Detecting Earth-Mass Planets with Gravitational Microlensing

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1. Introduction

The recent discovery of several giant planets (Mayor & Queloz 1995, Marcy & Butler 1996) has confirmed the existence of planets orbiting main sequence stars other than the Sun. Two of these first 3 giant planets have orbits that were unexpected, and this together with the surprising discovery of planets in a pulsar system (Wolszczan & Frail 1992) demonstrates the importance of observational studies of extra-solar planetary systems. Indirect ground based techniques which detect the reflex motion of the parent star through accurate radial velocity measurements or astrometry are likely to have sensitivity that extends down to the mass of Saturn (\( \sim 100M_\oplus \)) (Butler et al. 1996) or even down to \( 10M_\oplus \) (Shao & Colavita 1992) with interferometry from Keck or the VLT. There is great interest in searching for planets with a masses similar to that of the Earth, and NASA’s new ExNPS program (Elachi 1995) seeks to build a spacecraft capable of imaging nearby Earth mass planets in the infrared. In order to ensure the success of such a mission, we will need to have at least a rough idea of how prevalent planets with masses close to that of the Earth really are.

A ground based gravitational microlensing survey system sensitive to planets down to \( 1M_\oplus \) has been proposed by Tytler (1995). This project would involve both a microlensing survey telescope to detect microlensing events in progress and a world-wide network of follow-up telescopes that would monitor the microlensing lightcurves on a \( \sim \) hourly timescale in search of deviations due to planets. Existing microlensing surveys (Alcock et al. 1993, Aubourg et al. 1993, Udalski et al. 1993, and Alard 1995) have recently demonstrated real time microlensing detection capability (Alcock et al. 1996, Udalski et al. 1994), and two world-wide microlensing follow-up collaborations (Allbrook et al. 1995 and Pratt et al. 1995) are now in operation, but to detect Earth mass planets, more capable survey and follow-up systems will be required.

In this paper, we provide the theoretical basis for this enterprise by calculating realistic microlensing lightcurves and detection probabilities for planets as small as \( 1M_\oplus \). Previous authors (Mao & Paczyński 1991, Gould & Loeb 1992, and Bolatto & Falco 1994) have considered the deviations from the single lens lightcurve due to planets using the point source approximation. This is a poor approximation for planets in the \( 1-10M_\oplus \) mass range, so we have calculated planetary-binary lensing event lightcurves for realistic finite size source stars, and we show that planets in the \( 1-10M_\oplus \) mass range can cause deviations from the standard single lens lightcurve with amplitudes larger than \( 10\% \) which last for a couple hours or more. We calculate planetary detection probabilities based upon a set of assumed event detection criteria and a simple planetary system model loosely based upon the solar system.

2. Microlensing

The only observable feature of a microlensing event is the time variation of the total magnification of all the lens images due to the motion of the lens with respect to the observer and source. The characteristic transverse scale for a lens of mass \( M \) is given by the Einstein ring radius which is the radius of the ring image obtained when the source, lens and observer are collinear. It is given by

\[
R_E = 2\sqrt{\frac{GM D}{c^2}} = 4.03 AU \sqrt{\left(\frac{M}{M_\odot}\right) \left(\frac{D}{2kpc}\right)},
\]

where \( D \) is the “reduced distance” defined by \( 1/D = 1/D_1 + 1/D_2 \). \( D_1 \) and \( D_2 \) are the distances from the observer to the lens and from the lens to the source respectively. For a point mass lens, the amplification of a microlensing event is given by

\[
A = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}, \quad u = \sqrt{u_{\text{max}}^2 + (2(t - t_0)/\hat{I})^2},
\]

where \( u \) is the separation of the lens from the source-observer line of sight in units of \( R_E \), and \( t_0 \) and \( \hat{I} \) refer to the time of peak amplification and the Einstein diameter crossing time respectively.

