INTERNATIONAL CENTRE FOR
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VANISHING CORRECTIONS
ON INTERMEDIATE SCALE AND IMPLICATIONS
FOR UNIFICATION OF FORCES

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I Introduction

Grand unified theories based upon SUSY $SU(5), SO(10)$, non-SUSY $SO(10)$ with intermediate symmetries and those inspired by superstrings have been the subject of considerable interest over recent years. In order to solve the strong CP problem through Peccei-Quinn mechanism and achieve small neutrino masses [1] necessary to understand the solar neutrino flux [2] and/or the dark matter of the universe, an intermediate scale seems to be essential [3]. Such a scale might correspond to the spontaneous breaking of gauged B-L contained in intermediate gauge symmetries like $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ (\(\equiv G_{213}\)) and $SU(2)_L \times SU(2)_R \times SU(4)_C$ (\(\equiv G_{224}\)) with [3-6] or without [7] parity, or even others like $SU(2)_L \times U(1)_{B-L} \times SU(4)_C$ and $SU(2)_L \times U(1)_{B-L} \times SU(3)_C$. But it is well known that the predictions of a grand unified theory are more [8] or less [6, 9] uncertain predominantly due to threshold [10] and gravitational smearing effects [11, 12] originating from higher dimensional operators. The uncertainty in the intermediate scale prediction naturally leads to theoretical uncertainties in the neutrino mass predictions through seesaw mechanism. Therefore, an intermediate scale, stable against theoretical uncertainties would be most welcome from the point of precise predictions on neutrino masses.

Another problem in SUSY GUTs having supergrand desert is the requirement of \(\alpha_s(M_Z) \geq 0.12\) to achieve unification at $M_{\tilde{g}} \approx 2 \times 10^{16}$ GeV. Even though the problem is alleviated by unknown GUT threshold and gravitational corrections [13], realization of a natural grand unification scale $M_{\tilde{g}} \approx M_{\tilde{u}} \approx 5.6 \times 10^{17}$ GeV requires the presence of some lighter string states which could be the extra gauge bosons or Higgs scalars of a unifying symmetry, exotic vector-like quarks and leptons with nonconventional hypercharge assignments [14-16], or a $SU(3)_C$--octet and weak $SU(2)_C$-triplet in the adjoint representation of the standard gauge group [17]. But, in the absence of an intermediate symmetry, the neutrino mass predictions may fail short of the solar flux requirements by 2-3 orders. Assuming boundary conditions at the string scale to be different from a GUT-boundary condition, attempts have been made to bring down the values of intermediate scales relevant for larger neutrino masses [18].

The presence of a $G_{224P}$ intermediate gauge symmetry, having only two couplings for \(\rho > M_{\tilde{g}}\), would always guarantee gauge unification, and a demonstration of $M_1 \approx 10^{12} - 10^{14}$ GeV with $M_{\tilde{g}} \approx M_{\tilde{u}}$ in SUSY inspired $SO(10)$, would solve at least two of the major problems: the string scale unification with $\alpha_s(M_Z) \approx 0.11$ and neutrino masses
needed for solar neutrino flux.

It has been shown recently that in all GUTs where \( G_{224M} \) breaks spontaneously at the highest intermediate scale, the \( \sin^2 \theta_W(M_Z) \) prediction is unaffected by GUT-threshold and multiloop (two-loop and higher) radiative corrections emerging from higher mass scales [6]. As a single intermediate symmetry is more desirable from minimal consideration, we confine to the single \( G_{224M} \) symmetry in two step breakings of all possible GUTs including \( SO(10) \) and prove a theorem showing that all higher-scale corrections to the intermediate-scale (\( M_I \)) prediction vanish. In SUSY \( SO(10) \) inspired by superstrings [19], we find that \( M_I \approx 10^{12} - 10^{14} \) GeV is possible with \( M_{\mu} \approx M_{\text{GUT}} \), provided certain states in the predicted spectrum are light.

II Theorem on vanishing corrections on the intermediate scale

We now state the following theorem and provide its proof.

