A dark matter solution from the supersymmetric axion model

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Abstract

We study the effect of the late decaying saxino (the scalar superpartner of the axion) and find out that there is a possible dark matter solution from a class of supersymmetric extensions of the invisible axion model. In this class of models, the saxino which decays into two axions acts as the late decaying particle which reconciles the cold dark matter model with high values of the Hubble constant. Recent observations of the Hubble constant are converging to $H_0 = 70 - 80 \text{ km sec}^{-1} \text{ Mpc}^{-1}$, which would be inconsistent with the standard mixed dark matter model. This class of models provides a plausible framework for the alternative cold dark matter plus late decaying particle model, with the interesting possibility that both cold dark matter and the extra radiation consist of axion.

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Cosmological and astrophysical data accumulated in past several years made it possible to test the theories of large scale structure formation in our universe. The COBE-DMR discovery of cosmic microwave background (CMB) anisotropy provided the means for the accurate normalization of theories of structure formation. The study of galaxy correlations supplied complementary data on smaller scales. With the scale invariant initial spectrum, they manifested that the pure hot dark matter model cannot explain the small scale structure of the universe and the pure cold dark matter (CDM) model shows best fit at the values \( \Omega_0 h = 0.25 \) \[1\] \((\Omega_0 = \text{(the present energy density)}/(\text{the critical energy density}) \text{ and } h = \text{(the present value of the Hubble constant)}/100 \text{ km sec}^{-1} \text{ Mpc}^{-1})\), which is much smaller than the presently favored values. The mixed dark matter (MDM) model attracted broad attention because it gives good fit for \( \Omega_\nu = 0.2 \) and \( \Omega_{\text{CDM}} = 0.8 \) at \( h = 0.5 \) \[2\].

For better test of models, however, the accurate values of \( \Omega_0 \) and \( h \) have been required. The observed value of \( \Omega_0 \) is ranging from 0.2 to \( \sim 1 \), still plagued by large systematic uncertainty. Several recent investigations of the Hubble constant tend to give higher values, in the range of \( h = 0.7 - 0.8 \) \[3\]. If we stick to the theoretical prejudice, \( \Omega_0 = 1 \), we have at least two serious problems. First, we cannot fit the power spectrum curve even in the MDM model. Second, it would be inconsistent with the estimated lower bound on the age of the universe from the oldest globular clusters. An \( \Omega_0 = 1 \) universe has an age of only \( t_0 = 6.5 h^{-1} \) Gyr, giving \( t_0 < 9.3 \) Gyr for \( h > 0.7 \). On the other hand, the observed value of the age of the oldest globular cluster is around 15 Gyr \[4\]. Still there are a few possible way to relax this bound, but it seems not to be less than 11 Gyr \[5\]. Assuming that the data from the globular cluster can be relaxed by some mechanism and considering error bars in Hubble constant observations, \( h = 0.6 \) would be marginally allowed. However, the standard MDM model is inconsistent with large scale structure data even for \( h = 0.6 \).

Introduction of the small cosmological constant corresponding to \( \Omega_\Lambda = 0.7 - 0.8 \) alleviates the universe age crisis mildly and can revive good features of the CDM model with \( \Omega_\Lambda + \Omega_{\text{CDM}} = 1 \) (the CDM+\( \Lambda \) model). However, such a small value of cosmological constant is still a theoretically knotty subject. One way to keep the cosmological constant to be zero and
maintain good features of the CDM model is introducing the late decaying particle (LDP) (the CDM+LDP model) [6]. In the CDM+LDP model, the LDP decays into very light and weakly interacting particles in the late stage of universe. This new radiation dominates the radiation energy of universe and would delay the beginning of the matter dominated epoch, which is necessary for the CDM model to be consistent with structure formation data even at the high values of $h$. But there are severe restrictions on the mass, lifetime and interactions of the LDP coming from structure formation data and preserving the successful predictions of nucleosynthesis and the CMB spectrum. We find that these can be met for the axion supermultiplet in supersymmetric extensions of the well-known axion model. It can provide cold dark matter with the late decaying particle consistently in a class of models.

