Can Asymptotic Series Resolve the Problems of Inflation?

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Abstract

We discuss a cosmological scenario in which inflation is driven by a potential which is motivated by an effective Lagrangian approach to gravity. We exploit the recent arguments [1] that an effective Lagrangian $L_{\text{eff}}$ which, by definition, contains operators of arbitrary dimensionality is in general not a convergent but rather an asymptotic series with factorially growing coefficients. This behavior of the effective Lagrangian might be responsible for the resolution of the cosmological constant problem. We argue that the same behavior of the potential gives a natural realization of the inflationary scenario.
1 Introduction

Inflation [2] [3] is an attractive paradigm for early Universe cosmology. Not only does it provide a mechanism to render the observable part of the Universe approximately spatially flat and homogeneous, but it also gives rise to a mechanism for the origin of inhomogeneities on galactic and super-galactic scales [4]. In spite of these successes, however, a convincing realization of inflation in the context of fundamental physics is still missing. In most models of inflation, the existence of a new fundamental scalar field $\phi$ must be postulated, and, in addition, its potential $V(\phi)$ is tightly constrained by cosmological considerations. The presence of such scalar fields also makes the cosmological constant problem worse since the value of the minimum of $V(\phi)$ is not set by any particle physics considerations.

In this Letter we suggest an inflationary scenario in which the inflaton, the scalar field $\phi$ giving rise to inflation, is a scalar condensate $\Phi$ arising in an effective field theory description of gravity. We determine its potential, making use of an asymptotic series analysis [1] of the effective field theory and show that some of the problems of inflation are resolved in a natural manner. A second crucial ingredient of our mechanism is a dynamical infrared cutoff.

Note that both gravitational effects and condensates have previously been invoked to generate inflation. In fact, the first model of inflation [5] made use of an $R^2$ correction term in the gravitational action. More recently, a realization [6] of inflation based on a special class of higher derivative gravity models implementing the Limiting Curvature Hypothesis [7] was proposed. Condensates were also considered in the past [8], [9] as a mechanism to drive inflation.

The first crucial new aspect of our analysis is the asymptotic series analysis of the effective field theory [1] in order to discover the potential for the condensate. It turns out that the potential is very different from the potentials typically used to generate inflation. Since expectation values of condensates of massless fields such as the graviton are infrared divergent, it is required to introduce an infrared cutoff. In the context of cosmology, this infrared cutoff is not arbitrary. The corresponding length scale is constrained by physical considerations (see e.g. [10]). Taking into account this time dependent infrared cutoff is the second key new aspect of this work.

Let us recall some general arguments [1] regarding the effective description of gravity at the Planck scale. We assume that at some stage in the very early Universe the gravity field as well as other relevant fields (e.g. scalar fields) take on nonzero vacuum expectation values (VEV), which we shall call condensates $^3$. In gravity this condensation process is likely to occur at the Planck scale, and it can be viewed in analogy with the phenomenon of gluon condensation in QCD which takes place at a scale of 1 GeV [11]. We denote any relevant condensate by $\langle \Phi \rangle$. It might stand for a scalar field like the ones

$^3$The condensates will disappear at late times, as discussed later in the text.
cosmologists often introduce to describe inflation (the inflaton), the dilaton or a gravitational field $\langle R \rangle$ itself (the restriction to scalar condensates is just for notational convenience). The natural scale for such a condensate is, of course, the Planck scale. For the higher dimensional operators $Q^n$ which depend on $\Phi$ we assume that there is a factorization rule which allows us to estimate the higher order condensates in the following way: $\langle Q^n \rangle \sim \langle \Phi^n \rangle \sim \langle \Phi \rangle^n$. This assumption is not crucial for our purposes.

As discussed in [1], the fact that the perturbative expansion of arbitrary Greens functions for the fundamental field theory is asymptotic [12], with factorially growing coefficients, leads to the conclusion that the effective Lagrangian constructed from the high dimensional operators is also an asymptotic series in $\Phi$. Hence, given the above assumptions, the vacuum expectation value of the energy of the condensate field can be expressed as an asymptotic series in $\langle \Phi \rangle$:

$$\langle H_{eff} \rangle = \sum_{n=0}^{n=\infty} n! (-1)^n a_n \langle \Phi \rangle^n,$$

with coefficients $a_n \sim 1$. We can use the Borel representation formula to evaluate the asymptotic series for $\langle H_{eff} \rangle$.