**Theorem:** In all two step breakings of grand unified theories, the mass scale \( (M_I) \) corresponding to the spontaneous breaking of the intermediate gauge symmetry \( SU(2)_L \times SU(2)_R \times SU(1)_X \times P [g_{2L} = g_{2R}] \) has vanishing contributions due to every correction term emerging from higher scales \( (\mu > M_I) \).

To prove the theorem we consider the two-step breaking pattern in SUSY or nonSUSY GUTs,

\[
GUT \xrightarrow{M_0} G_{224M} \xrightarrow{M_I} G_{113} \xrightarrow{M_L} U(1)_{em} \times SU(3)_C
\]

which may or may not originate from superstrings. Following the standard notations, we use the following renormalization group equations (RGEs) for the gauge couplings \( \alpha_i(\mu) = g_i^2(\mu)/4\pi \).

\[
M_Z \lesssim \mu \lesssim M_I
\]

\[
\frac{1}{\alpha_i(M_I)} = \frac{1}{\alpha_i(M_Z)} + \frac{\alpha_i}{2\pi} \ln \frac{M_I}{M_Z} + \theta_i - \Delta_i, \quad i = Y, 2L, 3C
\]  

\[
M_I \lesssim \mu \lesssim M_{\mu}
\]

\[
\frac{1}{\alpha_i(M_{\mu})} = \frac{1}{\alpha_i(M_I)} + \frac{\alpha_i}{2\pi} \ln \frac{M_{\mu}}{M_I} + \theta_i' - \Delta_i', \quad i = 2L, 2R, 4C
\]

where \( \Delta_i \) includes threshold effects at \( \mu = M_Z (\Delta_i^Z) \) due to the top-quark and Yukawa couplings and superpartners in SUSY theories. It also includes threshold effects \( (\Delta_i') \) due to heavy particles near the intermediate scale.

\[
\Delta_i = \Delta_i^Z + \Delta_i'
\]

\[
\Delta_i' = \Delta_i^{YR0} + \Delta_i^{G_0}, \quad i = 2L, 2R, 4C
\]

The second term in the r.h.s. of (1)-(2) is the usual one-loop (multiloop) contribution.

The GUT threshold \( \Delta_i^Z \), gravitational corrections \( \Delta_i^{YR0} \), or the string threshold effects \( \Delta_i^{G_0} \), when the model is based upon string inspired \( SO(10) \) [20], are contained in \( \Delta_i' \).

\[
\Delta_i'^0 = \Delta_i^{YR0} + \Delta_i^{G_0} \quad i = 2L, 2R, 4C
\]

In nonSUSY and SUSY GUTs, the \( \Delta_i^{YR0} \) may emerge from higher dimensional operators scaled by the Planck mass [11] leading to a nonrenormalizable Lagrangian

\[
\mathcal{L}_{\text{R0}} = -\frac{\eta^{(1)}}{2M_{Pl}} \text{Tr}(F_{a\mu}F^{a\mu}) - \frac{\eta^{(2)}}{2M_{Pl}^2} \text{Tr}(F_{a\mu}F^aF^{a\mu}) + \ldots
\]

where \( M_{Pl} \) = Planck mass, and \( \phi = \text{Higgs field} \) which is responsible for breaking the GUT symmetry to \( G_{224M} \). For example, in \( SO(10) \), \( \phi = 54 \). These operators lead to the modifications of the GUT scale boundary conditions on gauge couplings,

\[
\alpha_i^G(M_I) = \alpha_i^{G_0}(M_I) + \alpha_i^V(M_I) = \alpha_i^G(M_I)(1 + \epsilon_i) = \alpha_i^G
\]

which imply

\[
\Delta_i^{YR0} = \frac{\Delta_i}{\alpha_G} \quad i = 2L, 2R, 4C
\]

where \( \alpha_i^G = \text{GUT coupling and } \epsilon_i = \text{known functions of the parameters } \eta^{(i)}, \text{the vacuum expectation value of } \phi, \text{and } M_{Pl} \).

