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Photons interaction with a homogeneous constant magnetic field.

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1. Analyticity properties in the variable $\sqrt{e^2 h_{\mu\nu} h_{\mu\nu}}$

In the previous papers ¹⁾ we have studied the interaction of a photon field with a homogeneous and constant electromagnetic field obeying to the conditions $\tilde{h}_{\mu\nu} \tilde{h}_{\mu\nu} = 0^{*})$, $h_{\mu\nu} h_{\mu\nu} > 0$.

It has been shown that at the order α and by using the Green functions exact in the $h_{\mu\nu}$ field ²⁾ such an interaction can be described in terms of two indices of refraction, for which one can show to hold the dispersion relations in the photon energy, which are valid for the material media. The absorptive part of the index of refraction is connected with the existence of the process $\gamma \rightarrow e^+ e^-$ and its threshold occurs at the photon energy $\omega = 2m$. We have also shown that for the frequencies under the threshold of the process $\gamma \rightarrow e^+ e^-$ both indices of refraction can be developed in power series in $\sqrt{e^2 h_{\mu\nu} h_{\mu\nu}}$ which converge only asymptotically. We show now that when $\omega > 2m$, the development in power series of

$\sqrt{e^2 h_{\mu\nu} h_{\mu\nu}}$, loses also this property then it loses any meaning independently of the magnitude of $\sqrt{e^2 h_{\mu\nu} h_{\mu\nu}}$.

In fact, if one had used the perturbative method not only on the α variable but also on the $\sqrt{e^2 h_{\mu\nu} h_{\mu\nu}}$ variable, the absorptive part of the refraction indices would have been identically zero because the Fourier transform of $h_{\mu\nu}$, $h_{\mu\nu}(k)$, is different from zero only at $\tilde{k} = k_0 = 0$ and the process $\gamma \rightarrow e^+ e^-$ is forbidden at each order; and this requires that the power series development in $\sqrt{e^2 h_{\mu\nu} h_{\mu\nu}}$ has zero radius of convergence. This result is related to the vacuum polarization due to a constant and homogeneous electric field. In a perturbative approach in the electric field variable no pair creation occurs and the vacuum is stationary; on the contrary

*) $\tilde{h}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} h_{\alpha\beta}$, where $\epsilon_{\mu\nu\alpha\beta}$ is the complete antisymmetric tensor of fourth rank.

a calculation which uses the Green function modified by the presence of the electric field shows at the same time the vacuum instability and the vanishing radius of convergence of the perturbative series in the electric field ²⁾.

II. Branch points of the indices of refraction.

From the integral expression of reference ¹⁾ the branch points of the index of refractions occur at the photon energies

$$\omega_n = 2 \sqrt{m^2 + n \sqrt{e^2 h_{\mu\nu} h_{\mu\nu}}} \quad (1)$$

On the other hand the energy levels of an electron in a magnetic field are

$$\pm \sqrt{p_{||}^2 + m^2 + n \sqrt{e^2 h_{\mu\nu} h_{\mu\nu}}} \quad (2)$$

where $p_{||}$ is the unquantized momentum along the direction of the magnetic field and n is a positive integer or zero. As it must be from (2) follows (1) if one considers that the electron pairs interaction has been neglected, together with the possibility that more complicated processes than $\gamma \rightarrow e^+ e^-$ contribute to the absorptive part.

III. Time life of the process: $\gamma \rightarrow \gamma' + \gamma''$

The results we have now discussed follow from the calculation of the vacuum polarization current in the order $\alpha^{1/2}$. One can go on and calculate the terms of the vacuum current bilinear in the radiation field $\langle J_\mu(x) \rangle_0$, which gives rise to a Hamiltonian of the form

$$H = \int d^4x : \langle J_\mu(x) \rangle_0 A_\mu(x) :$$

where $A_\mu(x)$ is the radiation field and this Hamiltonian is also responsible of the process $\gamma \rightarrow \gamma' + \gamma''$.