Defining a function $f(s)$ through

$$a_n n! = \int_0^\infty f(s) s^{n-2} \exp(-\frac{1}{s}) ds$$

(for all values of n) we obtain

$$\langle H_{eff} \rangle = \int_0^\infty \frac{f(s) ds}{s(s + \langle \Phi \rangle)} \exp(-\frac{1}{s})$$

If the coefficients $a_n$ are not changing too fast, then the function $f(s)$ will be slowly varying, which we will assume to be the case in the following.

An example of an effective Lagrangian of the above form arises in QCD (see [1]). The effective Lagrangian for collective degrees of freedom (e.g. colorless mesons) is, after integrating out high frequency quark and gluon degrees of freedom, an asymptotic series when expanded in powers of the collective field.

It is known that the higher order quantum gravity corrections to physical observables in a de Sitter phase of an expanding Universe are in general infrared divergent. In particular, this divergence is observed in the vacuum correlator $\langle \phi^2 \rangle$ (see the recent papers [13]-[14] on this subject and references to previous publications therein). This divergence leads to corrections to expectation values which have power-law and not exponential dependence on time $t$. This means that nonperturbative dynamics should be taken into account. However, such dynamics is as yet unknown. Therefore we follow

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$^4$The assumption that the series is Borel summable is, as we argued in [1], not crucial for our analysis.
[1] and introduce a phenomenological cutoff parameter \( \epsilon(t) \) into the vacuum expectation value (VEV)

\[
\langle \Phi \rangle \to \frac{\langle \Phi \rangle}{\epsilon(t)},
\]

in order to account for the new physics responsible for removing the infrared divergences mentioned above. In the context of cosmology, we naturally expect this physics and therefore \( \epsilon \) to depend on time.

The introduction of the new field \( \epsilon(t) \) is an absolutely crucial ingredient of our approach. This is, as mentioned earlier, an attempt to take into account infrared physics which is not fully understood. It is possible that the function \( \epsilon(t) \) is related to the zero modes. These modes should be treated separately, and their dynamics may be described by a new classical effective field \( \epsilon(t) \).

At this point, the dynamics of this new field is not known, and the field magnitude is also uncertain. However (and this is our main point) we do know that one and the same function (one and the same physics) is responsible for explaining both phenomena under discussion: inflation and the cosmological constant problem. It may be the case (in the spirit of stochastic chaotic inflation [3]) that the initial conditions in different universes give rise to different behaviors of the function \( \epsilon(t) \), and that in some universe some of the conditions to be discussed in the next section are not satisfied. In this case inflation would continue forever, resulting in a completely empty and cold Universe with nonzero cosmological constant. For our Universe the function \( \epsilon(t) \) has some specific properties, in particular it is close to zero today. Exactly such a function, satisfying some constraints, we keep in mind in what follows\(^5\).

Making use of (4), the integral which describes the vacuum energy becomes

\[
\langle H_{\text{eff}} \rangle \sim \int_0^\infty \exp\left(-\frac{1}{s} \frac{f(s) ds}{s(s + \langle \Phi \rangle / \epsilon(t))} \right) \sim \epsilon \to 0
\]

and goes to zero if \( \epsilon \) tends to zero. Thus, the cosmological constant problem associated with the field driving inflation is reduced to the problem of analyzing the function \( \epsilon(t) \). As we demonstrated in [1], the effect (5) does not depend crucially on our assumptions concerning both the factorization properties of the condensates \( \langle \Phi^n \rangle \sim \langle \Phi \rangle^n \) and the exact factorial dependence of the coefficients. Both of these effects presumably can be modeled (as we have done) by introducing in formula (3) a smooth function \( f(s) \) whose moments exactly reproduce the given \( n \)–dependence of the coefficients. If this function is mild enough, it will not destroy the relation (5), but might change some numerical coefficients. Besides that, a condensate might have both a

\(^5\)We are grateful to Andrei Linde for very useful discussions regarding this subject.
singular piece proportional to \( \langle \Phi \rangle_{\epsilon(t)} \) and a regular part, i.e. be proportional to \( \langle \Phi \rangle_{\epsilon(t)} + \text{const.} \) As can be seen, this does not invalidate the relation (5).