Using suitable combinations of gauge couplings and Eqs. (1)-(2), we obtain the following analytic formulas.

\[
\frac{\alpha_i}{M_I} = \left\{ \frac{L_iB_i - L_iA_i}{D} + \frac{(J_iB_i - K_iA_i)}{D} + \frac{(K_iA_i - J_iB_i)}{D} \right\}
\]

\[
\frac{\alpha_i^G}{M_I} = \left\{ L_iA_i - L_iB_i \right\} + \frac{(K_iA_i - J_iB_i)}{D} + \frac{(J_iA_i - K_iA_i)}{D}
\]

where

\[
D = A_iB_i - A_iB_i
\]

\[
L_i = \frac{16\pi}{3a(M_Z)} \left[ \frac{a(M_Z)}{a(M_Z)} - \frac{3}{8} \right]
\]
\[ L_y = \frac{16\pi}{3\alpha(M_Z)} \left[ \sin^2\theta_W(M_Z) - \frac{3}{8} \right] \] (10)

\[ A_U = 2a_{2L} - a_{2R} + a_{4C} \]
\[ B_U = \frac{5}{3} a_{2L} - a_{2R} - \frac{2}{3} a_{4C} \]
\[ A_L = \frac{8}{3} a_{2L} + \frac{5}{3} a_{2R} - \frac{3}{2} a_{4C} - A_U \]
\[ B_L = \frac{5}{3} (a_{2L} - a_{2R}) - B_U \]
\[ J_\phi = 2\pi \left[ \theta_{2L} + \frac{5}{3} \theta_{2R} + \frac{8}{3} \theta_{4C} + \theta_{3L} + \theta_{3R} + 2\theta_4 - 2\theta_{4C} \right] \]
\[ K_\phi = 2\pi \left[ \frac{5}{3} (\theta_{2L} - \theta_{2R}) + \theta_{3L} + \theta_{3R} - \frac{5}{3} \theta_{4C} \right] \]
\[ J_\Delta = 2\pi \left[ \Delta_{2L} + \frac{5}{3} \Delta_{2R} + \Delta_{4C} + \Delta_{4R} - 2\Delta_{4C} \right] \]
\[ K_\Delta = 2\pi \left[ \frac{5}{3} (\Delta_{2L} - \Delta_{2R}) + \Delta_{4R} + \frac{2}{3} \Delta_{4C} - \frac{5}{3} \Delta_{4L} \right] \] (11)

The first, second, and the third terms in the r.h.s. of (8)-(9) represent the one-loop, the multiloop, and the threshold effects, respectively. Each of these contains contributions originating from lower scales \( \mu = M_Z - M_t \), and higher scales \( \mu > M_t - M_t \). We now examine the contributions to \( \frac{\alpha}{M_Z} \) term by term. In the presence of the \( G_{224p} \) gauge symmetry for \( \mu > M_t, \sigma_{2L}(\mu) = \sigma_{2R}(\mu) \). Then Eq.(2) gives

\[ a_{2L} = a_{2R} \]
\[ \theta_{2L} = \theta_{2R} \]
\[ \Delta_{2L} = \Delta_{2R} \] (12)

where the \( G_{224p} \) symmetry implies

\[ \Delta_{2L}^{V} = \Delta_{2R}^{V} \]
\[ \Delta_{2R}^{RRO} = \Delta_{2L}^{VRO}, \Delta_{2L}^{RRO} = \Delta_{2R}^{VRO} \] (13)

The restoration of left-right discrete symmetry in the presence of \( SU(4)_C \) in \( G_{224p} \) plays a crucial role in giving rise to vanishing contribution due to every type of higher scale corrections.

\( (A) \) One-loop contributions

Using (12) we find that \( B_U \) and \( A_U \) are proportional to each other,

\[ B_U = \frac{2}{3} (a'_{2L} - a'_{4C}) = \frac{1}{3} A_U \] (14)
\[ D = \frac{5}{3} (a'_{2L} - a'_{4C}) A_U - \left( \frac{8}{3} a_{2C} - a_{2L} + \frac{5}{3} a_{4C} \right) B_U \]
\[ = \frac{4A_U}{9} (3a_{2L} + 2a_{2C} - 5a_{4C}) \] (15)

Then \( B_U \) or \( A_U \) cancel out from the denominator and the numerator of the one-loop term in (9) leading to

\[ \left( \frac{\ln \frac{M_t}{M_Z}}{\ln \frac{M_t}{M_Z}} \right)_{\text{one loop}} = \frac{12\pi}{\alpha_d} \left( \sin^2\theta_W - \frac{1}{2} + \frac{\alpha}{3\alpha_s} \right) \]
\[ \frac{d}{d} = 3a_{2L} + 2a_{2C} - 5a_{4C} \] (16)

