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Quantum Coherence and Skyrmions in Bilayer Quantum Hall System

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ABSTRACT

Quantum coherence in a bilayer quantum Hall system is analyzed based on a microscopic Chern-Simons gauge theory with the lowest Landau level projection, where the dynamics is governed by the $W_\infty \times SU(2)$ algebra. We present a systematic method to calculate the current and static correlation functions. It is pointed out that Skyrmion excitations may be detected by measuring the Hall current distribution. We also emphasize that the FQH state is a condensed state but not a coherent state of the bosonized electron field. The interlayer coherence is due to the coherent mode in the CP^1 field.

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1. INTRODUCTION

The FQH state is realized in a strong magnetic field B . An electron makes a cyclotron motion with magnetic radius $\ell_B = \sqrt{\hbar c / eB}$. When the filling factor ν is less than one, all electrons are accommodated in the lowest Landau level (LLL). To make a consistent theory it is necessary to make the LLL projection [1], which projects away the kinetic term of the electron Hamiltonian. After the projection the system is dynamical because the X and Y coordinates of the guiding center do not commute.

When two of these layers are laid parallel and close enough, an interlayer coherence develops spontaneously [2,3] and we expect Josephson-like phenomena [2,4] to occur. Skyrmions are also expected to appear as new topological excitations [3] in this coherent mode.

To analyze the FQH system it is convenient to use bosonized electrons [5] based on the Chern-Simons (CS) gauge theory. The Landau-Ginzburg (LG) theory has been proposed and produced many interesting results [5,2,6]. However, the LLL projection is scarcely understood in this approach. There exists also another criticism [7]. To overcome these problems we propose a microscopic CS gauge theory with the LLL projection. We show that the FQH state is a condensed state but not a coherent state of bosonized electrons: therefore, the mean field does not exist as a bona fide classical field. The LG theory is incompatible with the LLL projection, though many of the results [5,2,6] can be justified within the present microscopic theory. Quantum coherence in bilayer system is due to the coherent mode in the CP^1 field. Our field-theoretical results are consistent with those in the single-mode approximation (SMA) [3]. In this paper we use the unit $c = \hbar = 1$ and take the length unit to be ℓ_B .

2. BOSONIZED ELECTRONS

We consider a bilayer electron system in strong magnetic field. We denote the electron field at the layer $\alpha (= 1, 2)$ by $\psi_\alpha(\mathbf{x})$. The Hamiltonian is given by

$$H = \frac{1}{2M} \sum_{\alpha} \int d^2x \psi_{\alpha}^{\dagger}(\mathbf{x})(P_x^2 + P_y^2) \psi_{\alpha}(\mathbf{x}) + \frac{1}{2} \sum_{\alpha, \beta} \int d^2x d^2y V_{\alpha\beta}(\mathbf{x} - \mathbf{y}) \psi_{\alpha}^{\dagger}(\mathbf{x}) \psi_{\beta}^{\dagger}(\mathbf{y}) \psi_{\alpha}(\mathbf{x}) \psi_{\beta}(\mathbf{y}), \quad (1)$$

where P_k is the covariant derivative, $P_k = -i\partial_k + eA_k^{\text{ext}}$ with $A_k^{\text{ext}} = \frac{1}{2}\epsilon_{kj}x_j B$ and $V_{\alpha\beta}(\mathbf{x} - \mathbf{y})$ the Coulomb interaction. We include the tunneling interaction H_T later.

The bilayer system has various phases. We concern only about the one possessing interlayer coherence, that is the so-called Halperin (m, m, m) phase. It is chosen [2] by introducing the bosonized-electron field ϕ_{α} by an operator phase transformation with a common phase χ , $\phi_{\alpha}(\mathbf{x}) = e^{i\chi(\mathbf{x})}\psi_{\alpha}(\mathbf{x})$. By this transformation the covariant derivative is modified as, $P_k \rightarrow \tilde{P}_k \equiv P_k + C_k$, where the CS field $C_k(\mathbf{x}) \equiv \partial_k \chi(\mathbf{x})$ is determined by the CS constraint, $\epsilon_{jk}\partial_j C_k = 2\pi m\rho$, in terms of the density $\rho = \rho_1 + \rho_2$ with $\rho_{\alpha} = \phi_{\alpha}^{\dagger}\phi_{\alpha}$. Here, m is an odd integer.