A few remarks are in order. The vanishing of the vacuum energy is a consequence of the asymptotic nature of the effective Lagrangian and of the infrared properties of the VEVs (which are hidden in the dynamics of the function \( \epsilon(t) \)) and it does not require any fine tuning of parameters. The vanishing of the vacuum energy (5) can be interpreted (after inflation, when all relevant condensates presumably go to zero) as the vanishing of the cosmological constant, which is the only relevant operator in the effective Lagrangian (all other terms are marginal or irrelevant operators).

We close this section with the following remark: the strong infrared dependence of the vacuum condensate \( \langle \Phi \rangle \) is not a unique property of quantum gravitational effects in de Sitter space. Two-dimensional QCD with a large number of colors also exhibits a strong infrared dependence of a form similar to what we have postulated in (5). In particular, the so-called mixed vacuum condensates can be exactly calculated in this theory in the chiral limit \((m_q \to 0)\). They exhibit the following dependence on the infrared parameter \(m_q\) [15]:

\[
\frac{1}{2^n} \langle \bar{q}(ig\epsilon_{\lambda\sigma}G_{\lambda\sigma}\gamma_5)^n q \rangle = \left(-\frac{g^2\langle \bar{q}q \rangle}{2m_q}\right)^n \langle \bar{q}q \rangle,
\]

where \(q\) is a quark field and \(G_{\lambda\sigma}\) is a gluon field of \(QCD_2(N = \infty)\). The chiral condensate \(\langle \bar{q}q \rangle\) in this theory can be calculated exactly [16]. It does not vanish without contradicting Coleman’s theorem. The very important feature of this formula: it diverges in the chiral limit \(m_q \to 0\), where the parameter \(m_q\) plays the role of the infrared regulator of the theory. Now, if we consider the asymptotic series constructed from these condensates, then

\[
\sum_{n=0}^{\infty} (-)^n n! a_n \langle \bar{q}(ig\epsilon_{\lambda\sigma}G_{\lambda\sigma}\gamma_5)^n q \rangle \sim \int_0^{\infty} \exp\left(-\frac{1}{s}\right) \frac{ds f(s)}{s(s + \frac{1}{m_q})} \sim m_q \to 0, \quad (7)
\]

(with a function \(f(s)\) related to the coefficients \(a_n\) as before by (2)) we would get a result of zero for this series, in spite of the fact that each term on the left hand side diverges in the chiral limit and irrespective of the precise behavior of the coefficients \(a_n\) in the formula (7)! It should be clear that this result is a direct consequence of the asymptotic nature of the series. Although this example is only a toy model, it gives us a hint of what might happen in real Nature.

2 The Condensate as an Inflaton

In this section we will discuss the possibility that inflation can be realized within the same framework of an asymptotic series analysis of the Effective
Lagrangian which we briefly reviewed in the Introduction. We shall demonstrate that inflation can be obtained in a very natural way, driven by the same condensate $\langle \Phi \rangle$ which we discussed earlier.

The general ideology is the following. One could expect that at a very early epoch a phase transition similar to the confinement-deconfinement transition in QCD occurs. In the case of QCD we know that a gluon condensate is formed in the infrared region and disappears at higher temperatures. Such a behavior of a condensate is a welcome property for the inflationary scenario (see Ref.[8] where the idea of the inflaton being a condensate was also considered, but from a different point of view). If an analogous phenomenon could occur for $\langle \Phi \rangle$ it would provide the chance to describe inflation without introducing any new fields and specific potentials for them. Rather, inflation would be described in terms of the nontrivial dynamics of the gravitational field itself. A nonzero condensate can lead to inflation because it provides a nonzero potential. The vanishing of the condensate with time (as a consequence of the dynamics of all relevant fields) will automatically lead to a termination of the epoch of inflation.

