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Abstract

A new wall-current monitor has been developed in order to reinforce the beam-monitoring system in the PF 2.5-GeV linac for the KEK B-Factory. A prototype monitor was tested for its performance and characteristics. The experimental results in terms of both bench tests and beam tests by single-bunch electron beams were analyzed on the basis of equivalent-circuit models. The frequency response of the monitor agreed well with a lumped equivalent-circuit model for both time- and frequency-domain measurements. The position dependence and its frequency characteristics of the monitor also agreed well with a distributed equivalent-circuit model for both time- and frequency-domain measurements. The rise time of the monitor was about 3ns, which indicated a poor response for short-pulse beams (<1ns). The reason could be attributed to the stray inductance of the ceramic solid resistor and not very good frequency response of the ferrite core.

1. Introduction

A so-called wall-current monitor (WCM), which detects induced wall currents around the inner surface of a vacuum pipe, is widely used for charged-particle beam diagnostics[1], and is very important for obtaining precise information concerning the amount of beam current flowing inside a vacuum pipe, and to support easy tunings of the beam orbit. The wall current is also useful for obtaining the transverse position of the beam centroid relative to the geometric center of the vacuum pipe because of the strong spatial dependence of the wall current along the azimuthal direction. This spatial dependence applied to the beam-position monitor which has been carried out at several laboratories[2]. A detailed theoretical analysis of the WCM using an equivalent-circuit model has been reported[3,4]. However, the position dependence of the WCM as a function of the pickup frequency has not been discussed. The authors have reported on the preliminary experimental results[5], in which no theoretical analysis was performed in detail. The purpose of this work is to derive detailed formulae for the frequency response and the position dependence of the monitor on the basis of theoretical models.

The PF 2.5-GeV linac is now being upgraded for the KEK B-Factory (KEKB)[6]. A new beam-intensity monitor utilizing induced wall currents has been developed in order to reinforce the beam-monitoring system of the linac. A prototype WCM, which was based on a previous study[7], was tested concerning both its characteristics and performance. The experimental results were analyzed by means of a new analysis method based on equivalent-circuit models. Particularly, the precise beam-position dependence and the frequency response of the WCM gave important information about its performance.

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This paper is organized as follows: The inner structure of the monitor is described in Section 2. Sections 3 and 4 treat the frequency response and the position dependence of the monitor, respectively, and a new analysis method for the experimental results on both bench and beam tests is given in these sections. Section 5 concludes this paper.

2. Wall-current monitor

Figures 1 (a) and (b) show a cross-sectional view and a schematic drawing of the prototype monitor, respectively.

![Fig. 1 (a), (b)]

The monitor comprises a disk-shaped ceramic solid resistor, a Mn-Zn toroidal ferrite core inside an aluminum case, and four pickup BNC-type receptacles with π/2 rotational symmetry. The aluminum case needs to cover the monitor components in order not to affect the pickups by the electromagnetic noise mainly generated by a high-power klystron modulator. An insulated short gap (12mm wide) in a vacuum pipe (60.5mm in inner diameter) interrupts the return current on the inner surface of the pipe wall. Each component is separable into two pieces in order to be easily mounted around the pipe. The monitor is electrically in contact with the outer surface of the pipe by RF contactors. The Mn-Zn ferrite core[8] was used to feature a high magnetic permeability (μ=4000 at frequency (<1MHz)) and a comparatively high Curie temperature (>140°С). The resistor[9] is made of a ceramic (A12O3+SiO2) mixed with carbon powders sintered at temperatures above 1000°С as electric conductor. The resistance was selected to be 1Ω so as to have a good frequency response for short-pulse beams. Figure 2 shows a variation of the magnetic permeability(μ) of the ferrite core as a function of the frequency.

![Fig. 2]

The induced wall-current signal is picked-up by measuring the voltage drop between the ends of the resistor with a short wire connected to the BNC-type receptacles.

3. Frequency response and pulse-width dependence of the WCM

3.1 Equivalent-circuit model using lumped-circuit constants

The equivalent-circuit model of the monitor is shown using lumped-circuit constants in Fig. 3 (a).

![Fig. 3 (a), (b)]

In this figure, R is the resistor value, including the stray inductance (Ls), C is the capacitance of the insulated gap, L is the inductance of the ferrite core, which is dependent upon frequency (as shown in Fig. 2), and the current source shows a beam. The current-source model can be replaced with the voltage-source model based on Norton's theorem[10], as shown in Fig. 3 (b).