The fact that \( a'_{i}(i = 2L, 2R, 4C) \) are absent from (16) demonstrates that the scale \( M_t \) is independent of the one-loop contribution to the gauge couplings at higher scales, \( \mu = M_t - M_t \). But these coefficients do not cancel out from \( \frac{\alpha}{M_Z} \), which assumes the form:

\[ \ln \frac{M_t}{M_Z} = \frac{12\pi}{\alpha_d} \left( \sin^2\theta_W - \frac{1}{2} + \frac{\alpha}{3\alpha_s} \right) + X \] (17)

\[ X = \frac{6\pi}{\alpha_d} \left[ a_{2C} \left( 1 - \frac{8}{3} \sin^2\theta_W \right) + a_{2L} \left( \frac{5}{3} \frac{\alpha}{\alpha_s} - 1 + \sin^2\theta_W \right) \right] \]
\[ + \frac{5}{3} a_{4C} \left( \sin^2\theta_W - \frac{\alpha}{\alpha_s} \right) \left( a'_{4C} - a'_{2L} \right) \]

The first term in the r.h.s. of (17) is the one-loop contribution in (16).

We also note that for any standard weak doublet \( \left( H \right) \)

\[ a_{2C}^{(H)} = 0, \quad 2a_{4C}^{(H)} = 5a_{4C}^{(H)} \]

which keeps the one-loop term in (16) unchanged. Thus the scale \( M_t \) is predominantly unaffected by the presence of any number of light doublets with masses \( < M_t \), degenerate or nondegenerate.

\( (B) \) Two-loop and higher-loop effects:

Using the second term in the r.h.s. of (9), (14) and (15), the coefficients \( a'_i \) and terms containing \( \theta' \) cancel out, leading to

\[ \left( \frac{\ln \frac{M_t}{M_Z}}{\ln \frac{M_t}{M_Z}} \right)_{\text{multiloop}} = \frac{K_{\alpha \alpha U - J_\phi B_{\phi}}}{D} \]
\[ = \frac{2\pi}{d} (5\theta_{4} - 3\theta_{2L} - 5\theta_{2C}) \] (19)
showing that all multiloop contributions to the gauge couplings originating from \( \mu = M_I - M_J \) are absent in \( \ln \frac{M_I}{M_J} \). But these multiloop effects do not cancel out from the unification mass,

\[
\left( \ln \frac{M_U}{M_Z} \right)_{\text{multiloop}} = \left( \ln \frac{M_I}{M_J} \right)_{\text{multiloop}} + X_\nu
\]

(20)

where the first term in the r.h.s. of (20) is the same as in (19),

\[
X_\nu = \frac{9\pi}{4d(a_{4c} - a_{2L})} \times \left\{ \left[ \frac{5}{3} (\theta_{2L} + \frac{5}{3} \theta_1 - \frac{8}{3} \theta_{3c}) + \frac{10}{3} (\theta_{2L} - \theta_{3c}) \right] \right. \\
\left. \left( a_{2L} - a_\nu \right) - \left( \frac{8}{3} a_{3c} - \frac{5}{3} a_{2L} - \frac{8}{3} a_\nu \right) \left[ \frac{5}{3} (\theta_1 - \theta_{2L}) + \frac{2}{3} (\theta_{3c} - \theta_{2L}) \right] \right\}
\]

(21)

(C) Threshold effects

Including threshold effects at \( \mu = M_E, M_J \) and \( M_K \), we separate \( J_\Delta \) and \( K_\Delta \) into three different parts

\[
J_\Delta = J_\Delta^L + J_\Delta^A + J_\Delta^Z \\
K_\Delta = K_\Delta^E + K_\Delta^A + K_\Delta^Z
\]

where

\[
J_\Delta^L = 2\pi \left( \Delta_{2L}^L + \Delta_{3L}^L - 2\Delta_{4c}^L \right), \\
J_\Delta^A = 2\pi \left( \Delta_{2L}^A + \frac{5}{3} \Delta_1^A - \frac{8}{3} \Delta_{3c}^A \right), \quad i = I, Z, \\
K_\Delta^L = 2\pi \left( \Delta_{2L}^L + \frac{2}{3} \Delta_{4c}^L - \frac{5}{3} \Delta_{2L}^L \right), \\
K_\Delta^A = \frac{10\pi}{3} \left( \Delta_1^A - \Delta_{2L}^A \right), \quad i = I, Z
\]