3. PSEUDOSPIN STRUCTURE

We assign "up" and "down" pseudospins to the electrons belonging to the upper and lower layers, respectively. The pseudospin operator is defined locally by $S^a(\mathbf{x}) = \frac{1}{2}\Phi^{\dagger}\tau^a\Phi$, where τ^a is the Pauli matrix and $\Phi = (\phi_1, \phi_2)^T$ is the SU(2) field. A local SU(2) transformation, $e^{-i\vartheta}$, is defined with $\vartheta = \sum_{a=1}^3 \int d^2x f^a(\mathbf{x}) S^a(\mathbf{x})$. The operator S^3 describes the density difference between the two layers, and called *out-of-phase* density. On the other hand, the operator ρ generates the U(1) phase transformation, and called *in-phase* density.

The bosonic field $\phi_{\alpha}(\mathbf{x})$ may be decomposed into

$$\phi_{\alpha}(\mathbf{x}) = \phi(\mathbf{x})n_{\alpha}(\mathbf{x}), \quad \phi(\mathbf{x}) = e^{i\xi(\mathbf{x})}\sqrt{\rho(\mathbf{x})}, \quad (2)$$

where ξ is the in-phase angle, and $\mathbf{n} = (n_1, n_2)^T$ is the CP¹ field subject to $\mathbf{n}^{\dagger}(\mathbf{x})\mathbf{n}(\mathbf{x}) = 1$. It follows that $S^a(\mathbf{x}) = \frac{1}{2}\mathbf{n}^{\dagger}(\mathbf{x})\tau^a\mathbf{n}(\mathbf{x})\rho(\mathbf{x})$. We shall see later that the ground state is an eigenstate of the density operator $\rho(\mathbf{x})$ and a coherent state of the CP¹ field $\mathbf{n}(\mathbf{x})$,

$$\rho(\mathbf{x})|g_0\rangle = 2\rho_0|g_0\rangle, \quad \mathbf{n}(\mathbf{x})|g_0\rangle = \mathbf{n}_0|g_0\rangle, \quad (3)$$

where ρ_0 is a constant and we may choose $\mathbf{n}_0 = \sqrt{1/2}(1, 1)^T$. The state $|\Phi\rangle = e^{i\vartheta}|g_0\rangle$ is also a coherent state of the CP¹ field, and is parametrized as

$$n_{\alpha}(\mathbf{x})|\Phi\rangle = e^{\pm i\vartheta(\mathbf{x})/2} \sqrt{\frac{1 \pm \sigma(\mathbf{x})}{2}} |\Phi\rangle, \quad (4)$$

Here, ϑ stands for the out-of-phase angle. We obtain

$$S^a(\mathbf{x}) = \frac{1}{\rho_0} \langle \Phi | S^a(\mathbf{x}) | \Phi \rangle = \delta^{a1} - \epsilon^{1ab} f^b(\mathbf{x}) + \dots, \quad (5)$$

in the power series expansion of f^a , where $f^2 = \sigma$ and $f^3 = \theta$ up to the first order in f^a .