In a “planetary lensing event,” the majority of the lightcurve is described by eq. (2), but in the region of the planetary deviation we must consider the binary lens case (Schneider et al., 1992, Rhie 1996). If \( \omega \) and \( \epsilon \) denote (in complex coordinates) the source and image positions in the lens plane, the binary lens equation is given by

\[
\omega = z - \frac{1 - \epsilon}{\bar{x} - \bar{x}_s} - \frac{\epsilon}{\bar{x} - \bar{x}_p},
\]

where \( \epsilon \) is the fractional mass of the planet, and \( x_s \) and \( x_p \) are the positions of the star and planet respectively. We work in units of the Einstein radius, \( R_E \),
of the total mass $M$. Eq. (3) has 3 or 5 solutions ($z$) for a given source location, $\omega$.

The Jacobian determinant of the lens mapping (3) is

$$J = 1 - \left| \partial_z \xi \right|^2 ; \quad \partial_z \xi = \frac{1 - \epsilon}{(z - x_s)^2} + \frac{\epsilon}{(z - x_p)^2},$$

and the total amplification of a point source is obtained by summing up the absolute value of the inverse Jacobian determinant calculated at each image:

$$A = \sum_i |J_i|^{-1}. \quad (5)$$

The curve defined by $J = 0$ is known as the critical curve, and the lens mapping (3) transforms the critical curve to the caustic curve in the source plane. By eq. (5), a point source which lies on a caustic will have an infinite magnification. (The singularity at $J = 0$ is integrable, so finite sources always have finite magnifications.) When the source star is in the region of the caustic curve, the magnification will differ noticeably from the single lens case allowing a “planetary” signal to be detected. If the planet mass is of order 1-10 $M_\oplus$, the “caustic region” is comparable to the size of the source star, and the point source approximation is not appropriate.

3. Planetary Lightcurves

Because lensing conserves surface brightness, the magnification of an image is just the ratio of the image area to the source area (which is given by eq. (4) for a point source). For a finite size source, we calculate the lens magnification in the image plane where it is given by the sum of the image area weighted by the limb darkened source profile assumed to have the form: $I(\theta)/I(0) = 1 - 0.6(1 - \cos \theta)$. This avoids the magnification singularities on the caustics in the source plane. We integrate over the images as follows: First, we determine the location of the “center” of each image which is usually the image of the center of the star. If a portion of the stellar disk is inside a caustic when the center is outside, then we must also include an additional double image of the included portion of the star. We then cycle over the included images and build a numerical integration grid centered on each image. These grids are expanded until all the grid boundary points are outside of the images. Sometimes these image grids can include more than one image, and in these cases, the redundant grids are dropped from the calculation.

The source radius projected to the lens plane is given by $r_s \equiv R_\Delta D_d/(R_E(D_d + D_h))$ normalized to the Einstein ring radius. Figure 1 shows some examples of planetary lightcurves calculated for source size $r_s = 0.003$, planetary mass fraction $\epsilon = 10^{-5}$ & $10^{-4}$, and separations of $\ell \equiv |x_p - x_s| = 0.8 \& 1.3$. For a typical Galactic lens of $0.3 \, M_\odot$, the mass fractions $\epsilon = 10^{-5}$ & $10^{-4}$ correspond to planet masses of 1 & 10 $M_\oplus$ respectively. The insets show the effects of varying the source size $r_s$ over the values 0.003, 0.006, 0.013, and 0.03. $r_s$ values of 0.003 and 0.006 correspond to a main sequence turn-off source star lensed by lenses in the disk and bulge respectively while values of 0.013 and 0.03 correspond to a clump giant source lensed by lenses in the disk and bulge. (We have assumed $R_\Delta = 3 R_\odot$ for turn-off stars and $R_\Delta = 13 R_\odot$ for clump giants.)

Figure 3 shows two dimensional plots of the magnification ratio $(A/A_\Delta)$ of the planetary binary lens case to the single lens case for the planetary parameters $\epsilon = 10^{-4}$, $r_s = 0.003$, and $\ell = 0.8 \& 1.3$. Source trajectories are represented by straight lines across these figures, and the $\epsilon = 10^{-4}$ lightcurves shown in Figure 1 correspond to source paths which cross close to the center of these 2-d plots at an angle of $\sin^{-1}0.6 = 36.9^\circ$ from the horizontal lens axis.