The axion is originally introduced to solve the strong CP problem. The axion model has an interesting cosmological effect. The axion produced from the initial vacuum misalignment has very small momentum and therefore is a good candidate of the cold dark matter.

In the supersymmetric extension of the axion model, the axion supermultiplet ($\Phi$) consists of the axion ($a$), its real scalar superpartner saxino ($s$), and the fermionic superpartner axino ($\tilde{a}$). The cosmological impacts of these two additional weakly interacting particles were studied before [7,8]. In this paper, we present unappreciated possibility of saxino cosmology, in which the axino decays to two axions and acts as a LDP.

After supersymmetry breaking, the axino and the saxino get their masses. In global supersymmetry, they remain massless at the tree-level. In the supergravity model with supersymmetry broken in the hidden sector, they are expected to gain masses of order $m_{3/2}$. However the axino and saxino masses are dependent on the specific forms of the axion sector superpotential as well as the Kähler function in the context of supergravity [9]. A model-dependent analysis is necessary to evaluate to the masses of axion supermultiplet. With the minimal Kähler function, the saxino mass is roughly of order of $m_{3/2}$ in most cases. In no scale supergravity, it remains massless at the tree level and gains radiative corrections of order $10 \sim 100\text{MeV}$.

The effective Lagrangian describing the interactions of the axion supermultiplet is given
\[ \mathcal{L} = \sum_i \nu_i^2 \exp \left[ \frac{q_i (\Phi + \bar{\Phi})}{F_a} \right] \left. \frac{\alpha_c}{16\pi F_a} \Phi W_\alpha W^\alpha \right|_{D} + \left. \frac{\alpha_c}{16\pi F_a} \Phi W_\alpha W^\alpha \right|_{F} = \left( 1 + \frac{\sqrt{2x}}{F_a} \right) \left( \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{1}{2} \partial_\mu s \partial^\mu s + \frac{i}{\alpha} \gamma_\mu \partial^\mu \tilde{a} \right) + \frac{\alpha_c}{8\pi F_a} \left( a F_{\mu \nu}^a \bar{F}^{a \mu \nu} + \bar{a} \gamma_\mu \sigma_{\mu \nu} \bar{a} F^{a \mu \nu} + s F_{\mu \nu}^a F^{a \mu \nu} + \cdots \right) - \frac{x}{F_a} \partial_\mu a \frac{\partial^\mu a}{4} \tilde{a} + \cdots. \]  

(1)

where \( F_a \) is the axion decay constant and \( \alpha_c \) is color coupling constant. The model-dependent parameter \( x \) is given by

\[ x = \sum_i q_i^2 v_i^2 / F_a^2 \]

where \( q_i \) and \( v_i \) are the \( U(1)_{PQ} \) charges and the VEVs of the fields in the axion sector. It is of order 1, in general. At present particle phenomenology, astrophysical and cosmological observations restrict the range of the axion decay constant to be

\[ 10^{10} \text{GeV} \lesssim F_a \lesssim 10^{12} \text{GeV}. \]

(2)

From these couplings we obtain the saxino decay widths to two different channels

\[ \Gamma_{s \rightarrow 2g} = \frac{\alpha_c^2 m_s^3}{128 \pi^3 F_a^2}, \]

(3)

\[ \Gamma_{s \rightarrow 2a} = \frac{x^2 m_s^3}{64 \pi F_a^2}, \]

(4)

where \( m_s \) is the saxino mass. Depending on the model dependent parameter \( x \), the main decay mode can be changed and the effect of the saxino decay appears quite different. When \( s \rightarrow 2g \) is the main decay mode, the lifetime is given by

\[ \tau_s = 2.6 \times 10^5 \text{sec} \left( \frac{\alpha_c}{0.1} \right)^2 \left( \frac{F_a}{10^{11} \text{GeV}} \right)^2 \left( \frac{100 \text{MeV}}{m_s} \right)^3. \]

(5)

The resulting gluons thermalize and dump some amount of entropy to thermal bath [8].

