Application of sum rules to heavy baryon masses

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Abstract

Model independent sum rules are applied to recent measurements of heavy c-baryon and b-baryon masses. The sum rules are generally satisfied to the same degree as for the light (u,d,s) baryons.

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Recent measurements of the masses of a number of heavy baryons (with c or b quarks) now make it possible to test model independent sum rules that were derived some time ago using fairly minimal assumptions within the quark model.[1] The sum rules depend on standard quark model assumptions, and the additional assumption that the interaction energy of a pair of quarks in a particular spin state does not depend on which baryon the pair of quarks is in. No assumptions are made about the type of potential, and no internal symmetry is assumed. A more detailed discussion of the derivation of the sum rules is given in Ref.[1] In a previous paper, we applied an isospin breaking sum rule to the Σc charge states.[2] In the present paper we test sum rules that connect baryon states of different isospin, which we characterize as medium strong energy difference sum rules.

Before looking at the heavy baryon masses, we review the application of the model independent sum rules to the light baryons. There, the following sum rules hold [3]

$$\frac{1}{3}(\Omega^* - \Delta^{++}) = \Xi^* - \Sigma^+ = \Xi^0 - \Sigma^+,$$

$$(147 \pm 1) \quad (149 \pm 1) \quad (125)$$

$$2\bar{N} + 2\bar{\Xi} - 3\Lambda - \bar{\Sigma} = \Omega^* + \bar{\Delta} - \bar{\Xi} - \bar{\Sigma}.$$

$$( -26) \quad (-14 \pm 2)$$
The baryon symbol has been used for its mass, and, except for the $\Delta$, a star indicates spin $\frac{3}{2}$. A bar over the symbol represents an average over the particular isospin multiplet. Where specific charges are indicated, these could be changed using the isospin breaking sum rules in Ref.[1]. The experimental values in MeV for each sum is given below each equation.

If SU(3) symmetry were broken only by a small octet component, then each side of Eq. (2) would equal zero, corresponding to the Gell-Mann Okubo formula, and equal spacing in the decuplet. The deviations of each side in Eq. (2) from zero indicate that the light baryon interactions violate this symmetry by about 10 MeV. The deviation between the two sides indicates that the light baryon wave functions have SU(3) breaking differences that also affect the masses by about 10 MeV. Similar SU(3) breaking should be expected in heavy baryon masses, even if the heavy spin-spin interactions are more SU(3) symmetric. The $\Xi^0 - \Sigma^+$ term in Eq. (1) indicates SU(6) breaking in the light baryon wave functions, resulting in a $\sim 20$ MeV breaking in the mass. We use the terminology “symmetry breaking in the wave function” because the sum rules allow any amount of breaking in the interactions, but do rest on “baryon independence” for each quark-quark interaction energy. This is a slightly weaker assumption than full SU(3) symmetry of the wave function, which would require each individual wave function to be SU(3) symmetrized. Instead, we use wave functions with no SU(3) symmetry as described in Ref.[3].

For extension to heavy baryons, it is convenient to use the equalities in Eq. (1) to replace Eq. (2) by

\begin{align}
(\Delta^+ - p) &= (\Sigma^{*0} - \Sigma^0) + \frac{3}{2}(\Sigma^0 - \Lambda^0) = (\Xi^c - \Xi) + \frac{3}{2}(\Sigma^0 - \Lambda^0). \\
\text{(297)} & \quad \text{(307)} & \quad \text{(330)}
\end{align}

Equation (3) shows a spread of $\sim 30$ MeV corresponding to the SU(3) and SU(6) breaking of the light baryon wave functions, and demonstrates the ambiguity that will arise when it is extended to heavy baryons.