4. Planetary Detection Probabilities

Let us define a reasonable set of planetary detection criteria: First of all, the microlensing event must be discovered by the microlensing survey system, and then the planetary deviation must be detected by the microlensing follow-up system. The follow-up system is assumed to observe each lensed star about once per hour with an accuracy of 0.5-1% so that moderate amplitude deviations can be detected. Then, we require that the lightcurve deviate from the single lens lightcurve by more than 4% for a period longer than $\hat{t}/400$ which is about 2.4 hours for a typical event lasting $\hat{t} = 40$ days. This deviation must occur after the event has been detected by the survey system which we take to be after magnification $A = 1.58$ has occurred. (This is the 0.5 magnitude event detection threshold.) Using these detection criteria, we examine all detectable (i.e. $A_{\text{MAX}} > 1.58$) events for a fixed lens star-planet separation, $\ell$, and determine what fraction of these events pass the selection criteria. This
involves examining all possible lines (or lightcurves) on the 2-d deviation plots like the ones shown in figure 3. (These simulated lightcurves are assigned uniform distributions in \(u_{\text{min}}\) and angle.) Our probability results for the 4% threshold are shown in Figure 2 for fractional masses of \(\epsilon \approx 10^{-4} \& 10^{-5}\) and a variety stellar radii. We can see from figure 2 that the detection probability is highest for separations close to the Einstein ring radius, but for \(\ell = 1\) detection is difficult if \(r_1\) is large or if \(\epsilon\) is small. A similar effect also occurs for \(\ell < 1\) and moderately large \(r_s\). This can be understood by considering the amplification contours in the source plane. From Figure 3(a) we can see that the \(\ell < 1\) lens system has a region of negative deviation in the center of two regions containing the caustics which have a positive deviation. A large source star will cover much of this region so that the positive and negative deviations will tend to cancel in the integral over the entire source. For \(\ell = 1\), the positive and negative deviation regions are more closely packed together and this effect is even stronger.

5. A Model Planetary System

In order to translate the results displayed in figure 2 to the probability of detecting planets, we must make some assumptions about the planetary systems that we are searching for. For simplicity, let us define a simple “factor-of-2” model planetary system which has a distribution of star-planet separations that is uniform in \(\log(\ell)\) and has on average one planet for every factor of two in separation from the parent star in the region of interest. The region of interest, or the “lensing zone,” is the interval in \(\ell\) where the lensing detection probability is high: \(0.6 < \ell < 1.5\). For stellar lenses disk or bulge in the mass range \(0.1-1 M_\odot\), the lensing zone will cover about a factor of 2 in transverse distance somewhere in the range \(0.6-6.0\) AU. A virtue of the “factor-of-2” model is that the distribution of planets is not changed by orbital inclination or phase. For a planetary system like our own where the planets’ semi-major axes fall in the range \(0.4-40\) AU, it would be very rare for the orbital parameters to conspire to move the outer planets inside the lensing zone, so our assumption that the planetary system always extends through the lensing zone is reasonable. We have used the “factor-of-2” model to calculate the probability of planet detection for a variety of stellar radii and detection thresholds assuming that the planets have a unique mass. The results are summarized in Table 1.

Table 1 can be used to estimate the number of planets that might be detected with a second generation microlensing survey and follow-up system similar to that discussed by Tytler (1995). Such a system might be able to discover 200 lensed turn-off stars and 50 lensed clump giants per year! We’ll assume that practical difficulties such as weather serve to reduce the actual detection probability to 50% of the theoretical values shown in Table 1. Then with planetary detection thresholds of 4% for turn-off stars and 2% for clump giants (which are brighter), we would expect to detect about 19 \(10 M_\odot\) planets or 3 \(1 M_\odot\) planets per year if every lens system had a “factor-of-2” planetary system with planets of these masses. (These numbers are roughly independent of the fraction of the lenses in the bulge or disk.) If a third of all lenses have no planets, a third have \(1 M_\odot\) planets and the remaining third have \(10 M_\odot\) planets, then we would expect to detect 6 \(10 M_\odot\) planets and a single \(1 M_\odot\) planet every year. More than half of the \(10 M_\odot\) planetary lightcurves and a third of the \(1 M_\odot\) lightcurves would have deviations larger than 10%. Clearly, a null result from an eight year survey of this magnitude would be a highly significant indication that planetary systems like our own are rare.