(22)

Using the parity restoration constraint gives

\[
K_\Delta^A = \frac{4\pi}{3} (\Delta_{4c}^L - \Delta_{2L}^L) = -\frac{1}{3} J_\Delta^L
\]

and

\[
J_\Delta^L B_0 - K_\Delta^A A_0 = 0
\]

(23)

Using (23) in the third term in (9) gives

\[
\left( \ln \frac{M_I}{M_J} \right)_{\text{threshold}} = -\frac{9}{4d} \left( K_\Delta^L + \frac{J_\Delta^A}{3} + K_\Delta^A + \frac{J_\Delta^Z}{3} \right)
\]

(24)

Thus, it is clear that the would-be dominant source of uncertainty due to GUT-threshold effects has vanished from \( \ln \frac{M_I}{M_J} \) which contains contributions from only lower thresholds at \( \mu = M_Z \) and \( \mu = M_J \). But the GUT threshold contributions do not cancel out from \( \ln \frac{M_I}{M_J} \) which has the form

\[
\left( \ln \frac{M_I}{M_J} \right)_{\text{threshold}} = \left( \ln \frac{M_I}{M_J} \right)_{\text{threshold}} + X_\Delta
\]

(25)

where

\[
X_\Delta = 2\pi \left( \Delta_{2L}^E - \Delta_{3L}^E \right) \\
\times \left[ \left( J_\Delta^L + K_\Delta^A \right) \left( \frac{8}{3} a_{3c} - a_{2L} - \frac{8}{3} a_\nu \right) + \frac{9}{4d} \right] \\
\left[ \left( J_\Delta^L + K_\Delta^Z \right) \left( a_{2L} - a_\nu \right) / (a_{4c} - a_{2L}) \right]
\]

(26)

(D) Gravitational smearing and string threshold effects

In the presence of left-right discrete symmetry in G224P, \( \Delta_{2L}^{NR} = \Delta_{3L}^{NR} \) and \( \Delta_{4c}^{NR} = \Delta_{3c}^{NR} \) holds. The analysis of Sec.(C) holds true in these cases also leading to

\[
J_\Delta^{NR} B_0 - K_\Delta^{NR} A_0 = 0 \\
J_\Delta^{NR} B_0 Y - K_\Delta^{NR} A_V = 0
\]

(27)

Thus the theorem is proved demonstrating explicitly that \( \ln \frac{M_I}{M_J} \) does not have any modification due to corrections to the gauge coupling constants at higher scales for \( \mu > M_I \). When the Higgs scalars, fermions or gauge bosons of the full G224P representations are taken into account, their contributions to \( \ln \frac{M_I}{M_J} \) vanish exactly. The origin behind all cancellations is the G224P symmetry and the relation between the gauge couplings,

\[
\frac{1}{a_\nu(\mu)} = \frac{3}{5} \frac{1}{a_{2L}(\mu)} + \frac{2}{5} \frac{1}{a_{4c}(\mu)}, \quad \mu \geq M_I
\]
Since no specific particle content has been used in proving the vanishing corrections, the theorem holds true without or with SUSY and also in superstring based models.

Another stability criterion on $M_f$ with respect to contributions from lower scale corrections is that, up to one-loop level, it remains unchanged by the presence of any member of light weak doublets having masses from $M_2$ to $M_f$.

The other by product of this analysis is on the stability of $M_f$ with respect to $16_H + \overline{16}_H$ pairs. In all correction terms for $\ell \ell (M_f/M_2)$ the higher scale one-loop coefficients appear in the combination $\alpha'_4 - \alpha'_6$. We note that for any $16_H$ (or $\overline{16}_H$)

$$\langle \alpha'_4 \rangle \approx (\alpha'_6)_{\nu_{10}}$$

which keeps the value of $\alpha'_4 - \alpha'_6$ unaltered. Thus, the value of $M_f$ is almost unaffected by the presence of any number of pairs of $16_H + \overline{16}_H$ between $\mu = M_f - M_{H,1}$. This has relevance for SUSY $SO(10)$ and string inspired models.

