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QUARK-LEPTON YUKAWA-UNIFICATION AT LOWER MASS SCALES ¹

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ABSTRACT

In the nonsupersymmetric two-Higgs-doublet-standard model, we find t - b - τ Yukawa unification consistent with the CERN-LEP data, experimental values of m_b , m_τ , and $m_t = 160$ GeV-190 GeV at lower mass scales: $M_c \simeq 3 \times 10^8$ GeV- 3×10^9 GeV. We also show how such quark-lepton unification scales can be reconciled with $SU(2)_L \times SU(2)_R \times SU(4)_C$ -intermediate breaking in SO(10) including threshold and gravitational corrections.

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Considerable attention has been paid to the study of implications of quark-lepton unification through $SU(4)_C$ -gauge hypothesis [1]. If $SU(4)_C$ occurs as a part of left-right gauge group such as $SU(2)_L \times SU(2)_R \times SU(4)_C$ ($\equiv G_{224}$) with parity ($g_{2L} = g_{2R}$) or without it ($g_{2L} \neq g_{2R}$) [2], or even as $SU(2)_L \times U(1)_{B_R} \times SU(4)_C$, it can undergo spontaneous symmetry breaking to the standard model (SM) gauge group, directly or by more than one step, such that weak interaction phenomenology at lower-energies remains close to that of the SM. In addition, rare-kaon decays mediated by $SU(4)_C$ -gauge bosons might be observable by low-energy experiments in near future [3,4], if the relevant symmetry breaking scale is not too high. When embedded in a grand unified theory (GUT), the existence of a G_{224} -breaking scale near $10^6 - 10^7$ might lead to deservable proton decays [5] or neutron-antineutron oscillations [6]. If neutrinos are Majorana particles, a G_{224} -breaking scale near $10^9 - 10^{11}$ GeV yields small neutrino masses necessary to understand solar neutrino puzzle by MSW mechanism or dark matter of the universe [7].

Almost five years ago, an interesting possibility of relating the $SU(4)_C$ -breaking scale to the top quark mass was advanced by demanding unification of the three Yukawa couplings of the third generation fermions at the spontaneous symmetry breaking scale of the intermediate gauge symmetry $SU(2)_L \times SU(2)_R \times SU(4)_C$ ($\equiv G_{224}$) in SO(10) grand unification, but by using [8],

$$\sin^2 \theta_W = 0.226 \pm 0.005$$

$$\alpha_s = 0.100 - 0.117$$

With the standard model (SM) gauge group containing one Higgs-doublet below the intermediate scale, it was shown that either the running mass of bottom-quark must be greater than 4.35 GeV, or the top-quark pole mass must be less than 80 GeV. However, with two-Higgs doublets in SM, it was noted that for $\tan \beta \simeq 35$ -45, the three Yukawa couplings meet in the allowed range of the intermediate scale $M_c = 10^{12.2} - 10^{13.6}$ GeV yielding the bounds, $80 \text{ GeV} < m_t < 180 \text{ GeV}$. Such a high scale of quark-lepton unification would predict branching ratios for rare kaon decays twelve orders smaller than the current experimental limit.

More recently, the question of Yukawa unification at the GUT-scale has been analysed in the context of nonSUSY SO(10), where partial unification of gauge couplings is achieved through G_{224} intermediate symmetry [9].

The purpose of this brief report is to examine whether the idea of quark-lepton unification through $SU(4)_C$ is realizable at mass scales significantly lower than those investigated

earlier but consistent with the most recent determinations of the input parameters [10]

$$\begin{aligned}
\sin^2 \theta_W(M_z) &= 0.2316 \pm 0.0003 \\
\alpha_s(M_z) &= 0.118 \pm 0.007 \\
\alpha^{-1}(M_z) &= 127.9 \pm 0.1 \\
m_b &= 4.25 \text{ GeV} \\
m_\tau &= 1.785 \text{ GeV} \\
m_t &= 174 \pm 15 \text{ GeV}
\end{aligned} \tag{1}$$

We also examine how such a scale is accommodated within $SO(10)$ through G_{224} -intermediate breaking.