Unless the process $\gamma \rightarrow e^+e^-$ which cannot occur if one uses the perturbative theory, the process $\gamma \rightarrow \gamma' + \gamma''$ can occur without the intervention in the energy momentum balance of the constant and homogeneous field provided that the two end photons are emitted in the same direction of the initial photon; although this is far from being a convincing justification of the procedure, we will limit ourselves to the calculation of the S-matrix element of the transition $\gamma \rightarrow \gamma' + \gamma''$ in the lowest order of the external field. The phase space ρ_N of N photons with energy E and momentum Q is

$$\rho_N \equiv \lim_{m^2 \rightarrow 0} \int \prod_i d^3p_i \delta(E - \sum_i \sqrt{p_i^2 + m^2}) \delta(Q - \sum_i p_i) = \frac{\pi^{(N-1)}}{2^{N-1}} \frac{(E - |Q|)^{3N-4}}{(2N-1)!} \sum_{m=0}^{2N-2} \frac{(2N-4-m)!}{m! (2N-2-m)! (3N-4-m)!} \left(\frac{2|Q|}{E - |Q|} \right)^m$$

*)

*) This formula has been obtained by Dr. R. Hagedorn, to whom I express my thanks.

which shows that in the limit $E - |Q| \rightarrow 0$, ρ_N is different from zero only if $N=2$; i.e. the process in which a photon decays into N photons in the presence of external sources which do not contribute to the energy momentum balance has a phase space different from zero only if the final photons are two.

The matrix element of the process $\gamma \rightarrow \gamma' + \gamma''$ in the vacuum vanishes on account of the charge conjugation invariance, which entails the following transformation properties of the electrons propagator

$$\gamma \rightarrow \gamma^T, \quad S_F(x) \rightarrow S_F^T(-x) \quad (3)$$

where T means transposition.

In the presence of a constant and homogeneous electromagnetic field (3) is true only if the transposition is applied also to the tensorial indices; i.e. if the magnetic field is reversed. In terms of circular polarization we get the following squared matrix elements, where k, k', k'' are the components of the momenta of the three photons perpendicular to the direction of the magnetic field

$$|\langle \alpha k', \beta k'' | S | \alpha k \rangle|^2 = \frac{\pi^4}{648} \frac{e^8 h_{\mu\nu} h_{\mu\nu}}{2 m^4 \omega \omega' \omega''} k''^2, \quad \alpha \neq \beta$$

$$|\langle \alpha k', \alpha k'' | S | \beta k \rangle|^2 = \frac{\pi^4}{648} \frac{e^8 h_{\mu\nu} h_{\mu\nu}}{4 m^4 \omega \omega' \omega''} k^2$$

where each of α, β denotes the two circular polarization of the photons.

The fact that only the components of the momenta normal to the direction of the magnetic field is effective for the decay follows from the angular momentum conservation. In fact, if the photon field is propagating along the direction of the magnetic field the rotation invariance around the axis defined by the magnetic field direction implies that the angular momentum must balance among the three photons independently of the magnetic field, and if one analyses the photons in terms of circular polarization, convinces itself that this is impossible.

The time life of the process is

$$\tau^{-1} = \frac{1}{3\pi} \frac{e^8 h_{\mu\nu} h_{\mu\nu} \omega}{2 m^4 (576 \pi^2)^2}$$

where one expects that this formula is true only when the photon energy ω is small.

All that has been said depends critically on the fact that the photon mass is zero; the detection of the process

$$\gamma \rightarrow \gamma' + \gamma''$$

in the case in which the external source does not contribute to the energy momentum balance, would be a proof of the zero mass of the photon.

REFERENCES

- 1) A. Minguzzi, Nuovo Cimento 4, 476 (1956)
6, 501 (1957)
9, 145 (1958)
- 2) J. Schwinger, Phys. Rev. 82, 664 (1951)