Previous studies on the saxino decay have been focused on the saxino heavier than 10 GeV because it has been believed that the saxino should decay before nucleosynthesis. The reason is that the high energy radiation from the late decaying saxino would destroy the light nuclei which were made during the nucleosynthesis era. However when \( x \gg 10^{-3} \), as expected in many models, the main decay mode of the saxino is \( s \rightarrow 2a \) with the lifetime
\[
\tau_s = 1.3 \times 10^3 \text{sec } x^{-2} \left( \frac{F_a}{10^{11} \text{GeV}} \right)^2 \left( \frac{100 \text{MeV}}{m_s} \right)^3.
\]  

(6)

The crucial difference is that the resulting axions interact very weakly and cannot be thermalized. They do not dump entropy to thermal bath of the universe and do no harm to the light nuclei made during nucleosynthesis. However, their effect appears gravitationally through their energy density. It affects the result of nucleosynthesis, which makes us consider the different range of the saxino mass.

For further discussion, we need to know the relic abundance of the saxino. There are two distinct contributions: thermal and non-thermal. First we consider the thermal relic. Without inflation, the saxino would be hot thermal relic which decouples at the temperature, as estimated by Rajagopal et al. [7],

\[
T_{\text{dec}} \simeq 10^9 \text{GeV} \left( \frac{F_a}{10^{11} \text{GeV}} \right)^2 \left( \frac{\alpha_c}{0.1} \right)^{-3}.
\]  

(7)

Then its relic density is given by

\[
Y_s \equiv \frac{n_s}{s} = 0.278 \frac{g_{\text{eff}}}{g_{\ast s(T_{\text{dec}})}} \simeq 1.2 \times 10^{-3},
\]  

(8)

where \(n_s\) is the number density of the saxino and \(s = (2\pi^2/45)g_{\ast s}T^3\) is the entropy density.

In the inflationary scenario, the reheating temperature is an important parameter in our discussion because the decoupling temperature of the saxino is rather high. At present, the relevant upper bound on the reheating temperature comes from the gravitino production and is quoted as [11]

\[
T_{\text{reh}} \lesssim 10^9 \text{GeV}
\]  

(9)

for reasonable values of the gravitino mass. If we use the MSSM value \(\alpha_c(10^{11} \text{GeV}) \simeq 1/20\) in the Eq. (7), the reheating temperature seems lower than the decoupling temperature. However, in the heavy quark axion model, the heavy quark and saxino coupling is about \((\text{mass of heavy quark})/F_a\), which is much stronger than any other interaction with saxino. In this case, the axion will not decouple until the heavy quark decouples from the heat.
bath. Therefore the decoupling temperature could be lower than reheating temperature. Though the decoupling temperature is greater than the reheating temperature, saxino could be produced from the thermal bath through the reactions like $q\bar{q} \leftrightarrow sg$ and $gg \leftrightarrow sg$. Then the relic density is estimated to be

$$Y_s \simeq 0.75 \times 10^{-4} \left( \frac{\eta \alpha_c^3}{10^{-4}} \right) \left( \frac{F_a}{10^{11} \text{GeV}} \right)^{-2} \left( \frac{T_{\text{reh}}}{10^9 \text{GeV}} \right),$$

where $\eta$ is phase space factor times the number of channels, and correction from the thermal effects which is roughly of order 1.

As non-thermal relic, there can be a coherent oscillation of the saxino field. The coherent oscillation begins when the supersymmetry breaking saxino mass becomes comparable to the Hubble parameter. We estimate the relic density from coherent oscillation to be

$$Y_s \simeq 5 \times 10^{-8} \left( \frac{m_s}{10 \text{MeV}} \right)^{-1/2} \left( \frac{s_1}{10^{11} \text{GeV}} \right)^2,$$

where $s_1$ is the initial value of the saxino when the oscillation begins. The misalignment production is negligible for the saxino when $F_a < 10^{13} \text{ GeV}$.