We use the renormalization group equations (RGEs) for the top-quark, bottom quark, and the τ -lepton Yukawa couplings in the two-Higgs doublet SM in the range of mass scales, $m_t \leq \mu \leq M_c$,

$$\begin{aligned}
16\pi^2 \frac{dh_t}{dt} &= h_t \left[\frac{9}{2} h_t^2 + \frac{1}{2} h_b^2 - \sum_i c_i^{(t)} g_i^2 \right] \\
16\pi^2 \frac{dh_b}{dt} &= h_b \left[\frac{9}{2} h_b^2 + \frac{1}{2} h_t^2 + h_\tau^2 - \sum_i c_i^{(b)} g_i^2 \right] \\
16\pi^2 \frac{dh_\tau}{dt} &= h_\tau \left[\frac{5}{2} h_\tau^2 + 3h_b^2 - \sum_i c_i^{(\tau)} g_i^2 \right]
\end{aligned} \tag{2}$$

where $t = \ell n \mu$, h_j = Yukawa coupling of the j th fermion, and

$$\begin{aligned}
c_i^{(t)} &= \left(\frac{17}{20}, \frac{9}{4}, 8 \right), \\
c_i^{(b)} &= \left(\frac{1}{4}, \frac{9}{4}, 8 \right), \\
c_i^{(\tau)} &= \left(\frac{9}{4}, \frac{9}{4}, 8 \right), \quad i = Y, 2L, 3C
\end{aligned}$$

For $\mu > m_t$, the integration of (2) gives

$$\begin{aligned}
h_t(\mu) &= \frac{h_t(m_t)}{A_t} e^{\frac{9}{2} I_t + \frac{1}{2} I_b}, \\
h_b(\mu) &= \frac{h_b(m_t)}{A_b} e^{\frac{1}{2} I_t + \frac{3}{2} I_b + I_\tau}, \\
H_\tau(\mu) &= \frac{h_\tau(m_t)}{A_\tau} e^{3 I_b + \frac{5}{2} I_\tau}, \\
I_i &= \int_{\ell n m_t}^{\ell n \mu} \frac{h_i^2(t)}{16\pi^2} dt, \quad i = t, b, \tau, \\
A_f &= \prod_i \left(\frac{\alpha_i(\mu)}{\alpha_i(m_t)} \right)^{\frac{c_i^{(f)}}{2\alpha_i}}, \quad f = t, b, \tau
\end{aligned} \tag{3}$$

In the earlier work [8] only h_t was included inside the square brackets, in the r.h.s. of (2), but we take into account the contributions of all the three Yukawa couplings of the third generation.

The functions $A_f(f = t, b, \tau)$ are obtained by integrating out the gauge coupling contributions in (2). Using terms up to one-loop out of the two-loop approximation,

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(m_t)} - \frac{a_i}{2\pi} \ell n \frac{\mu}{m_t} - \frac{1}{4\pi} \sum_j \frac{b_{ij}}{a_j} \ell n \frac{\alpha_j(\mu)}{\alpha_j(m_t)} \tag{4}$$

where

$$a_i = \begin{pmatrix} \frac{21}{5} \\ -3 \\ -7 \end{pmatrix}, \quad b_{ij} = \begin{pmatrix} \frac{104}{25} & \frac{18}{5} & \frac{144}{5} \\ \frac{8}{5} & 8 & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix}$$

Using mass-operator renormalization of $SU(3)_C \times U(1)_{em}$ theory below M_z , the QCD rescaling factors for $m_b(m_t)$ and $m_\tau(m_t)$ are obtained as [9]

$$\begin{aligned}
\eta_b &= 1.53 \pm_{-0.06}^{+0.07} \\
\eta_\tau &= 1.015
\end{aligned}$$

where $m_i(m_t) = m_i/\eta_i$, $i = b, \tau$. The renormalization group equations for gauge couplings from $M_z - m_t$ gives

$$\begin{aligned}
\alpha_1^{-1}(m_t) &= 58.51, \quad \alpha_2^{-1}(m_t) = 30.15 \\
\alpha_3^{-1}(m_t) &= 9.30 \pm 0.5
\end{aligned} \tag{5}$$

For a given ratio of the vacuum expectation values of the two Higgs scalars, $\frac{v_b}{v_d} = \tan \beta$, we computed the three values of Yukawa couplings at $\mu = m_t$ using

$$\begin{aligned}
h_t(m_t) &= \frac{m_t(m_t)}{174 \sin \beta} \\
h_b(m_t) &= \frac{m_b(m_t)}{174 \cos \beta} \\
h_\tau(m_t) &= \frac{m_\tau(m_t)}{174 \cos \beta}
\end{aligned} \tag{6}$$

Using the initial values from (5) and (6) as inputs in (2) and (3), we have plotted $h_i(\mu)$ vs $\log_{10} \mu$, such that the solutions for $h_i(\mu)$ are consistent with eqs. (2)–(4).

