanks M. Peskin for asking whether lepton flavor violation mixing angles might be directly probed at the NLC. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE–AC03–76SF00098 and in part by the NSF under grant PHY–95–14797. The work of N. A.-H. is supported by NSERC. J.L.F. is supported by a Miller Institute Research Fellowship.