A topological current, $Q_{\mu} = \frac{1}{2\pi} \epsilon_{\mu\nu\lambda} \partial_{\nu} \mathbf{n}^{\dagger} \partial_{\lambda} \mathbf{n}$, may be constructed from the CP¹ field, which conserves trivially, $\partial_{\mu} Q_{\mu} = 0$. For the coherent state (4), we obtain

$$Q_0(\mathbf{x}) = \frac{1}{4\pi} \epsilon_{ij} \partial_i \sigma \partial_j \theta = \frac{1}{4\pi} \epsilon_{ij} \partial_i f^2 \partial_j f^3 + \dots \quad (6)$$

Topological excitations are Skyrmions [6]. If we take $\sigma = (\kappa r^{2n} - 1)/(\kappa r^{2n} + 1)$ and $\theta = n\varphi$ (φ is the azimuthal angle), the coherent state (4) describes a Skyrmion with scale κ and topological charge $Q \equiv \int d^2x Q_0(\mathbf{x}) = n$. The Skyrmion spin texture minimizes the nonlinear σ -model Hamiltonian (20) in the SU(2)-invariant limit.

4. LOWEST LANDAU LEVEL PROJECTION

The coordinate $\mathbf{x}$ of the bosonized electron is decomposed into the guiding center $\tilde{X}$ and the relative coordinate $\tilde{R}$, where $\tilde{R} = (\tilde{P}_y, -\tilde{P}_x)$, $\tilde{X} \equiv (\tilde{X}, \tilde{Y}) = \mathbf{x} - \tilde{R}$, with $[\tilde{X}, \tilde{Y}] = -i$, $[\tilde{P}_x, \tilde{P}_y] = i$. Here, we have used quantities with check because they are bosonized-electron variables. The same formulas given below follow also for electron variables without check. Two independent sets of harmonic oscillators, $\tilde{a} \equiv \frac{1}{\sqrt{2}}(\tilde{P}_x + i\tilde{P}_y)$ and $\tilde{b} \equiv \frac{1}{\sqrt{2}}(\tilde{X} - i\tilde{Y})$, are defined, with $[\tilde{a}, \tilde{a}^{\dagger}] = [\tilde{b}, \tilde{b}^{\dagger}] = 1$.

The LLL projection is to suppress the relative motion of the electron, which is to impose the condition

$$\tilde{a}\phi_{\alpha}(\mathbf{x})|\Psi\rangle \propto (\tilde{P}_x + i\tilde{P}_y)\phi_{\alpha}(\mathbf{x})|\Psi\rangle = 0, \quad (7)$$

on the state in the Hilbert space.

In various places we deal with operators of the form $O = \int d^2x f(\mathbf{x})\rho(\mathbf{x})$. The state $O|\Psi\rangle$ does not belong to the lowest Landau level even if $|\Psi\rangle$ does, because

$$\tilde{a}\phi_\alpha(\mathbf{x})O|\Psi\rangle = \tilde{a}f(\mathbf{x})\phi_\alpha(\mathbf{x})|\Psi\rangle \neq 0. \quad (8)$$

To make the LLL projection of O it is necessary to project the c-number function $f(\mathbf{x})$, whose result we denote by $\hat{O}$ and $\hat{f}(\mathbf{x})$, respectively.

We do this in a systematic way. We first make a Fourier transformation of $f(\mathbf{x})$, $f(\mathbf{x}) = (2\pi)^{-2} \int d^2q e^{i\mathbf{x}\mathbf{q}} f(\mathbf{q})$. The problem of the LLL projection is reduced to that of the plane wave $e^{i\mathbf{x}\mathbf{q}}$. We separate the coordinate $\mathbf{x}$ into the guiding center $\tilde{X}$ and the relative coordinate $\tilde{R}$, or equivalently into the two sets of operators $(\tilde{a}, \tilde{a}^\dagger)$ and $(\tilde{b}, \tilde{b}^\dagger)$. We make normal ordering as

$$\begin{aligned} e^{i\mathbf{q}\mathbf{x}} &= e^{-(1/4)\mathbf{q}^2} \exp\left[\frac{1}{\sqrt{2}}\mathbf{q}\tilde{a}^\dagger\right] e^{i\mathbf{q}\tilde{X}} \exp\left[-\frac{1}{\sqrt{2}}\mathbf{q}^\dagger\tilde{a}\right], \\ e^{i\mathbf{q}\tilde{X}} &= e^{-(1/4)\mathbf{q}^2} \exp\left[\frac{i}{\sqrt{2}}\mathbf{q}^\dagger\tilde{b}^\dagger\right] \exp\left[\frac{i}{\sqrt{2}}\mathbf{q}\tilde{b}\right]. \end{aligned} \quad (9)$$