For smaller $\tan \beta$ in the range of 5 – 10, the ratio $\frac{h_b}{h_\tau}$ is found to approach unity for $\mu = 10^{8.3} - 10^9$ GeV; but there is no possibility of the three Yukawa couplings having a common meeting point.

In the next step we searched the parameter space in the large $\tan \beta$ region taking the allowed values of the top quark mass from 160 GeV to 190 GeV. Our results are

shown in Fig. 1 where unification of the three Yukawa couplings is clearly exhibited for $m_t = 160$ GeV, 170 GeV, 180 GeV and 190 GeV with $\tan\beta = 52.80, 55.89, 58.94$ and 61.94 , respectively. The unification scale increases with top quark mass from $M_c = 10^{8.5}$ GeV to $M_c = 10^{9.5}$ GeV.

In Fig. 2 we have shown the variation of the unification scale $\mu(\equiv M_c)$ as a function of $\tan\beta$ where in the larger (smaller) $\tan\beta$ region, the values of μ and $\tan\beta$ correspond to the unification of three (two) Yukawa couplings. The dotted, solid and dashed lines correspond to $m_t = 160$ GeV, 170 GeV and 180 GeV, respectively. From the larger $\tan\beta$ region of Fig. 2, it is clear that, for a fixed $\tan\beta$, the unification scale increases with m_t ; and for a fixed m_t , the unification scale increases with $\tan\beta$, although the rate of increase is different in different regions of the curve. In Fig. 3 we show how, for smaller values of $\tan\beta$, the ratio $\frac{k_b}{k_\tau}$ approaches unity at $M_c = 10^{8.4}$ GeV ($m_t = 170$ GeV, solid line) and at $M_c = 10^9$ GeV ($m_t = 180$ GeV, dotted line).

So far no specific intermediate gauge group embedding $SU(4)_C$ has been assumed. The unification of the three Yukawa couplings is a consequence of low-energy data and the two Higgs doublet standard model. In order to embed such a scenario in $SO(10)$, the most attractive candidate is the intermediate gauge group $SU(2)_L \times SU(2)_R \times SU(4)_C$ with parity broken at the GUT scale [2].

$$SO(10) \xrightarrow{\frac{M_U}{210}} G_{224} \xrightarrow{\frac{M_C}{126}} SM \xrightarrow{\frac{M_Z}{10}} U(1)_{em} \times SU(3)_C$$

where from M_z to M_c we have now two weak doublets ϕ_u and ϕ_d instead of one as in earlier analyses [11-13]. The Yukawa coupling for every fermion of the third generation in the presence of G_{224} symmetry has the form [9]

$$16\pi^2 \frac{dh_f}{dt} = h_f(6h_f^2 - \sum_i c_i^{(f)} g_i^2)$$

$$c_i^f = \left(\frac{9}{4}, \frac{9}{4}, \frac{45}{4} \right) \quad (12)$$

and the RGEs for the gauge couplings for $\mu > M_c$ are

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_U)} - \frac{a'_i}{2\pi} \ell n \frac{\mu}{M_c} - \frac{1}{4\pi} \sum_j \frac{b'_{ij}}{a'_j} \ell n \frac{\alpha_j(\mu)}{\alpha_j(M_c)}, \quad (13)$$

$$i = 2L, 2R, 4C$$

where

$$a'_i = \begin{pmatrix} -3 \\ \frac{11}{3} \\ \frac{-23}{3} \end{pmatrix} \quad b'_{ij} = \begin{pmatrix} 8 & 3 & \frac{45}{2} \\ 3 & \frac{584}{3} & \frac{765}{2} \\ \frac{9}{2} & \frac{153}{2} & \frac{643}{6} \end{pmatrix}$$

Integrating the fermion Yukawa coupling in (12) with the help of one-loop approximation from (13) and using the unification value at M_c , for $h_f(M_c)$, we have plotted $h_f(\mu)$ as a function of μ as shown in Fig. 4 by the dashed line for $\mu = M_c - 10^{18}$ GeV. It is clear that h_f uniformly decreases and attains the value of 0.35 and 0.27 at $\mu = M_U = 10^{15.9}$ GeV and $\mu = 10^{18}$ GeV, respectively.