III Predictions in nonSUSY $SO(10)$

The stability of $M_f$ in nonSUSY $SO(10)$ under the variation of $\eta^{(1)}$ in (5) was demonstrated in Ref.[21] by accurate numerical estimation. According to the present theorem $\ell \ell (M_f/M_2)$ is not only independent of the 5-dimensional operator and $\eta^{(1)}$, but also of other higher dimensional operators in (5) and parameters arising from the GUT scale. Similarly, the vanishing GUT threshold correction to $M_f$ obtained in the accurate numerical evaluation of Ref.[22] is a part of the present theorem. Imposing the parity restoration criteria for $\mu \geq M_f$ [23], the minimal nonSUSY $SO(10)$ with $\mathbb{14}$, $126$ and 10 representations, $\sin^2 \theta_W = 0.2316 \pm 0.0063, \alpha_s(M_Z) = 0.118 \pm 0.007$, and $\alpha^{-1}(M_Z) = 127.9 \pm 0.1$ predicts [21-23].

$$M_f = 10^{13.6^{+0.16}_{-0.15}} \text{GeV},$$

$$M_{\chi} = 10^{10.02^{+0.25}_{-0.29}} \text{GeV}$$

where the first (second) uncertainties are due to those in the input parameters (threshold effects). In the case of $M_{\chi}$, the threshold uncertainties are due to those at $M_f$ and $M_f$ thresholds only. The third uncertainty due to 5-dimensional operator in (5), which is absent in $M_f$, has been calculated for $\eta^{(1)} = \pm 5(\pm 10)$. In spite of addition of a number of extra $126$ and $10$ dimensional Higgs fields to build a model for degenerate and seesaw contributions to the neutrino masses in $SO(10)$ introducing $SU(2)_L$ horizontal symmetry, the scale $M_f$, according to the present theorem, is identical to that in the minimal model with the same predictions on the nondegenerate neutrino masses [24]. The proton lifetime predictions in the minimal model including NRO contribution is

$$\tau_{p \to e^+ e^-} = 1.44 \times 10^{32.1^{+0.71}_{-0.75}} \text{yr.s}$$

which might be tested by the next generation of experiments.

IV Intermediate scale in SUSY $SO(10)$

In the conventional SUSY $SO(10)$ employing the Higgs supermultiplets $\mathbb{14}$, $16_H \oplus \overline{16}_H$ and $10$, in the usual fashion, it is impossible to achieve $M_f$ substantially lower than $M_{\chi}$. When $256 \oplus 126_H$ are used instead of $16_H \oplus \overline{16}_H$, no intermediate gauge group containing $SU(3)_C \oplus SU(2)_L \neq \eta_{12}$ have been demonstrated [25, 26] by using extra light $G_{2215}$-submultiplets not needed for spontaneous symmetry breaking, but predicted to be existing in the spectrum [19].

In the present analysis, in addition to the usual $\mathbb{14}$ with all components at the GUT scale, the pair $16_H + \overline{16}_H$ with desired components at $G_{2215}$ breaking scale, and the bidoublet $\omega(2,2,1) \subset 10$ near $M_f$ while (2,2,6) is at $M_{\chi}$, we examine the effects of other components in $\mathbb{14}$, or in $16_H + \overline{16}_H$ not absorbed by intermediate scale gauge bosons, being lighter and having masses between $1 \text{TeV} - M_{\chi}$.