In table I, we list the measured heavy baryon masses that will be used in the sum rules. We indicate the expected baryon assignments in table I. The $\Xi^{*+}_c$ is the spin $\frac{1}{2}$ usc baryon having the u-s quarks in a spin 1 state. We extend Eq. (3) to charmed baryons by changing the s-quark into a c-quark.
<table>
<thead>
<tr>
<th>Baryon</th>
<th>Mass (MeV)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Σ⁺⁺</td>
<td>Λ⁺⁺ + 168.0 ± 0.3</td>
<td>[4]</td>
</tr>
<tr>
<td>Σ⁺</td>
<td>Λ⁺ + 168.7 ± 0.4</td>
<td>[4]</td>
</tr>
<tr>
<td>Σ⁺⁺⁺</td>
<td>Λ⁺⁺⁺ + 245 ± 7</td>
<td>[5]</td>
</tr>
<tr>
<td>Ξ⁺⁺</td>
<td>2563±15</td>
<td>[6]</td>
</tr>
<tr>
<td>Ξ⁺⁰</td>
<td>2643.3±2.2</td>
<td>[7]</td>
</tr>
<tr>
<td>Ω⁺</td>
<td>2700±3</td>
<td>[8]</td>
</tr>
<tr>
<td>Σ⁻</td>
<td>Λ⁻ + 173 ± 9</td>
<td>[9]</td>
</tr>
<tr>
<td>Σ⁺⁻</td>
<td>Λ⁺⁻ + 229 ± 6</td>
<td>[9]</td>
</tr>
</tbody>
</table>

Table 1: Heavy baryon masses used in the sum rules.

This leads to\[10\]

\[
(S\Sigma^{*0} - \Lambda^0) + \frac{1}{2}(S\Sigma^0 - \Lambda^0) = (S\Sigma^{*+} - \Lambda^+ + \frac{1}{2}(S\Sigma^+ - \Lambda^+)
\]

(307) (330 ± 7)

We have used the light Sigma baryons for the left hand side of Eq. (4). Use of other combinations could change the left hand side, as indicated in Eq. (3), but the Sigma combinaton is the most reasonable since they are most similar to their charmed counterparts. Equation (4) is written in terms of differences from the Λ mass which is how the Σ and Σ⁺ masses are measured. We have had to use the measured Σ⁺⁺⁺ mass for the Σ⁺⁺⁺ mass in Eq.(4), but that difference is probably small.

Changing the c-quark in any c-baryon sum rule to a b-quark leads to the corresponding sum rule for b-baryons. Applying this to Eq. (4) leads to

\[
(S\Sigma^{*0} - \Lambda^0) + \frac{1}{2}(S\Sigma^0 - \Lambda^0) = (S\Sigma^{*0} - \Lambda^0) + \frac{1}{2}(S\Sigma^0 - \Lambda^0)
\]

(307) (316 ± 10)

The sum rules in Eqs. (4) and (5) are satisfied to about the same extent as the light baryon sum rules.

In Ref.[2] we used a sum rule to predict the Ξ′ mass which has now been measured. This permits a test of the sum rule, which we write here as
\[
\Sigma^+ + \Omega^{*-} - \Xi^0 - \Xi^{*0} = \Sigma^{++} + \Omega_c^0 - 2\Xi_c^{*+}.
\]

(15) \hspace{1cm} (27 \pm 30)

Again, we have used the combination of light baryon masses most similar to the corresponding charmed baryons. However there is ambiguity resulting from using various combinations of light baryon masses, so that the left hand side of Eq. (6) could vary between -3 and +27 MeV.

The spin \( \frac{3}{2} \) counterpart of Eq. (6) can be used to predict the as yet unmeasured \( \Omega_c^{*0} \) mass

\[
\Omega_c^{*0} = \Omega_c^0 + 2(\Xi_c^{*+} - \Xi_c^{*+}) - (\Sigma_c^{+++} - \Sigma_c^{+++}) = 2783 \pm 30,
\]

(7)

where we have used the most similar c-baryon combination rather than using any light baryons. This increases the error on the prediction, but is the more reasonable choice.

We see that, especially when the most reasonable combination of light baryons is taken, the medium strong energy difference sum rules are satisfied at least as well for the heavy baryons as for the light quark baryons. However the situation is not as nice for the isospin breaking mass differences in the case of the \( \Sigma_c \). In Ref.[2] we showed that the \( \Sigma_c \) sum rule is violated by three standard deviations, while the corresponding light baryon sum rule is satisfied. Since sum rules in disagreement are of more concern than those which are satisfied, resolving the \( \Sigma_c \) mass differences is of prime importance.
References

[10] Equation (4) is a combination of several equations in Ref. [1].