Here, $\tilde{a}$ acts on $\phi(\mathbf{x})|\Psi\rangle$ in (8) and vanishes due to the LLL condition (7). We eliminate $\tilde{a}^\dagger$ by inserting the completeness condition $1 = \sum |\Psi_i\rangle\langle\Psi_i|$ of the LLL states $|\Psi_i\rangle$. Then, in place of (8), we get

$$\tilde{a}\phi_\alpha(\mathbf{x})\hat{O}|\Psi\rangle = \tilde{a}\hat{f}(\mathbf{x})\phi_\alpha(\mathbf{x})|\Psi\rangle = \hat{f}(\mathbf{x})\tilde{a}\psi_\alpha(\mathbf{x})|\Psi\rangle = 0,$$

where $\hat{f}(\mathbf{x}) = (2\pi)^{-2} \int d^2q f(\mathbf{q}) e^{-(1/4)\mathbf{q}^2} e^{i\mathbf{q}\tilde{X}}$, and

$$\hat{O} = \int d^2x \hat{f}(\mathbf{x})\rho(\mathbf{x}) = \int \frac{d^2q}{(2\pi)^2} f(-\mathbf{q})\hat{\rho}_\mathbf{q}, \quad (10)$$

$$\hat{\rho}_\mathbf{q} \equiv e^{-(1/4)\mathbf{q}^2} \int d^2x e^{-i\mathbf{q}\tilde{X}} \rho(\mathbf{x}). \quad (11)$$

The LLL projection of S^a is similarly defined, which we denote by $\hat{S}_\mathbf{q}^a$ in the momentum space.

5. $W_\infty \times \text{SU}(2)$ ALGEBRA

After the LLL projection, the dynamics arises solely from the commutation relation $[\tilde{X}, \tilde{Y}] = -i$, which leads to the W_∞ algebra [1]. With the pseudospin incorporated, the $W_\infty \times \text{SU}(2)$ algebra governs the dynamics,

$$\begin{aligned} [\hat{\rho}_\mathbf{p}, \hat{\rho}_\mathbf{q}] &= 2i\hat{\rho}_{\mathbf{p}+\mathbf{q}} e^{(1/2)\mathbf{p}\mathbf{q}} \sin(\mathbf{p} \wedge \mathbf{q}), \\ [\hat{S}_\mathbf{p}^a, \hat{\rho}_\mathbf{q}] &= 2i\hat{S}_{\mathbf{p}+\mathbf{q}}^a e^{(1/2)\mathbf{p}\mathbf{q}} \sin(\mathbf{p} \wedge \mathbf{q}), \\ [\hat{S}_\mathbf{p}^a, \hat{S}_\mathbf{q}^b] &= i\varepsilon^{abc} \hat{S}_{\mathbf{p}+\mathbf{q}}^c e^{(1/2)\mathbf{p}\mathbf{q}} \cos(\mathbf{p} \wedge \mathbf{q}) \\ &\quad + \frac{i}{2} \delta^{ab} \hat{\rho}_{\mathbf{p}+\mathbf{q}} e^{(1/2)\mathbf{p}\mathbf{q}} \sin(\mathbf{p} \wedge \mathbf{q}), \end{aligned} \quad (12)$$

where $\mathbf{p} \wedge \mathbf{q} = \frac{1}{2} \varepsilon_{ij} p_i q_j$.