The adjoint representation $45$ contains the left-handed triplet $\sigma_3(3,1,1)$, the right-handed triplet $\sigma_6(1,3,1)$ and also $\sigma^{(0)}(1,1,1,1)$ under $G_{2215}$. Under the standard gauge group, $\sigma_6$ and $\sigma^{(0)}$ decompose as

$$\sigma_6(1,3,1) = \left[ \sigma_6^{(1)}(1,1,1,1) + \sigma_6^{(2)}(1,-1,1,1) + \sigma_6^{(3)}(1,0,0,1) \right]$$

$$\sigma^{(0)}(1,1,1,1) = \left[ \sigma^{(0)}_3(1,2,3,1) + \sigma^{(0)}_5(1,-2,3,1) + \sigma^{(0)}_6(1,0,2,1) + \sigma^{(0)}_8(1,0,0,1) \right]$$

The representation $16_H$ contains the $G_{2215}$ submultiplets $\chi^{(0)}(2,1,4)$ and $\chi^{(R)}(1,2,4)$ and the latter decomposes under SM gauge group as

$$\chi^{(R)}(1,2,4) = \chi^{(R)}_3(1,2,3,1) + \chi^{(R)}_5(1,-2,3,1) + \chi^{(R)}_6(1,0,2,1)$$
To make the model simpler we assume some of these lighter components from 45 or the pair $16_{Y} \oplus 16_{C}$ to be either at $M_{C} \simeq 1$ TeV while others are at $M_{Y}$. In that case all the equations for $\ell_{Y}^{n}, \theta_{Y}^{n}, \ell_{C}^{n}, \theta_{C}^{n}, \theta_{C}^{t}$ derived in Sec. II hold with the replacements:

$$
\ell_{Y}^{n} \rightarrow \ell_{Y}^{n} \frac{M_{I}}{M_{C}}, \quad \ell_{C}^{n} \rightarrow \ell_{C}^{n} \frac{M_{I}}{M_{C}}, \quad \theta_{Y}^{n} \rightarrow \theta_{Y}^{t} \frac{M_{I}}{M_{C}}, \quad \theta_{C}^{n} \rightarrow \theta_{C}^{t} \frac{M_{I}}{M_{C}}, \quad \theta_{C}^{t} \rightarrow \theta_{C}^{t} \frac{M_{I}}{M_{C}}.
$$

We present them here only up to one-loop. The two-loop, threshold, and gravitational corrections will be estimated elsewhere [27]

$$
\left( \ln \frac{M_{I}}{M_{C}} \right)_{\text{two-loop}} = \frac{12 \pi}{\alpha_{C}} \left( \sin^{2} \theta_{Y} - \frac{1}{2} - \frac{\alpha_{s}}{3 \alpha_{c}} \right) - 9 \ln \frac{M_{I}}{M_{C}} + \frac{12 \pi}{\alpha_{C}} \left( \sin^{2} \theta_{C} - \frac{1}{2} - \frac{\alpha_{s}}{3 \alpha_{c}} \right) - 9 \ln \frac{M_{I}}{M_{C}} + \frac{12 \pi}{\alpha_{C}} \left( \sin^{2} \theta_{C} - \frac{1}{2} - \frac{\alpha_{s}}{3 \alpha_{c}} \right) - 9 \ln \frac{M_{I}}{M_{C}}
$$

where

$$
\left( \ln \frac{M_{I}}{M_{C}} \right)_{\text{two-loop}} = \left( \ln \frac{M_{I}}{M_{C}} \right)_{\text{two-loop}} + X_{C} + Y
$$

in (15)-(16). In addition, there are contributions to the mass scales due to two-loop calculations from $M_{2} - M_{C}$. We present them here only up to one-loop. The two-loop, threshold, and gravitational corrections will be estimated elsewhere [27]

$$
\left( \ln \frac{M_{I}}{M_{C}} \right)_{\text{two-loop}} = \frac{12 \pi}{\alpha_{C}} \left( \sin^{2} \theta_{Y} - \frac{1}{2} - \frac{\alpha_{s}}{3 \alpha_{c}} \right) - 9 \ln \frac{M_{I}}{M_{C}} + \frac{12 \pi}{\alpha_{C}} \left( \sin^{2} \theta_{C} - \frac{1}{2} - \frac{\alpha_{s}}{3 \alpha_{c}} \right) - 9 \ln \frac{M_{I}}{M_{C}} + \frac{12 \pi}{\alpha_{C}} \left( \sin^{2} \theta_{C} - \frac{1}{2} - \frac{\alpha_{s}}{3 \alpha_{c}} \right) - 9 \ln \frac{M_{I}}{M_{C}}
$$