From the relic abundance of the saxino, we obtain strong bound on the saxino mass comes from nucleosynthesis. To preserve successful nucleosynthesis, the additional energy density at the time of nucleosynthesis should be smaller than, say, $\Delta N_\nu$ times the energy density of one neutrino species. For the saxinos and decay produced axions, this corresponds to $mY < 1.3\Delta N_\nu(T_D/g_{*s(T_D)})$ for $T_D > T_{NS}$ and $mY < 1.3\Delta N_\nu(T_{NS}/g_{*s(T_{NS})})$ for $T_D < T_{NS}$ where $T_D = 0.55g_{*T_D}\sqrt{M_P\Gamma}$ is the temperature at decay time and $T_{NS} \approx 0.8 \text{ MeV}$ the temperature at the time of nucleosynthesis. Combining these, we obtain the axino mass bound

$$m_s > \frac{4 \text{TeV}}{x^2 \Delta N_\nu^2} \left( \frac{g_{*T_D}}{100} \right)^{5/2} \left( \frac{Y_s}{10^{-3}} \right)^2 \left( \frac{F_a}{10^{11} \text{GeV}} \right)^2,$$

or

$$m_s < 107 \text{MeV} \Delta N_\nu \left( \frac{Y_s}{10^{-3}} \right)^{-1}.$$
If we adopt $\Delta N_e = 0.3$ [12], the upper bound in the Eq. (13) would be 30 MeV in the hot thermal relic case. Though the low reheating temperature weakens it, it is still difficult to get such a small mass in models where the saxino gets its mass at tree-level. However in models like no scale supergravity models, where the axino and the saxino have zero tree level masses and get masses by radiative corrections, the $10 \sim 100$ MeV mass of the saxino naturally occurs in connection with the $\sim$ keV order axino mass.

Then the mass, lifetime, decay products and relic abundance of the saxino nicely fit with the requirements of the LDP. The condition the late decaying particle should satisfy is [6]

$$\left( \frac{\tau}{\text{sec}} \right) \left( \frac{m_Y}{\text{MeV}} \right)^2 \simeq 0.55 \left[ \left( \frac{h}{0.25} \right)^2 - 1 \right]^{3/2}.$$  

(14)

For the hot saxino thermal relic, using the Eq. (8), it becomes

$$\frac{190}{x^2} \left( \frac{m_s}{10\text{MeV}} \right)^{-1} \left( \frac{F_a}{10^{11}\text{GeV}} \right)^2 \simeq 5.7$$

(15)

for the $h = 0.6$ case. Considering the bounds on $F_a$ and $m_s$ (the Eqs. (2) and (13)), this can be met in models with $x \simeq 1$ and $F_a \sim 10^{10}$ GeV. If the reheating temperature is lower than decoupling temperature of saxino, we should use the Eq. (10). Then we obtain

$$\frac{7.0}{x^2} \left( \frac{m_s}{100\text{MeV}} \right)^{-1} \left( \frac{F_a}{10^{11}\text{GeV}} \right)^{-2} \left( \frac{T_{\text{reh}}}{10^9\text{GeV}} \right)^2 \simeq 5.7.$$  

(16)

In this case, depending on the reheating temperature $T_{\text{reh}}$, the condition can be satisfied in models with small $x$ and $F_a \simeq 10^{12}$ GeV.

The CDM+LDP model is distinguished from the CDM+$\Lambda$ model by the existence of the intermediate matter domination era before the final matter domination era. This temporary matter domination occurs because the saxino dominates the universe before it decays into the axions. Due to the temporary matter domination, there is one more length scale [6]

$$\lambda_{\text{EQ1}} \simeq 8 \times 10^{-2} \left( \frac{\text{MeV}}{m_Y} \right) \text{kpc}.$$  

(17)

Objects on this and smaller scales would collapse at high red shift and the CDM+LDP model has more structure on these scales. For hot saxino thermal relic, this scale is
\[
\lambda_{EQ1} \simeq 6.7 \text{kpc} \left( \frac{m_s}{10 \text{MeV}} \right)^{-1},
\]

and for regenerated saxinos,

\[
\lambda_{EQ1} \simeq 11 \text{kpc} \left( \frac{m_s}{100 \text{MeV}} \right)^{-1} \left( \frac{F_a}{10^{11} \text{GeV}} \right)^2 \left( \frac{T_{\text{reh}}}{10^9 \text{GeV}} \right)^{-1}.
\]

The existence of this scale might be better in explaining the abundance of the high red shift quasars and globular clusters.