6. GROUND STATE

The Coulomb term in the Hamiltonian (1) needs the LLL projection. After the LLL projection it becomes

$$\hat{H}_C = \frac{1}{2} \int \frac{d^2q}{(2\pi)^2} V_+(\mathbf{q}) \hat{\rho}_{-\mathbf{q}} \hat{\rho}_\mathbf{q} + 2 \int \frac{d^2q}{(2\pi)^2} V_-(\mathbf{q}) \hat{S}_{-\mathbf{q}}^3 \hat{S}_\mathbf{q}^3, \quad (13)$$

where $V_\pm(\mathbf{q})$ is the Fourier transformation of the potential $V_\pm(\mathbf{x})$, with $V_\pm = \frac{1}{4}(V_{11} + V_{22} \pm 2V_{12})$. The potential V_- [V_-] part has the global $\text{SU}(2)$ [$\text{SO}(2)$] invariance.

The $\text{SU}(2)$ -invariant term is minimized by requiring

$$\hat{\rho}_\mathbf{q}|g\rangle = (2\pi)^2 2\rho_0 \delta(\mathbf{q})|g\rangle. \quad (14)$$

With this choice of the ground state it is shown that the gapless mode is absent in the in-phase density fluctuation, as leads to the incompressibility of the FQH system at $\rho = 2\rho_0$. With respect to the $\text{SO}(2)$ -invariant term, because of the existence of a gapless mode in the out-of-phase density fluctuation [2,8], the ground state cannot be an eigenstate of S^3 . It should satisfy

$$\begin{aligned} \langle g|\hat{S}_\mathbf{q}^3|g\rangle &= 0, \quad \langle g|\hat{S}_\mathbf{q}^1|g\rangle = (2\pi)^2 \rho_0 \cos \theta_0 \delta(\mathbf{q}), \\ \langle g|\hat{S}_\mathbf{q}^2|g\rangle &= -(2\pi)^2 \rho_0 \sin \theta_0 \delta(\mathbf{q}), \end{aligned} \quad (15)$$

where θ_0 is a constant angle. The ground state is infinitely degenerate. We choose the ground state $|g_0\rangle$ with $\theta_0 = 0$. Then, the conditions (14) and (15) are equivalent to (3). It follows [2] from the CS constraint and the LLL condition that the above ground state is realized only at the filling factor $\nu \equiv 4\pi\rho_0/eB = 1/m$.

In the state where the density distribution is fixed the phase becomes arbitrary. Indeed, the ground-state condition (14) implies $\langle g_0|\phi_\alpha(\mathbf{x})|g_0\rangle = 0$ due to the phase factor $e^{i\xi(\mathbf{x})}$ in (2), where $e^{i\xi(\mathbf{x})}$ is interpreted as the creation operator [9] of an antivortex at $\mathbf{x}$ which carries m units of flux. The ground state is a condensed state but not a coherent state of bosonized electrons. Hence, the mean-field $\phi_\alpha^{cl}(\mathbf{x}) \equiv \langle \phi_\alpha(\mathbf{x}) \rangle$ is not a bona fide classical field. The same conclusion is made also for the monolayer FQH system. This property makes the FQH system very unique. Its origin is the LLL projection because such a ground state is realized only in the absence of the kinetic term.

7. SKYRMION EXCITATIONS

We consider a state $|\hat{\Phi}\rangle = e^{i\hat{\phi}}|g_0\rangle$, where $\hat{\phi}$ is the LLL projected generator. We evaluate $\langle \hat{\Phi}|\hat{S}_q|\hat{\Phi}\rangle$ and denote its Fourier transformation by $\rho_0\tilde{s}^a(\mathbf{x})$. Using the algebra (12) and the conditions (14) and (15) it is easy to see

$$\tilde{s}^a(\mathbf{x}) = \delta^{a1} - \varepsilon^{1ab}e^{-(1/2)\nabla^2}f^b(\mathbf{x}) + \dots \quad (16)$$

We find $\tilde{s}^a(\mathbf{x}) = s^a(\mathbf{x})$ with (5) for a sufficiently smooth spin texture, which we assume. It is also easy to find

$$\tilde{\rho}(\mathbf{x}) = 2\rho_0 + \nu Q_0(\mathbf{x}) + \dots, \quad (17)$$

where $\tilde{\rho}(\mathbf{x})$ is the Fourier transformation of $\langle \hat{\Phi}|\hat{\rho}_q|\hat{\Phi}\rangle$ and $Q_0(\mathbf{x})$ is the Skyrmion density. A Skyrmion is a topological excitation realized in the coherent mode. This equation says that the Skyrmion carries electric charge $-e\nu Q$, which is a fractional charge in general.