How one could describe such a scenario in a more quantitative way? The honest answer is that we do not know, because such a description is inevitably based on quantum gravity at the Planck scale. However, one may try to use some phenomenological input to the problem. Thus, we suggest to consider the VEV of the energy (5) as an effective potential of the theory with the condensate $\langle \Phi \rangle$. As we discussed earlier, this condensate $\langle \Phi \rangle$ is not necessarily the condensate of a scalar field (which may not exist as a fundamental field in the theory), but rather an effective description of the strength of the interactions of all relevant fields. Therefore, with the motivation given above, we propose to interpret the potential (5) as an effective potential $V_{\text{eff}}(\Phi)$ which is the essential element of an inflationary cosmology:

$$V_{\text{eff}}(\Phi) = \tilde{c} \int_0^\infty \exp\left(-\frac{1}{s} \frac{f(s) ds}{s(s \sigma_d + \frac{\Phi}{\epsilon(t)})}\right),$$

with the constant $\tilde{c}$ setting the energy scale at which inflation takes place. We normalize $\epsilon(t)$ such that initially $\epsilon(0) = 1$. More precisely, we interpret this potential as an effective interaction which is an inherent element of the classical equation of motion describing the evolution of the condensate [3], [17]:

$$\ddot{\Phi} + 3H \dot{\Phi} + V'_{\text{eff}}(\Phi) = 0,$$

where $\dot{\Phi}$ denotes the derivative with respect to $\Phi$.

The main question addressed in this Letter is whether the condensate $\langle \Phi \rangle$ (denoted by $\Phi$ from now on) can act as the inflaton. We will show that the standard conditions for the inflationary potential (like the “slow rolling” condition) can easily be satisfied for a potential of the form (8). Before we
proceed, let us note that the potential \((8)\) is very different from one what people usually consider. In particular, it is very flat, has no minimum for finite \(\Phi\), and for large \(\Phi\) goes like \(1/\Phi\). However, these properties should not be considered as a sign that the theory is ill defined. As we discussed before, \(V_{\text{eff}}(\Phi)\) is not a fundamental potential describing a fundamental field. It must be considered as an effective one. Thus, there is no requirement for it to have a global minimum. This potential, by definition, is not designed to quantize a theory starting from the well-defined vacuum state. Rather, by construction, it should be thought as giving the effective strength of all relevant interactions.

We now turn to the analysis of inflation driven by a condensate with potential \(V(\Phi)\) given by \((8)\). In order to obtain a viable inflationary cosmology, several requirements must be satisfied. In order to obtain inflation at all, the “slow rolling” conditions must be obeyed. After inflation, there must be a period in which the energy of the inflaton is efficiently transferred to matter fields (this is the “reheating” requirement). A stringent constraint on all inflationary theories is that they not produce an amplitude of density perturbations in excess of the observational limits (the “fluctuation problem”). Finally, it must be verified that there are well justified initial conditions which lead to inflation.

In our scenario, the condensate \(\Phi\) forms at some very early time with a value \(\Phi \simeq 0\). Spatial fluctuations in the value of \(\Phi\) will not dominate the energy density. Hence, provided that the “slow rolling” conditions are satisfied, inflation will set in, and the spatial gradients will be exponentially suppressed. Thus, soon after the formation of the condensate, its equation of motion is well described by \((9)\).

A sufficient condition for inflation is that the “slow rolling” conditions
\[
V' m_{\text{pl}} \ll \sqrt{48\pi V}\tag{10}
\]
and
\[
V'' \ll 24\pi V m_{\text{pl}}^{-2}\tag{11}
\]
are satisfied. The first condition expresses the requirement that the potential energy of \(\Phi\) dominate over the kinetic energy (and one thus gets an inflationary equation of state), the second criterion ensures that the \(\ddot{\Phi}\) term in the equation of motion \((9)\) can be neglected (and one thus gets a period of inflation of a sufficient number of Hubble expansion times in order to solve the horizon and flatness problems [2]).