Now we include threshold [11] and gravitational corrections to the two-loop approximation for the intermediate scale M_c and the GUT scale M_U , which are predicted up to one loop as

$$\ell n \frac{M_c}{M_z} = \frac{36\pi}{47\alpha} \left(\frac{7}{24} - \sin^2 \theta_W + \frac{2}{9} \frac{\alpha}{\alpha_s} \right)$$

$$\ell n \frac{M_U}{M_z} = \frac{15\pi}{47\alpha} \left(\frac{1}{10} + \sin^2 \theta_W - \frac{19}{15} \frac{\alpha}{\alpha_s} \right) \quad (14)$$

With the input parameters given in (1), the model predictions in two loop approximations are $M_U^0 = 10^{15.9 \pm 2} \text{ GeV}$, $M_c^0 = 10^{10.9 \pm 0.2} \text{ GeV}$, where the uncertainties are due to the input parameters. Threshold corrections are very close to the values obtained in ref. [11]. We now evaluate gravitational corrections due to the 5-dimensional operators [14] contributing to the nonrenormalizable term [15]

$$\alpha_{NRO} = -\frac{\eta}{2M_{Pl}} Tr(F_{\mu\nu} \phi_{(210)} F^{\mu\nu}) \quad (15)$$

When the 210 dimensional Higgs field acquires vacuum expectation values, the added presence of (15) in the renormalizable lagrangian leads to the modification of the boundary conditions on gauge couplings and consequently the gravitational corrections on the mass scales are

$$\delta_C^{NRO} \equiv \delta \ell n \frac{M_c}{M_z} = \frac{9}{94} \left(\frac{3\pi}{2\alpha_G^3} \right)^{1/2} \eta \frac{M_U^0}{M_{Pl}}$$

$$\delta_U^{NRO} \equiv \delta \ell n \frac{M_U}{M_z} = -\frac{15}{376} \left(\frac{3\pi}{2\alpha_G^3} \right) \eta \frac{M_U^0}{M_{Pl}} \quad (16)$$

Using GUT coupling constant, $\alpha_G^{-1} = 46.3 \pm 0.3$, $M_{Pl} = 10^{19} \text{ GeV}/2\pi = 1.6 \times 10^{18} \text{ GeV}$, and M_U^0 and M_c^0 at their two-loop values, we present δ_C^{NRO} and δ_U^{NRO} in Table 1 for $\eta = -10$ to 10 . We note that while positive (negative) values of η increase (decrease) M_c , the same values tend to decrease (increase) M_U . Using $\eta = \pm 5$ and the threshold corrections of ref. [11], our estimations on the mass scales are

$$M_c = 10^{10.9 \pm 2.7}_{-1.4} \pm 1.3 \pm 0.2$$

$$M_U = 10^{15.9 \pm 0.8}_{-1.7} \pm 0.6 \pm 0.2$$

where the first (second) uncertainty represents threshold (gravitational) corrections. It is clear that when threshold and 5-dim operator effects are taken into account, the intermediate scale encompasses the quark-lepton-Yukawa unification scales for the third generation fermions. Our analysis includes the most recent global fits to the CERN-LEP data encompassing $\alpha_s(M_z) = 0.111 - 0.125$ and effects of Yukawa couplings of all the three fermions of third generation. The quark-lepton unification at $M_U \simeq 3 \times 10^8$ GeV $\rightarrow 3 \times 10^9$ GeV has been demonstrated as a consequence of low-energy data and two-Higgs doublet standard model below the G_{224} -breaking scale. In this work we have not considered large threshold effects on Yukawa couplings and possibility of unification at still lower scales $M_U \simeq 10^6$ GeV, which might be more model-dependent [16].