6. Discussion

In the previous section, we have shown that a significant number of planets with masses down to \(1 M_\odot\) can be detected via gravitational microlensing if microlensing events towards the Galactic bulge are monitored ~ hourly with photometric precision of 0.5-1.0% which is readily achievable in crowded stellar images.

We can also use the results of our probability calculations to help determine the optimal planetary search strategy. For example, given a large number of events to monitor for planetary deviations and a limited amount of observing time, how long should we follow each event? The probabilities given in Table 1 assume that each event is followed from event detection at \(A = 1.58\) until \(A\) drops to 1.13, but if we stop the follow-up observations when \(A > 1.34\), then we will only be sensitive to planetary deviations from planets in the interval \(0.62 < \ell < 1.62\). Inte-

\footnote{These estimates are based upon an assumed \(2 \text{m}\) survey telescope which could monitor \(30 \text{ million} \) bulge stars over a \(250 \text{ day} \) bulge season with an assumed lensing rate of \(\Gamma = 2.4 \times 10^{-5} \text{ events/yr}\) and a 50% detection efficiency.}
igration over the curves in figure 2 indicates that this will reduce the chance of detecting a planet by 5-10% (for the “factor-of-2” model), but the total number of observations required drops by 27%. Thus, if the capacity of the follow-up system is saturated, it is best to concentrate follow-up observations on events with $A > 1.34$. This effect is basically geometric: planets that are outside the lensing zone ($\ell > 1.6$) tend to give rise to “isolated” events that aren’t associated with a stellar lensing event detected by the survey system.\footnote{Isolated planetary lensing events might be detected by microlensing surveys, but the detection efficiency and variable star background rejection would be quite poor.} It is optimal to search for planets at $\ell < 1.6$ where they would “modulate” a detectable stellar lensing lightcurve.

Our results also suggest that it will be easier to detect Earth mass planets by monitoring turn-off star lensing events than giant star events. (Gould and Welch (1995) have shown that combined infrared and optical observations may allow the detection of earth mass planets in giant star lensing events, however.)

We’ve established that low mass planets can be detected, but we should also address what can we learn about each planet that is discovered through microlensing. Planetary lightcurve deviations would be detected in real time so that observations can be repeated every few minutes during the planetary deviation. The lens parameters $\ell$ (the separation perpendicular to the line of sight in units of $R_E$) and the mass ratio $\epsilon$ can generally be determined from gross features of the lightcurve. $\ell$ is easily determined (up to a 2-fold ambiguity) from the amplification that the unperturbed lightcurve would have in the deviation region, and the mass ratio $\epsilon$ can be determined from the timescale of the planetary deviation. The 2-fold ambiguity in $\ell$ is also easily resolved in most cases by the shape of the lightcurve deviation as can be seen in figures 1 and 3. For $\ell < 1$, the the deviation region consists of positive deviation regions surrounding the two caustic curves with a long trench of negative deviations in between. This leads to light curves with regions of large negative perturbations surrounded by regions of smaller positive perturbations. For $\ell > 1$, the situation is reversed and the dominant perturbation is a central positive one which has regions of small negative perturbations on either side of it.

Another parameter that may be measured is the angular Einstein ring radius of the planet itself. This comes about because the ratio of the this radius to the angular radius of the star is the parameter which describes the finite source effects. For planets of Earth mass, the finite source effects are almost always important, so in principle, this parameter may be measurable in most events.