Let us first neglect the time dependence of \(\epsilon\). It is easy to show that for \(\Phi \ll \epsilon\) the slow rolling conditions are satisfied. Since \(f(s)\) is slowly varying and the exponential factor in \((8)\) cuts off the \(s\)-integral at \(s \simeq 1\), the potential \(V(\Phi)\) can be approximated as follows:
\[
V(\Phi) \simeq \tilde{c} \int_1^\infty dss^{-2}f(s) \sim \tilde{c}f(1).\tag{12}
\]
Analogous approximations for $V'(\Phi)$ and $V''(\Phi)$ show that the slow rolling conditions (10) and (11) are satisfied independent of the value of $\tilde{c}$.

The slow rolling conditions are satisfied also for large values of $\Phi$ ($\Phi > \epsilon$). In this case, the potential $V(\Phi)$ can be approximated as

$$V(\Phi) \simeq \tilde{c} \int_{1}^{s_{\Phi}} ds \frac{f(s)}{s^{\Phi}/\epsilon} \simeq \tilde{c} \frac{f(s_{\Phi})}{\Phi/\epsilon} \ln(s_{\Phi}),$$

(13)

where $s_{\Phi}$ is the value of $t$ for which

$$s_{\Phi} = \frac{\Phi}{\epsilon}.$$  

(14)

The quantities $V'(\Phi)$ and $V''(\Phi)$ can be estimated in a similar fashion, with the result that they are suppressed compared to (13) by $1/\Phi$ and $1/(2\Phi^{2})$, respectively. Thus, the inequalities (10) and (11) are again obeyed independent of the value of $\tilde{c}$.

From the above discussion is appears that the “slow rolling” conditions are satisfied for all values of $\Phi$. If this were true, the resulting cosmological scenario would be unacceptable since the scale factor would keep on inflating (albeit with a decreasing expansion rate), resulting in a completely empty and cold Universe today.

Post-inflationary reheating is a necessary requirement for a successful inflationary cosmology. In the usual models of inflation, the “slow rolling” approximation breaks down after a finite duration of inflation. Afterwards, the inflaton starts oscillating about its ground state. The inflaton oscillations induce parametric resonance [18], [19] in the fields which couple to $\Phi$, leading to rapid energy transfer from the inflaton to matter fields.

How can a graceful exit from inflation and reheating be realized in our scenario? The time dependence of the infrared cutoff $\epsilon$ may play a crucial role. At the beginning of inflation, we expect the effective infrared cutoff to be given by the Hubble radius (see [10] for the use of this infrared cutoff in a different context), i.e.

$$\epsilon(t) \propto H(t).$$

(15)

The value of $\epsilon$ will decrease as $\Phi$ increases and the Hubble expansion rate $H(t)$ decreases. Well into the inflationary epoch, our system must be analyzed as a two field problem, the fields being $\Phi(t)$ and $\epsilon(t)$. Our system then becomes similar to Linde’s two field inflation model [20], although the physical origin of the second field is quite different. We believe that there is no classical equation for $\epsilon(t)$ (beyond what is assumed in (15)). Its behavior is completely determined by the quantum nature of gravity. There is one more essential new element in comparison with Linde’s paper [20]: our effective potential (8) is very different from a standard potential for the scalar fields $\sim \Phi_{1}^{2} \phi_{2}^{2}$ which are finite polynomials with well defined minima. As we mentioned earlier,
our potential is not designed to quantize a theory starting from a well-defined vacuum state; rather it should be thought as an effective potential motivated by the form of the asymptotic series (1) of all relevant fields. Of course, one could start from the very beginning from the two field model given by potential (8) without any motivation concerning the origin of the $\Phi(t)$ and $\epsilon(t)$ fields. The technical analysis in this case would proceed along the lines of [20]. However, we believe that the motivation for the introduction of our fields is a very important issue even if we do not completely understand the dynamics of those fields.