8. TWO POINT FUNCTIONS

Two point functions such as $\langle \hat{\Phi}|\hat{\rho}_p\hat{\rho}_q|\hat{\Phi}\rangle$ cannot be evaluated by the algebraic relation alone. In so doing we need to deal with $\langle g_0|\{\hat{S}_p^a, \hat{S}_q^b\}|g_0\rangle$. We use the formula

$$\{\hat{S}_p^a, \hat{S}_q^b\} = -\varepsilon^{abc}\hat{S}_{p+q}^c \sin(\mathbf{p} \wedge \mathbf{q})e^{(1/2)\mathbf{p} \cdot \mathbf{q}} + \frac{1}{2}\delta^{ab}\hat{\rho}_{p+q} \cos(\mathbf{p} \wedge \mathbf{q})e^{(1/2)\mathbf{p} \cdot \mathbf{q}} + : \{\hat{S}_p^a, \hat{S}_q^b\} :. \quad (18)$$

Here, we normal order $:\hat{S}_p^a\hat{S}_q^b:$ explicitly with respect to ϕ_α and $\phi_\alpha^\dagger$. Then, we use (2) and analyze a small fluctuation around the ground state $|g_0\rangle$, to find that

$$\langle g_0|\hat{S}_p^a\hat{S}_q^b|g_0\rangle = (2\pi)^4\rho_0^2\delta^{1a}\delta^{1b}\delta(\mathbf{p})\delta(\mathbf{q}). \quad (19)$$

Various correlation functions are calculated by using these formulas as well as the algebraic relation (12).

9. COULOMB ENERGY

It is now straightforward to evaluate the Coulomb energy (13) of the state $|\hat{\Phi}\rangle$, $\Delta E_C \equiv \langle \hat{\Phi}|\hat{H}_C|\hat{\Phi}\rangle$, which turns out to be the energy of the nonlinear σ -model in the SU(2)-invariant limit,

$$\Delta E_C = \frac{1}{2}\rho_E \sum_{a=1}^2 \int d^2x [\nabla \tilde{s}^a(\mathbf{x})]^2 + \frac{1}{2}\rho_A \int d^2x [\nabla \tilde{s}^3(\mathbf{x})]^2 + 2\rho_0^2 \int d^2x d^2y \tilde{V}_-(\mathbf{x} - \mathbf{y}) \tilde{s}^3(\mathbf{x}) \tilde{s}^3(\mathbf{y}) + \dots \quad (20)$$

Here, $\tilde{V}_-(\mathbf{x})$ is the Fourier transformation of $e^{-\mathbf{q}^2/2}V_-(\mathbf{q})$, and $\rho_E \equiv \rho_s^+ - \rho_s^-$, $\rho_A \equiv \rho_s^+$ with

$$\rho_s^\pm = \frac{\nu}{32\pi^2} \int dq V_\pm(\mathbf{q}) q^3 e^{-(1/2)\mathbf{q}^2}. \quad (21)$$

Parameters ρ_A and ρ_E describe the pseudospin stiffness. This result agrees with the one [3] derived in the SMA. In deriving (20) the ground-state condition (14) is crucial. If the ground state were a coherent state of bosonized electrons ϕ_α , we would have obtained $\Delta E_C = 0$.

