twist division.

Thus compactification and realizing $SO(10)$ SUSY GUT in the observed sector under a $F \times F$ heterotic string. We find $Z_6$ orbifold models preserving the duality under a dual in any sense. The observed sector and the hidden sector are dual on the 10-dim.

**Abstract**

Dual is a promising key word in the particle physics at present. The string theory is

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$SO(10)$ Nodes on $Z_6$ Orbifold with Dual Wilson Line

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Duality appears in various phases of string theory. As for the gauge group of the ten dimensional heterotic string theory, the weight lattices of gauge degrees of freedom of $E_8$ (observable) sector and $E_8'$ (hidden) sector are dual so that it is natural for a four dimensional string theory with this duality to result from the theory.

It is possible to preserve the duality in the toroidal compactification which makes use of $Z_6$ translation group to reduce the space-time dimensions. When we represent the gauge degrees of freedom as $E_8 \times E_8'$ root momenta $P^I (I = 1...16)$ of the left-moving bosonic variables, the representation of the translation group in the gauge degrees of freedom is so-called a Wilson line $a^I$ and physical fields must satisfy the $Z_6$ singlet condition $\sum P^I a^I = 0 \mod 1$. Such a dual Wilson line as $a = (a, -a)$ where $a$ is an 8-d shift vector on top of the $E_8$ lattice brings about same gauge group with mutually dual weight lattice to the two sectors.

The toroidal compactification, however, leads to four dimensional string theories with gauge groups of rank 22 and $N=4$ supergravity. In order to break the large symmetry, we can follow the orbifold compactification. The scheme makes use of space group $S$ whose elements consist not only of a translation $v$ but also of a twist $\theta$ which is an element of point group. For the bosonized NSR fermions, the twist $\theta$ is expressed by $e^{2\pi i \xi}$ and $N=1$ supersymmetry requires that $\xi$ is given by

$$\xi = \frac{1}{N} (1, m, -1 - m, 0), \quad (1)$$

where $m$ is an integer and $N$ is the order of orbifold restricted to one of integers $3, 4, 6, 7, 8, 12$.

In fact, the twist breaks down also the duality and it is impossible to find out a dual embedding from the point group into $E_8 \times E_8'$ gauge degrees of freedom. The reason is as follows: we embed both point group and translation group of $S$ to the Cartan subalgebra
of $E_8 \times E_8'$ and express the representation as a shift vector $V_i^I = v^I + a^I (I = 1..16)$ on the $E_8 \times E_8'$ weight, where $v^I$ is a shift corresponding to a twist $\theta^I$ and $a^I$ is a Wilson line. We assume a dual shift $V_i = \frac{1}{\sqrt{N}} (x, -x)$, where $x$ is an 8-d vector on the $E_8$ weight lattice. Since $\frac{x}{2}$ is a weight of $E_8$, $x^I (I = 1..8)$ are all even or all odd integers in an orthonormal base and $x^2$ must be a multiple of eight. Also modular invariance condition of the orbifold partition function gives a restriction

$$N(V_i^2 - \xi^2) = 2k, \quad (2)$$

where $k$ is an integer. Substituting Eq.(1) to the Eq.(2), we obtain that

$$\sum_{I=1}^{8} (x^I)^2 = 4(kN + 1) + 4m(m + 1). \quad (3)$$

If the order $N$ was even, the r.h.s. was not a multiple of eight and the modular invariance was inconsistent with the duality.

On $Z_3$ and $Z_7$ orbifolds, we have looked for dual shifts which give non-abelian gauge groups of GUT models and MSSMs, i.e., $SU(5), SO(10)$ and $SU(3) \times SU(2)$. But no dual $V_i^I$ with dual Wilson lines which gives such groups exists.

It is possible, whether the order $N$ is primary or not, to map neither an overall element of $S$ nor a twist to a dual shift through a modular invariant embedding, because the modular invariance condition (2) for a shift $V_i^I$ applies also to the shift $v^I$. Here if we respect the duality, it is necessary to insist that space-time of extra dimensions is not directly compactified to an orbifold but to a torus $T^6$ with a dual Wilson line at first.

To verify the possibility, we investigate $SO(10)$ GUT models with a dual Wilson line on $Z_6$ orbifold. We choose the $SO(10)$ simple roots $R^I$ as

$$(0^4, 1, 0, -1, 0; 0^8), \ (0^6, 1, -1; 0^8), \ (0^5, 1, 0, 0; 0^8),$$

$$(0^3, 1, 0, -1, 0^2; 0^8), \ \frac{1}{2}(1, -1, -1, -1, -1, -1, 1, 0; 0^8), \quad (4)$$
and look for combinations of two shifts $V_{j_1}^l$ and $V_{j_2}^l$ which obey modular invariant condition (2) and $SO(10)$ invariant conditions that $R \cdot V_{j_i} = 0 \mod 1$ ($i = 1, 2$) and that $R' \cdot V_{j_1} \neq 0 \mod 1$ or $R' \cdot V_{j_2} \neq 0 \mod 1$ for roots $R'$ other than $SO(10)$. When we take $V_{j_1}^l$ as a shift $v^l$ corresponding to a twist, the Wilson line is given by $a^l = V_{j_2}^l - V_{j_1}^l$. We pick out combinations with a dual Wilson line and examine massless matter fields given by the shifts. The detail of massless conditions and $Z_6$ singlet condition that physical fields must satisfy is found Ref.3,4).

The $SO(10)$ of the roots (4) contains flipped $SU(5) \times U(1)_X \subset SU(3)_c \times SU(2)_L \times U(1)_Y$ so that axial anomaly related to weak hypercharge $Y$ of matter fields automatically vanishes. We can assign $\bf{16}$ of the $SO(10)$ to quark, lepton fields and assign $\bf{10}$ to two Higgs doublets, respectively$^4$.

We choose spectra which contain just three generations of $\bf{16}$ except for pairs of $\bf{16}$ and $\bf{16}^*$. We exclude models with chiral anomaly with respect to the gauge group of the hidden sector and so-called T-duality anomaly$^5$. Up to now, we have obtained three models on $Z_6$-I orbifold. For one of the models, we give the shift $v^l$, the Wilson $a^l$ and the spectrum of matter fields to the Table I, where $\theta^k$ represents the $k$-twisted sector. The gauge group of the hidden sector is $SO(10) \times SU(2)$.

(Table I)

When we suppose a model on torus with the dual Wilson line in the table, we get $N = 4$ dual $E_6 \times SU(3)$ model with matter fields of the adjoint representation in both observable and hidden sectors.

Also, the other two models are given by the Wilson line of the table, while the shifts
which lead to the models are $v = \frac{1}{11}(-3, -3, -1, 1, -1, 1, -1, -1; 3, -3, 5, 1, -1, 1, -1, -1),$
$v = \frac{1}{12}(-3, -3, -1, 1, -1, 1, -1, -1; 2, 2, -2, 2, 0, 0, -2, -2)$. The gauge groups of the
models are $SO(10) \times SO(10)' \times SU(2)'$ and $SO(10) \times SU(5)' \times (SU(2)')^2$.

References


Table I. Mass spectrum of a $SO(10) \times SO(10)^f \times SU(2)^f$ model through

$v = \frac{1}{12}(-3, -3, -1, 1, -1, 1, -1, -1; 6, 6, 2, -2, 2, -2, 2, 2)$ and

$a = \frac{1}{8}(3, 1, 3, -1, 1, -1, 1, -3, -1 - 3, 1, -1, 1, -1, -1)$.

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<td>2(1;1,2)</td>
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