There are indications [13] that the time dependence of expectation values of massless fields in an initial de Sitter state has corrections which depend on a power $p$ of time. Such a behavior is realized during the time when quantum corrections are in fact corrections, i.e. they are small. When they get large, we can no longer use a perturbative series. Rather, in order to describe $\epsilon(t)$ we must take into account all terms in the corresponding expansion at once. Clearly, we do not know how to do this at the moment. Instead, we assume a specific behavior for the function $\epsilon(t)$ with the following general feature: it is arbitrary function which decreases with time and vanishes at some finite time $t_c$. To be specific, we shall assume

$$\epsilon(t) = \epsilon(0)[1 - a(Ht)^p],$$

where $t = 0$ is the initial time, $p$ is an integer greater than 1, and $a$ is a constant much smaller than 1. For the two field system, the condition (10) required to obtain an inflationary equation of state becomes

$$\epsilon^2 + \dot{\Phi}^2 < 2V(\Phi).$$

Initially, the kinetic term for $\epsilon$ is subdominant. However, after a number of expansion times greater than $a^{-1}$, this kinetic term begins to dominate. Eventually it exceeds the potential energy. At this point, inflation ends.

We have thus shown that under certain assumptions about the time evolution of $\epsilon$, our model yields a graceful exit from inflation after a number of e-foldings which can be made large by decreasing the value of $a$. The specific manner in which the Universe reheats remains to be analyzed. We note that the vanishing of $\epsilon(t)$ at some time $t_c$ is an important feature of the function $\epsilon(t)$. This is because such a behavior enables us to obtain $\dot{\epsilon}^2 \gg H^4$ while $\epsilon$ itself is close to zero. After inflation ends, the Universe approaches the time $t_c$ when $\epsilon(t_c) = 0$. In the vicinity of this point our series (8) is completely reconstructed: condensates disappear and an effective potential in the form (8) does not make any sense. The existence of the function $\epsilon$ itself is also terminated. Right after this point we enter the stage when gravity is a weekly coupled theory and when the cosmological constant (vacuum energy) is almost zero.\(^6\)

\(^6\)The magnitude of the vacuum energy is proportional to $\epsilon$. Thus, the precise magnitude
Another way to obtain reheating (without invoking two field dynamics) is to postulate a phase transition once \( \Phi \) has reached some sufficiently large value, and to assume that at that point the energy in \( \Phi \) is released as thermal energy. It may also be possible to implement the “warm inflation” scenario [21] in which the dynamics of \( \Phi \) during inflation is driven by a thermal friction term \( \Gamma \dot{\Phi} \) in eq. (9) which dominates over the Hubble damping term, in which case there is continuous heating during inflation. However, we feel that the two field model described above is the most promising.

We believe that other problems which are inherent features of most inflationary models can also find their natural solutions in the approach suggested in this Letter. In particular, let us briefly mention the “fluctuation problem”. The only known solutions of this problem in the context of scalar field driven inflationary models involve introducing very small parameters into the scalar field potential. Otherwise, the predicted quantum fluctuations are in excess of those allowed by the bounds on cosmic microwave background (CMB) anisotropies.

If we were to treat \( \Phi \) as a fundamental scalar field and to apply the usual theory of quantum generation and classical evolution of fluctuations [22], [4], we would also encounter the “fluctuation problem” in our scenario. The magnitude \( \delta T/T \) of the predicted fluctuations in the CMB would be given by

\[
\frac{\delta T}{T} \sim \frac{HV'}{\dot{\Phi}^2},
\]

where the quantities are evaluated during the inflationary epoch at the time when the length scales which correspond to the angular scales of the CMB anisotropies being considered leave the Hubble radius. During the era of “slow rolling”, we can make use of the equation of motion (9) with \( \dot{\Phi} = 0 \) and equations (10) and (12) to estimate the quantities appearing in (18). Introducing the dimensionless constant \( c \) via \( \tilde{c} = cm_{pl}^5 \) we obtain the following estimate:

\[
\frac{\delta T}{T} \sim 10^2 f(1)^{1/2} c^{1/2}.
\]