10. HAMILTONIAN OF THE GOLDSTONE MODE

The ground state (15) is infinitely degenerate. When we choose one of them, the $SO(2)$ symmetry is spontaneously broken and the Goldstone mode appears. For a smooth spin texture we use (16) to rewrite (20) as $\Delta E_C = \int d^2x \mathcal{H}$ with

$$\mathcal{H} = \frac{\rho_E}{2}(\nabla\theta)^2 + \frac{\rho_A}{2}(\nabla\sigma)^2 + \frac{e^2\rho_0^2}{2C}\sigma^2, \quad (22)$$

where C is the capacity. Here, $|\sigma| \ll 1$ but θ is not small in general. This is the effective Hamiltonian governing the dynamics of the Goldstone mode.

The fields $\theta(\mathbf{x})$ and $\sigma(\mathbf{x})$ are classical fields. However, the canonical commutation relation, $[\rho_0\sigma(\mathbf{x}), \theta(\mathbf{y})] = i\delta(\mathbf{x} - \mathbf{y})$, follows naturally. To derive it we evaluate the equation of motion, $i(d\hat{S}_k^a/dt) = [\hat{S}_k^a, \hat{H}_C]$. We then take its expectation value by the state $|\hat{\Phi}\rangle = e^{i\hat{\Phi}}|g_0\rangle$. For a smooth spin texture, we use (16) to obtain

$$\frac{d}{dt}\theta = \frac{\rho_A}{\rho_0}\nabla^2\sigma - \frac{e^2\rho_0}{C}\sigma, \quad \frac{d}{dt}\sigma = -\frac{\rho_E}{\rho_0}\nabla^2\theta. \quad (23)$$

This set of equations agree precisely with the Heisenberg equations of motion of the Hamiltonian (22) provided that the above commutation relation is imposed.

Diagonalizing the Hamiltonian by the Bogoliubov transformation we obtain the excitation mode

$$H = \int d^2k E(\mathbf{k}) \alpha_{\mathbf{k}}^\dagger \alpha_{\mathbf{k}}, \quad E(\mathbf{k}) = \sqrt{\frac{\mu \mathbf{k}^2}{M^*} + \frac{\mathbf{k}^4}{4M^{*2}}}, \quad (24)$$

where $M^* = \rho_0/\sqrt{4\rho_A\rho_E}$ and $\mu = e^2\rho_E M^*/C$. The excitation mode has a linear dispersion relation as $\mathbf{k} \rightarrow 0$. Hence, the Goldstone mode is a superfluid mode.

11. ELECTRIC CURRENTS

We analyze the electric current $J_i^\alpha \equiv -(e/M)\psi_\alpha^\dagger P_i \psi_\alpha$ in each layer in external electric field E_i^α . Though some attempts has been made [10,3] on this problem they are incomplete. We presents a systematic method. We first note that J_i^α vanishes when it acts on the LLL state due to the LLL condition (7). We use an equivalent form of the

current. Making an infinitesimal local phase transformation, $\psi_\alpha \rightarrow e^{if^\alpha(\mathbf{x})}\psi_\alpha$, in the Hamiltonian (1), we obtain $\Delta H = (1/e) \int d^2x J_i^\alpha \partial_i f^\alpha$. We have thus extracted the current J_i^α by modifying the momentum P_i by $\partial_i f^\alpha$. Since it modifies the LLL projection, it is interpreted that the current flows via a Landau-level mixing. The modification of the Hilbert space may be neglected in the limit $f^\alpha \rightarrow 0$. Making the LLL projection we obtain the formula

$$\hat{J}_i^\alpha(\mathbf{k}) = \frac{ie(2\pi)^2}{k_i} \frac{\delta \Delta \hat{H}}{\delta f^\alpha(-\mathbf{k})} \Big|_{f \rightarrow 0}, \quad (25)$$

where $\Delta \hat{H} = -i[\hat{\Phi}, \hat{H}]$ with $\hat{\Phi}$ the LLL projected generator of the relevant phase transformation. It agrees precisely with the drift current discussed in Ref. [1] and gives the correct Hall current in the monolayer system.