The allowed values of intermediate scales for Yukawa unification predict the branching ratio for rare-kaon decays to be $B(K_L \rightarrow \bar{\mu}e) \simeq 10^{-16} - 10^{-18}$ and ν_τ as a candidate for hot and cold dark matter of the universe.

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References

- [1] J.C. Pati and A. Salam, Phys. Rev. **D10**, 275 (1974).
- [2] D. Chang, R.N. Mohapatra and M.K. Parida, Phys. Rev. Lett. **52**, 1072 (1984); Phys. Rev. **D30**, 1052 (1984).
- [3] N.G. Deshpande and R. Johnson, Phys. Rev. **D27**, 1193 (1984).
- [4] M.K. Parida and B. Purkayastha, Phys. Rev. **D53**, 1706 (1996).
- [5] J.C. Pati, Phys. Rev. **D29**, 1599 (1984).
- [6] R.N. Mohapatra and R.E. Marshak, Phys. Rev. Lett. **44**, 14316 (1980); R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
- [7] M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity (eds. D. Freedman et al., (North Holland, 1980); T. Yanagida, *Proceedings of the workshop on Baryon Number of the Universe*, Ed. Osawada et al. (KEK, 1979); R.N. Mohapatra and G. Senjanovic, Phys. Rev. **D23**, 165 (1981), Phys. rev. Lett. **44**, 912 (1980).
- [8] E.M. Freire, Phys. Rev. **D43**, 209 (1990).
- [9] N.G. Deshpande and E. Keith, Phys. Rev. **D50**, 3513 (1994).
- [10] P. Longacker and N. Polonsky, Phys. Rev. **D52**, 308 (1995); **D52**, 441 (1995); M. Shifman, Mod. Phys. Lett. **A10**, 605 (1995), L. Montanet et al., Phys. Rev. **D50**, 1173 (1994).
- [11] Dae-Gyu Lee, R.N. Mohapatra, M.K. Parida and M. Rani, Phys. Rev. **D51**, 229 (1995); R.N. Mohapatra and M.K. Parida, Phys. Rev. **D47**, 264 (1992).
- [12] N.G. Deshpande, E. Keith and P. Pal, Phys. Rev. **D46**, 2261 (1992).
- [13] Dae-Gyu Lee, Phys. Rev. **D50**, 207 (1994).
- [14] Q. Shafi and C. Wetterich, Phys. rev. Lett. **52**, 875 (1984); C.T. Hill, Phys. Lett. **135B**, 470 (1984).
- [15] P.K. Patra and M.K. Parida, Phys. Rev. **D44**, 2179 (1991).
- [16] R.R. Volkas, Phys. rev. **D53** (1996) (to appear).

Table 1. Gravitational corrections to mass-scale predictions by 5-dim operator in $SO(10)$ with $SU(2)_L \times SU(2)_R \times SU(4)_C$ intermediate symmetry

η	δ_C^{NRO}	δ_U^{NRO}	$\frac{M_C}{M_C^0}$	$\frac{M_U}{M_U^0}$
± 5	± 3.2	∓ 1.3	$10^{\pm 1.3}$	$10^{\pm 0.6}$
± 10	± 6.2	∓ 2.6	$10^{\pm 2.6}$	$10^{\mp 1.3}$

Figure Captions

Fig. 1. Variation of Yukawa couplings for the top quark (dashed line), the bottom quark (dotted line) and the τ -lepton (solid line) as a function of mass scale and $\tan \beta$ exhibiting quark-lepton unification at 3×10^8 GeV - 3×10^9 GeV.

Fig. 2. Variation of the quark-lepton unification scale (μ) as a functional of $\tan \beta$ for $m_t = 160$ GeV (dotted line), $m_t = 170$ GeV (solid line), and $m_t = 180$ GeV (dashed line) where, for small $\tan \beta$, $h_b = h_\tau$ but for larger $\tan \beta \geq 52$, $h_b = h_\tau = h_t$ has been realized.

Fig. 3. Variation of the ratio h_b/h_τ as a function of mass scale for smaller values of $\tan \beta$ and $m_t = 170$ GeV (solid line), $m_t = 180$ GeV (dotted line).

Fig. 4. Extrapolation of Yukawa couplings beyond the intermediate scale for $m_t = 160$ GeV to 190 GeV.

Fig.1

Fig.2

Fig.3

Fig.4