where

$$
Y = \frac{5}{8 \alpha_{C}} \left( a_{Y}^{0} - a_{Y}^{0} \right) \left[ (a_{2}^{0} - a_{Y}^{0}) \left( 3 a_{2}^{0} + 5 a_{Y}^{0} - 8 a_{C}^{0} \right) - (a_{2}^{0} - a_{Y}^{0}) \left( 3 a_{2}^{0} + 5 a_{Y}^{0} - 8 a_{C}^{0} \right) \right] \times \ln \frac{M_{I}}{M_{2}}
$$

and

$$
X_{C} = X_{C} \left( a_{C}^{0} - a_{C}^{0} \right) = 3 a_{2}^{0} + 2 a_{Y}^{0} - 5 a_{C}^{0}
$$

V Summary and conclusions

We have shown that there are scale corrections in the intermediate scale prediction ($M_{I}$), corresponding to the $G_{22M}$ gauge symmetry breaking, vanish exactly. Such corrections are due to one-loop, two-loop and all higher loop effects, GUT threshold and gravitational smearing effects originating from higher-dimensional operators. In string inspired SUSY GUTs, the string-loop threshold effects have also vanishing contributions on $M_{I}$. In nonSUSY $SO(10)$ models the intermediate scale has been predicted earlier and we emphasize that $M_{I} \simeq 10^{10}$ GeV is quite stable leading to more precise neutrino mass predictions. The predicted proton lifetime can be tested by future experiments. The $G_{22M}$ symmetry having only two gauge couplings guarantees unification, but the problem in SUSY $SO(10)$ is the realization of $M_{I} \ll M_{U}$. We find solutions to this problem with $M_{I} \simeq 5 \times 10^{12} - 2 \times 10^{14}$ GeV and $M_{U} \simeq M_{C} \simeq 6 \times 10^{17}$ GeV for small $\alpha_{S}(M_{Z})$ and certain states in the adjoint representation 45 and/or 16$_{Y}$ + 16$_{C}$ have masses near 1 TeV. The light states in 16$_{Y}$ + 16$_{C}$ may emerge naturally from the modes not absorbed by heavy $SU(2)_{R} \times SU(1)_{C}$ gauge bosons. String scale unification might be possible in case of another intermediate symmetry, such as $SU(2)$, with parity broken at the GUT scale, when the submultiplet $\sigma(1, 1, 0, 8)$ is at the intermediate scale [28]; but only in the present case of $G_{22M}$-intermediate symmetry, the scale $M_{I}$ has all higher scale corrections vanishing and neutrino mass predictions in SUSY $SO(10)$ are expected to be more precise.

Acknowledgements

The author thanks J.C. Pati, R.N. Mohapatra, G. Senjanovic and A.Yu. M. Smirnov for useful discussions. Financial support and hospitality from the Theory Group, International Centre for Theoretical Physics, Trieste, are gratefully acknowledged. The author also acknowledges the grant of a research project SP/S2/K-99/91 from the Department of Science and Technology, New Delhi.
References


Table 1. Predictions for mass scales in string inspired SO(10) model

<table>
<thead>
<tr>
<th>SM submultiplets</th>
<th>$M_2 - M_C$</th>
<th>$M_C - M_I$</th>
<th>$G_{2/45P}$ submultiplets</th>
<th>$a_i'$</th>
<th>$a_i'$</th>
<th>$M_I$ (GeV)</th>
<th>$M_I$ (GeV)</th>
</tr>
</thead>
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<tr>
<td>$\phi_{e, \nu_d}$</td>
<td>$\sigma_{L, R, \sigma}$</td>
<td>$\sigma_{L, \chi_L, \chi_L}$</td>
<td>$\chi_L, \chi_R, \chi_L$</td>
<td>$\tilde{\gamma}_{\nu, \nu}$</td>
<td>$\gamma_{R, \gamma}$</td>
<td>$\frac{47}{5}$</td>
<td>$\frac{7}{2}$</td>
</tr>
<tr>
<td>$\phi_{e, \nu_d}$</td>
<td>$\gamma_{L, \nu, \chi_{3, \chi}}$</td>
<td>$\gamma_{L, \chi_L, \chi_L}$</td>
<td>$\chi_L, \chi_R, \chi_L$</td>
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