The voltage (V₀(ω)) induced by the wall current is calculated by using the resistance division of the generator voltage(V_s) as follows:

\[
V₀(ω)= \frac{jωL}{-jω^3LL_sC-ω^2LRC+jω(L+L_s)+R}V_s,
\]

(1)
where $\omega$ is the angular frequency, $V_s$ being the generated voltage of the source; all the other parameters are indicated in the figure. If we replace $j\omega$ with a parameter $(s)$, we can obtain the pickup ($V_o(t)$) formula in the time domain by using of the inverse Laplace transform[11]. If we consider a step response for this circuit, the inverse Laplace transform of $V_s$ is represented by $1/s$. Thus, the initial response of the step pulse ($V_o$) can be obtained as follows:

$$V_o(t) = \mathcal{L}^{-1}\left[\frac{L}{s^3LC + s^2LRC + s(L+L_s) + R}\right],$$

(2)

where $\mathcal{L}^{-1}$ shows the inverse Laplace transform. Since it was difficult to analytically calculate eq.(2), the calculation was performed only numerically.

3.2 Experimental setup and results

First of all, the capacitance ($C$) of the insulated gap and the stray inductance ($L_s$) of the ceramic solid resistor were measured in order to decide the theoretical unknown parameters in eq.(2). The stray inductance was obtained by measuring its frequency response. The experimental setup is shown in Fig. 4.

![Fig. 4](image)

Both ends of the ceramic solid resistor contact the electrodes of the metal plates. The input signals ($V_i$) were provided by the CW generator through a BNC-type receptacle, and the pulse heights of the output signals ($V_o$) were measured by a fast digital sampling oscilloscope (Tektronics TDS684A). The 50$\Omega$ matched terminating resistor was connected with the input port in order to avoid any mismatched reflections. Aluminum foil covered the setup in order to make the ground potentials of the CW generator and the oscilloscope to be equal. Figure 5 shows a variation of the pulse height as a function of the frequency of the CW test pulses in the frequency range of 10 to 600 MHz.

![Fig. 5](image)

The data were fitted by a least-squares method using the theoretical curve obtained by the series-circuit model of resistance and inductance. The fitting function for the output voltage ($V_o$) was used as:

$$V_o = \frac{50}{50 + \sqrt{R^2 + \omega^2L_s^2}}V_i,$$

(3)

where $R$ ($2\Omega$) and $L_s$ were the resistance and stray-inductance values, respectively. The best-fitted values of the stray inductance ($L_s$) was 0.11$\mu$H. The capacitance ($C$) of the insulated gap was measured to be 35pF by an LCR meter (HP 4262A).

The pulse-width dependence and frequency response of the monitor, which correspond to the time-domain and frequency-domain measurements, respectively, were measured in the test bench. The experimental setup is shown in Fig. 6.

![Fig. 6](image)

3
It comprises pulse/CW generators (HP 8131A/Anritsu MG523B), a tapered coaxial calibration tube, and a digital sampling oscilloscope. The calibration tube is a 50-Ω coaxial transmission line. Fast test pulses/CW from the generators are provided into the calibration tube through an N-type connector. The picked-up pulse heights of the monitor were measured by the digital sampling oscilloscope as a function of the pulse width/CW frequency. Figure 7 shows the obtained result of the frequency response of the monitor.

Fig. 7

A theoretical curve was obtained by calculating the absolute value of the transfer function in eq.(1). Figure 8 represents the result of the pulse-width dependence of the monitor.

Fig. 8

The theoretical curve was obtained by a numerical calculation of the inverse Laplace transform in eq.(2). The experimental data agree well with the lumped equivalent-circuit model for both the frequency response and the pulse-width dependence.

4. Beam-position dependence of the WCM

4.1 Extended equivalent-circuit model using distributed-circuit constants

A highly relativistic charged-particle beam in a well-conducting vacuum pipe is accompanied by an electric field compressed along the transverse direction under Lorentz transformations. The spatial distribution of the induced wall-current round the inner surface of the circular pipe is strongly dependent upon the displacement from the beam center[4]. The induced wall-current distribution \( J(\theta, \phi) \) versus the beam position is represented as

\[
J(\delta, \theta, \phi) = \frac{I}{2\pi r} \frac{r^2 - \delta^2}{r^2 + \delta^2 - 2r\delta \cos(\phi - \theta)},
\]  

(4)

where \( I \) is the beam current and the other parameters are given in Fig. 1 (b). The spatial distribution of the induced wall current obeys this formula; however, the pickup is not simply subject to this distribution, because all of the wall currents do not pass straight through the resistor, but a part passes around the azimuthal direction through the aluminum case, which is short-circuited at both ends of a resistor (see Fig. 1 (a)). That is, an induced pickup is generated by the integrated summation of the wall currents passing around the azimuthal direction.