It is convenient to use the in-phase and out-of-phase currents, $J_i^\pm = (1/2)(J_i^1 \pm J_i^2)$. We denote the Fourier transformation of $\langle \hat{\Phi} | \hat{J}_i^\pm(\mathbf{k}) | \hat{\Phi} \rangle$ by $\bar{J}_i^\pm(\mathbf{x})$. The Hamiltonian consists of the electric-field term H_E , the Coulomb term H_C and the tunneling term H_T .

The electric-field term H_E is given by

$$\hat{H}_E = \int d^2x \hat{A}_0^\alpha(\mathbf{x}) \rho^\alpha(\mathbf{x}) = \int \frac{d^2q}{(2\pi)^2} A_0^\alpha(\mathbf{q}) \hat{\rho}^\alpha \cdot \mathbf{q}. \quad (26)$$

Using the formula (25) with $\Delta \hat{H} = \Delta \hat{H}_E$ we obtain

$$\bar{J}_i^{E+}(\mathbf{x}) = \frac{e^2\nu}{2\pi} \varepsilon_{ij} E_j^+(1 - 2\pi T_0(\mathbf{x})) + \dots, \quad (27)$$

$$\bar{J}_i^{E-}(\mathbf{x}) = \frac{e^2\nu}{2\pi} \varepsilon_{ij} E_j^- \bar{S}^3(\mathbf{x}) + \dots. \quad (28)$$

The first term in (27) is the familiar Hall current, while the second term represents the effect from the Skyrmion density $T_0(\mathbf{x})$. Thus, a Skyrmion excitation may be detected by measuring the Hall current distribution.

The Coulomb term (13) generates the out-of-phase current $\bar{J}_i^{C-}$ from the superfluid mode θ ,

$$\bar{J}_i^{C-}(\mathbf{x}) = e\rho_E \partial_i \theta(\mathbf{x}) + \dots. \quad (29)$$

The out-of-phase difference $\theta(\mathbf{x})$ induces a supercurrent to eliminate itself.

We proceed to include the tunneling interaction between the two layers. It annihilates one electron at one layer and creates one electron at the other layer at the same 2-dimensional point. The Hamiltonian density is

$$\mathcal{H}_T(\mathbf{x}) = -\lambda(\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1) = -2\lambda S^1(\mathbf{x}), \quad (30)$$

which is unchanged by the LLL projection. It generates only the out-of-phase current $\hat{j}_i^{T-}$, in such a way that

$$\hat{c}_i \bar{j}_i^{T1}(\mathbf{x}) = -\partial_i \bar{j}_i^{T2}(\mathbf{x}) = 2e\lambda\rho_0 \bar{s}^2(\mathbf{x}). \quad (31)$$

At each point $\mathbf{x}$ this amount of charge is exchanged between the two layers. Hence, we may identify

$$\bar{j}^z(\mathbf{x}) \equiv 2e\lambda\rho_0 \bar{s}^2(\mathbf{x}) = -2e\lambda\rho_0 \sqrt{1 - \sigma(\mathbf{x})^2} \sin \theta(\mathbf{x}) \quad (32)$$

as the tunneling current between the two layers.

12. CONCLUSION

Based on the CS gauge theory with the LLL projection, we have presented a systematic way to evaluate the current and correlation functions in the FQH system. We have derived the Hamiltonian governing the dynamics of the coherent mode in a bilayer quantum Hall system. We have already shown elsewhere [4] that such a Hamiltonian together with the tunneling term (30) leads to quantum coherent phenomena including the Josephson-like effect. We have also pointed out that a Skyrmion excitation may be detected by measuring the Hall current distribution. We wish to mention that the present results can be directly taken over to the monolayer quantum Hall ferromagnet [6]. We believe that our field-theoretical method is a powerful tool to analyze various aspects of the FQH effect. Details will be given in a forthcoming paper.

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