We should check one more constraint on the LDP. Though the main decay mode of the saxino is \( s \to 2a \), the radiative branching ratio should be sufficiently suppressed not to spoil the nucleosynthesis result by photodestruction and photoproduction of light nuclei. For \( m_s > 10 - 50 \text{ MeV} \) and \( \tau_s > 10^6 \text{ sec} \), it is roughly \( rm_s Y_s < 10^{-9} \text{ MeV} \) where \( r \) is the radiative branching ratio [13]. If the saxino is lighter than 100 MeV, the saxino cannot decay into hadrons, but can decay to two photons or two leptons. These decay channel is highly suppressed if the saxino is much heavier than 1 MeV. Decay to two photons is also suppressed by \( \alpha^2 \). Two lepton mode is suppressed by loop factor in the heavy quark axion model.

Finally we discuss the possible CDM candidates in the above context. Actually any CDM is good. But because the supersymmetric axion model has two candidates, the axion and the axino, in itself, we discuss only about them. Though the mass of the axion is very small \( (m_a \simeq \Lambda_0^2/\bar{F}_a) \), the large amount of axions can exist in our universe in the form of the coherent oscillation arising from the initial misalignment of the vacuum [14]. Its present energy density grows as \( F_a \) gets larger and becomes as large as the critical energy density around \( F_a = 10^{12} \text{ GeV} \). The upper bound on \( F_a \) comes from the critical energy density. When the reheating temperature is larger than the decoupling temperature of the saxino, \( F_a \simeq 10^{10} \text{ GeV} \) is necessary for the saxino to be the LDP. But when the reheating temperature is lower, \( F_a \simeq 10^{12} \text{ GeV} \) is desirable because the saxino can be the LDP even in models with small \( x \). Therefore, we have an interesting possibility that CDM consists of the axions, and the saxino acts as the LDP, which decays to provide additional radiation which consists again of axions.
The decoupling temperature and the relic number density of the axino are quite similar to those of the saxino. When \( T_{\text{reh}} > T_{\text{dec}} \), the stable \( \sim 2 \text{ keV} \) mass axino can be so called warm dark matter [7,9]. When \( T_{\text{reh}} \) is lower, the corresponding mass is

\[
m_\tilde{a} \simeq 30 \text{ keV} \left( \frac{F_a}{10^{11} \text{ GeV}} \right)^2 \left( \frac{T_{\text{reh}}}{10^9 \text{ GeV}} \right)^{-1},
\]

and the axino becomes CDM. Interestingly enough, this axino mass range for which the axino is CDM and the saxino mass range for which the saxino is a LDP simultaneously occur in models where the axino and the saxino get their masses by radiative corrections [7,9].

In conclusion, we point out that the axion supermultiplet which solves the strong CP problem can provide cold dark matter with \( \Omega_{\text{CDM}} = 1 \) and the late decaying particle simultaneously within the presently allowed parameter range if the 10 MeV \( \sim 1 \text{ GeV} \) saxino decays mainly into two axions. All the discussions are based on the flat, cosmological constant free universe. In this context, this model is one of a few possible models which satisfy the large scale structure and the large Hubble constant. It can survive the universe age problem if we allow the marginal value 11 Gyrs. If the cosmological and astrophysical observations are refined, the validity of this model will be proven in the near future.

**ACKNOWLEDGMENTS**

We thank K. Choi, E.J. Chun and P. Sikivie for helpful discussions. This work was supported in part by KOSEF and IFT at University of Florida(S.C.) and the Ministerio de Educación y Ciencia under research grant(H.B.K.).
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