The extended equivalent-circuit model using distributed-circuit constants is represented as follows:

Fig. 9

In this model, the inductive components along the azimuthal direction are introduced as distributed constants. It is understood that the end plate of the disk-shaped aluminum case has inductance along the azimuthal direction. Therefore, this model indicates that the WCM is dependent upon the position displacement of the beam and the frequency\((f)\) response in terms of the beam pulse width; that is, if the DC beams \((f \rightarrow 0)\) pass through the inside of the pipe, the inductive impedance along the azimuthal direction goes to zero, which shows that the position dependence of the pickups disappears. Moreover, if the beam-pulse width is very short \((f \rightarrow \infty, \text{ such as a single-bunch beam})\), the
inductive impedance becomes very large, and, thus, a strong position dependence
obeying the spatial distribution of the wall-current formula \( J(\theta, \delta, \phi) \) appears again.

In a quantitative discussion, this model is formulated by the following differential
equation derived from the conservation law of the wall-current flow:

\[
I_{n-1} - I_n - \frac{dQ_n}{dt} + i_n^R + i_n^L - J\lambda = 0,
\]
where \( I_n \) is the wall-current flow through the \( n \)-th inductor \( (l\lambda) \) along the azimuthal
direction, \( Q_n \) the stored charge in the \( n \)-th capacitor, \( i_n^R \) and \( i_n^L \) the current flows through
the \( n \)-th resistor and the \( n \)-th inductor of the ferrite core, respectively, the surface current
density \( J \) is the wall-current flow on the inner surface of the pipe, \( \lambda \) being the
infinitesimal step length along the \( \theta \) direction.
The voltage-drop relations at each distributed component are also satisfied as follows:

\[
\begin{align*}
\frac{R}{\lambda} i_n^R + \frac{L_s}{\lambda} \frac{\partial i_n^R}{\partial t} &= -V_n, \\
\frac{L_s}{\lambda} \frac{\partial i_n^L}{\partial t} &= -V_n, \\
V_{n-1} &= \frac{\partial}{\partial t} V_n, \\
Q_n &= C\lambda V_n,
\end{align*}
\]
where the parameters are defined in Fig. 9. Noting the above equations and again using
the differential equation, eq. (5) is expressed using the following formula \( (\lambda \to 0) \):

\[
\frac{\partial I}{\partial x} + C \frac{\partial V}{\partial t} \frac{i_n^R + i_n^L}{\lambda} = -J,
\]
where \( x \) shows the coordinate of the azimuthal direction; again using the differentiation of
the above differential equation in terms of time \((t)\), eq. (10) can be written as the
following second-order equation:

\[
\frac{1}{l} \frac{\partial^2 V}{\partial x^2} - C \frac{\partial^2 V}{\partial t^2} + \frac{\exp(-j\alpha)}{\sqrt{R^2 + \omega^2 L_s^2}} \frac{\partial V}{\partial t} + \frac{1}{L} V = \frac{\partial J}{\partial t},
\]

\[
\alpha = \tan^{-1}(L_s \omega / R).
\]
If the system is oscillatory and the desired solution is set to be \( V(x,t) = V(x)e^{i\omega t} \), the
differential equation for the position-displacement term \((V(x))\) is derived as follows:

\[
\frac{1}{l} \frac{d^2 V(x)}{dx^2} + \left[ \frac{\omega^2 C + j \omega \exp(-j\alpha)}{\sqrt{R^2 + \omega^2 L_s^2}} + \frac{1}{L} \right] V(x) = j\omega J(x).
\]
The differential equation (eq. (12)) can be solved as
\[ V(x) = \frac{I}{2\pi r \sqrt{\omega_0^2 r^2 + \delta^2 - 2r\delta \cos(\phi - \theta)}} \sin \left[ \sqrt{\omega_0^2 (\phi - \theta)} \right], \quad (13) \]

where

\[ \omega_0^2 = \frac{1}{\sqrt{R^2 + \omega^2 I^2}} \left[ \frac{\omega^2 C + j\omega \exp(-j\alpha)}{L} + 1 \right]. \quad (14) \]

The initial conditions to be applied to the solution of eq. (12) are \( V(x) = 0 \) and \( \partial V(x) / \partial x = 0 \) at \( x = 0 \), respectively.

The desired analytical solution obeys the integrated summation contributed by the spatial distribution of the wall-current flow at \( \phi = \psi \):

\[ V(x) = \frac{I}{2\pi r \sqrt{\omega_0^2 r^2 + \delta^2 - 2r\delta \cos(\phi - \psi - \theta)}} \int_0^\psi \frac{j\omega(r^2 - \delta^2) \sin \sqrt{\omega_0^2 (\phi - \psi - \theta)}}{r^2 + \delta^2 - 2r\delta \cos(\phi - \psi - \theta)} d\psi, \quad (15) \]

where \( \psi \) is the azimuthal angle for the wall-current flow. This is the desired analytical formula including the position dependence and frequency response of the pickup.

4.2 Experimental setup of the bench and beam test

Figure 10 (a) shows a schematic drawing of the test bench.