In summary, we have calculated realistic lightcurves for microlensing events where the lens star has a low mass planetary companion, and we have shown that planets with masses as small as $1M_\oplus$ can be detected via gravitational microlensing. Thus, gravitational microlensing is the only ground based method that has been shown to be sensitive to Earth mass planets.

Acknowledgements

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Table 1
Planetary Detection Probabilities

<table>
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<tr>
<th>$r_s$</th>
<th>$\epsilon$</th>
<th>$P(2%)$</th>
<th>$P(4%)$</th>
<th>$P(10%)$</th>
<th>$P(20%)$</th>
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</thead>
<tbody>
<tr>
<td>0.003</td>
<td>$10^{-4}$</td>
<td>0.188</td>
<td>0.144</td>
<td>0.094</td>
<td>0.052</td>
</tr>
<tr>
<td>0.006</td>
<td>$10^{-4}$</td>
<td>0.238</td>
<td>0.159</td>
<td>0.085</td>
<td>0.043</td>
</tr>
<tr>
<td>0.013</td>
<td>$10^{-4}$</td>
<td>0.201</td>
<td>0.118</td>
<td>0.052</td>
<td>0.014</td>
</tr>
<tr>
<td>0.03</td>
<td>$10^{-4}$</td>
<td>0.120</td>
<td>0.035</td>
<td>0.012</td>
<td>0.000</td>
</tr>
<tr>
<td>0.003</td>
<td>$10^{-5}$</td>
<td>0.060</td>
<td>0.034</td>
<td>0.014</td>
<td>0.004</td>
</tr>
<tr>
<td>0.006</td>
<td>$10^{-5}$</td>
<td>0.052</td>
<td>0.026</td>
<td>0.005</td>
<td>0.002</td>
</tr>
<tr>
<td>0.013</td>
<td>$10^{-5}$</td>
<td>0.019</td>
<td>0.008</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>0.03</td>
<td>$10^{-5}$</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
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</table>

Planetary detection probabilities $P$ are shown as a function of the deviation threshold for different values of the source star radius $r_s$, and planetary mass fraction $\epsilon$. Idealized “factor-of-2” planetary systems with one planet per factor of 2 in distance from the lens star are assumed. A planet is considered to be detected if it deviates from the single lens light curve by more than the threshold for a period of time longer than $\tilde{t}/400$. The $r_s$ values of 0.003 and 0.006 correspond to a turn-off source star with disk and bulge lenses respectively, while the $r_s$ values of 0.013 and 0.03 correspond to a giant source with disk and bulge lenses.
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Fig. 1.— Microlensing lightcurves which show planetary deviations are plotted for mass ratios of $\epsilon = 10^{-4}$ & $10^{-5}$ and separations of $\ell = 0.8$ & 1.3. The main plots are for a stellar radius of $r_\ast = 0.003$ while the insets show light curves for radii of 0.006, 0.013, and 0.03 as well. The dashed curves are the unperturbed single lens lightcurves, $A_0(t)$. For each of these lightcurves, the source trajectory is at an angle of $\sin^{-1} 0.6$ with respect to the star-planet axis. The impact parameter $u_{\text{min}} = 0.27$ for the $\ell = 0.8$ plots and $u_{\text{min}} = 0.32$ for the $\ell = 1.3$ plots.
Fig. 2.— The planetary deviation detection probability is plotted for different values of the planetary mass ratio, \( \epsilon \), and the stellar radii, \( r_s \). A planet is considered to be “detected” if the lightcurve deviates from the standard point lens lightcurve by more than 4\% for a duration of more than \( t/400 \). Only the portion of the lightcurve after the alert trigger at \( A = 1.58 \) is searched for planetary deviations.

Fig. 3.— This plate shows the magnification ratio between the planetary lensing case (\( A \)) and the single lens case (\( A_0 \)) as a function of source position for \( \epsilon = 10^{-4} \), \( r_s = 0.003 \) and \( \ell = 0.8 \) (a) & \( \ell = 1.3 \) (b).