Thus, unless \( f(1) \) happens to be extremely small (there is no reason to expect this to be the case), Planck scale inflation (\( c \sim 1 \)) seems to lead to the usual fluctuation problem in that \( \delta T/T \) is greater than one instead of \( 10^{-5} \). The reason for the fluctuations to be large is clear: at the Planck scale we have of the cosmological constant depends on the dynamics of the field \( \epsilon \). During the period of inflation the inertia of this field is very large because the kinetic term is very small for “slow rolling” conditions to be satisfied. The same inertia pushes the field \( \epsilon \) further and further to zero. It is not clear to us whether \( \epsilon \) will actually intersect the time-axis because of the same inertia, or our description in terms of a field \( \epsilon \) breaks down before the intersection takes place.
only one dimensional parameter. This parameter governs the dynamics of fluctuations. With the parameter of order one (in Planck units), an amplitude of fluctuations of order one will result.

In QCD we would encounter the same problem if we were to start from the wrong vacuum state. However, in QCD we know how to handle this problem, at least qualitatively\(^7\) We separate the strong and weak fluctuations. The strong fluctuations, which are presumably responsible for all nonperturbative phenomena, can be taken into account phenomenologically: they form a non-trivial background and nontrivial condensates which can not be calculated at the moment, but should be extracted from experiment. The weak fluctuations (which is the only relevant issue at the moment) in this background are in fact weak: standard perturbative techniques and the Wilson Operator Expansion are appropriate methods to calculate them.

The moral is simple: if one manages to describe the main vacuum background fields correctly, the quantum fluctuations in this true background will presumably be small, and it is these vacuum fluctuations in the gravitational field which will generate the CMB anisotropies. A strong quasi-classical vacuum configuration at the Planck scale is clearly of order one, however this is not the relevant configuration for the analysis of the CMB fluctuations. Of course, it is difficult to check such a treatment of the quantum fluctuations in gravity because we do not understand the quantum dynamics of the theory.

3 Conclusion

We have proposed an inflationary scenario in which inflation is driven by the dynamics of a condensate of gravitational fields which forms at some very early time in the evolution of the Universe (presumably at the Planck scale). We have interpreted the vacuum energy of this condensate as the effective potential which determines the dynamics of the condensate. The effective potential in turn is calculated using an asymptotic series approach discussed previously in Ref. [1].

We have shown that the “slow rolling” conditions (a key requirement for successful inflation) are automatically satisfied. The same dynamics of the infrared cutoff which is responsible for relaxing any contribution to the cosmological constant due to this condensate leads to a termination of the period of inflation and thus to a graceful exit from inflation\(^8\). We have also

\(^7\)If the dynamics of \(\Phi\) were dominated by friction instead of by the Hubble expansion, then the “fluctuation problem” could also be resolved as in Ref. [21] since the dominant fluctuations are thermal and not quantum.

\(^8\)In this respect, there are analogies with the proposal of Ref. [13]. However there are also differences: in Ref. [13] the infrared divergences act uniformly in space, whereas in our case the infrared dynamics is represented by a function \(\epsilon(t)\) which could in principle also depend on space. In general, this function need not be small. However, if inflation is successful, then the same inertia responsible for this success drives the function \(\epsilon\) (and
made some speculations concerning a mechanism by which the “fluctuation problem” could be solved. This mechanism relies on the fact that the field which drives inflation in our scenario is not a fundamental field, but rather a condensate with a complicated vacuum structure (like in QCD).

One of the nice features of our proposal is that inflation is tied to the way in which (at least part of) the cosmological constant problem is resolved. We believe that this is the most important new ingredient in the proposed inflationary scenario.

Acknowledgments

For useful discussions we are grateful to Andrei Linde, Paul Mende and Richard Woodard. One of us (R.B.) wishes to thank the Physics Department of the University of British Columbia for its hospitality during a visit when this work was initiated. The work of R.B. is supported in part by the U.S. Department of Energy under Grant DE-FG0291ER40688, Task A, the work of A.Z. is supported in part by the Natural Sciences and Engineering Research Council of Canada.

References


with it also the cosmological constant) to zero. A different behavior of $\epsilon(t)$ might lead to an empty and cold Universe with a nonzero cosmological constant.