The fast pulses/CW from the generators are sent to a broad-band transformer. Transformers used to match the cable impedance (50Ω) to the characteristic impedance of the pipe are attached to both ends of the pipe. One end of the transformer is terminated by a 50Ω resistor. A thin current-carrying wire (500μm φ) is stretched through the center of the monitor to simulate a beam. Leaving the wire and the matching sections fixed, the relative displacement between the monitor and the wire can be changed by moving the monitor with a precision micro-adjustable stage controlled by a personal computer (PC). The four induced pickups are sent to the digital sampling oscilloscope through 5-m-long RG-223 cables, which measures the four signal pulse heights simultaneously. The measured data are also sent to the PC through the GP-IB communication system.

An actual beam experiment of the monitor was carried out using a single-bunch electron beam (E=35 MeV, I=0.27 nC/bunch, pulse width~10ps (FWHM)) from the NERL linac[12]. The experimental setup is shown in Fig. 10 (b). At the exit of the linac, a slit (3mm φ) was inserted to ensure a small beam size. After the slit, a lead block with a hole (9mm φ) was placed so as to reduce any background showers from the slit. After the lead block, the monitor was placed on the same stage in order to change the position of the monitor relative to the beam. Behind the monitor, several lead blocks were also inserted so as to reduce any back scattering of the beam from carbon blocks used as a current monitor. A screen monitor was placed after the blocks in order to monitor the beam size. The picked-up signals were transmitted to the measurement room through 15-m-long coaxial cables (RG-223) and observed by the digital sampling oscilloscope.
4.3 Experimental results

Figure 11 (a) shows a variation of the pickup pulse height \( (V_i) \) versus the horizontal beam-position displacement (vertical displacement=0) as a function of the frequency. Figure 11 (b) indicates variations of the pickup pulse height \( (V_i) \) versus the horizontal beam-position displacement (vertical displacement=0) as a function of the pulse width.

All the data were normalized by the data of a beam center for each measurement in the figures. The strong position dependence approached the well-known wall-current spatial distribution (eq. (4)) for the shorter (or higher frequency CW) pulses. These results clearly show evidence of the propagation of the wall-current signal through the inductance distributed along the azimuthal direction.

The inductance per unit length for current flowing in the azimuthal direction was obtained to be \( l=6\times10^{-9} \text{H/mm} \) in this experiment. The inductance value agrees well with the calculation \( (l=4.6\times10^{-9} \text{H/mm}) \) within the error. The extended equivalent-circuit model using distributed-circuit constants agrees well with the experimental results within the experimental errors.

5. Conclusions

A new wall-current monitor has been developed in order to reinforce the beam-monitoring system in the PF 2.5-GeV linac for the KEKB project. A prototype monitor was tested for its performance and characteristics. The experimental results in terms of both bench tests and beam tests by single-bunch electron beams were analyzed on the basis of the equivalent-circuit model. The frequency responses of the monitor agreed well with the lumped equivalent-circuit model for both the time- and frequency-domain measurements. The position dependence and its frequency characteristics of the monitor also agreed well with the distributed equivalent-circuit model for both the time- and frequency-domain measurements. The stray inductance of the ceramic solid resistor was estimated to be about 0.11\( \mu \text{H} \) in this experimental analysis. The capacitance of the insulated gap was measured to be about 35pF. The rise time of the monitor was about 3ns, which indicated a poor response for short-pulse beams (<1ns). The reasons were understood to be attributed to the stray inductance of the ceramic solid resistor and not very good frequency response of the ferrite core.

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Fig. 1. (a) Cross-sectional view of the prototype wall-current monitor and (b) schematic drawing of the wall-current monitor. The axis of the coordinates for a beam and four pickups positions are shown in the figure.
Fig. 3. Lumped equivalent-circuit models of the wall-current monitor (a) on the use of the current source and (b) on the use of the voltage source.

Fig. 4. Experimental setup of a stray-inductance measurement of the ceramic solid resistor. Aluminum foil was wrapped around this setup in order to make the ground potentials of the CW generator and the oscilloscope to be equal.
Fig. 5. Frequency response of the ceramic solid resistor.

Fig. 6. Schematic drawing of the test bench for the wall-current monitor using an impedance-matched tapered coaxial line. The pulse/CW generator was used to measure the pulse-width dependence and the frequency response of the monitor, respectively.
Fig. 7. Frequency response of the wall-current monitor.

Fig. 8. Pulse-width dependence of the wall-current monitor.
Fig. 9. Extended equivalent-circuit model of the wall-current monitor using distributed-circuit constants.

Fig. 10. Schematic drawings of (a) the test bench and (b) the experimental setup of the beam test.
Fig. 11. Variation of the pickup pulse height ($V_p$) versus the horizontal beam-position displacement (vertical displacement=0) (a) as a function of the frequency and (b) as a function of the pulse width. All the data were normalized by the data of a center position for each measurement.