Probing Charge-Symmetry-Violating Quark Distributions in 
Semi-Inclusive Leptoproduction of Hadrons

J. T. Londergan, Alex Pang

*Dept. of Physics and Nuclear Theory Center, Indiana University, Bloomington, IN 47404*

A.W. Thomas

*Dept. of Physics and Mathematical Physics and Institute for Theoretical Physics, University of Adelaide, Adelaide, S.A., 5005, Australia*

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Abstract

Recent experiments by the HERMES group at HERA are measuring semi-inclusive electroproduction of pions from deuterium. We point out that by comparing the production of $\pi^+$ and $\pi^-$ from an isoscalar target, it is possible, in principle, to measure charge symmetry violation in the valence quark distributions of the nucleons. It is also possible in the same experiments to obtain an independent measurement of the quark fragmentation functions. We review the information which can be deduced from such experiments and show the “signature” for charge symmetry violation in such experiments. Finally, we predict the magnitude of the charge symmetry violation, from both the valence quark distributions and the pion fragmentation function, which might be expected in these experiments.
I. INTRODUCTION

It has long been been recognized that charge symmetry or isospin symmetry is respected in the strong interaction to a high degree of precision. Charge symmetry violation [CSV] in nuclear physics is generally good to within about 1% of the strong amplitude [1,2]. Therefore, in most analyses of strong interactions, it is reasonable to assume the validity of charge symmetry.

Charge symmetry has always been assumed in defining quark distribution functions. It is so common in quark/parton phenomenology that it is frequently not even mentioned as an assumption. Charge symmetry reduces by a factor of two the number of independent quark distributions one must define, and until recently there has been no compelling reason to suggest charge symmetry violation. On the other hand, there were no precise tests of charge symmetry in parton distributions, either.

At the nucleon level, charge symmetry appears as the symmetry under interchange of protons with neutrons. At the quark level, charge symmetry involves interchange of up and down quarks. Recently, the question of charge symmetry at the quark level has become of great interest. At the present time, we have a good quantitative knowledge of the valence and sea quark distributions in the nucleon. Furthermore, it is now possible to perform experiments whose precision, and range in \( x \), is sufficient to probe “small” components of quark distributions, and hence to test quantities like flavor symmetry of nucleon sea distributions, and charge symmetry of parton distributions, which were not accessible to direct experimental test in the past.

Interest in this general field has been sparked by the experimental evidence of violation of the Gottfried sum rule [3] reported by the New Muon Collaboration (NMC) [4]. Although this has widely been quoted as evidence of SU(2) flavor symmetry violation [FSV] in the proton sea [5]- [7] (i.e. \( \bar{u}^p(x) \neq \bar{d}^p(x) \)), an alternative explanation of that result could be from charge symmetry breaking in the proton sea, as has been pointed out in Refs. [8,9] (it could also arise from a linear combination of CSV and FSV effects in the nucleon sea).
Initial theoretical investigations of charge symmetry in valence quark distributions have
been carried out by Sather [10], and by Rodionov et al. [11]. Following the approach of Ref.
[12], they calculated valence quark distributions for the proton and neutron in a quark model
and in the MIT bag model, respectively. In both models the “majority” quark distributions
(i.e., $u^p(x)$ and $d^n(x)$) satisfied charge symmetry to within about 1%, while the “minority”
quark distributions ($d^p(x)$ and $u^n(x)$) were predicted to violate charge symmetry by 5% or
more at large $x$. More recently, Londergan et al. [13] proposed that pion-induced Drell-Yan
processes could be used to probe these CSV effects in nucleon parton distributions.

In this note, we point out that semi-inclusive pion production, from lepton deep inelastic
scattering on nuclear targets, could also be a sensitive probe of CSV effects in nucleon
valence distributions. Several authors have previously discussed the possibility of using semi-
inclusive scattering processes to address various physics issues [14–16]. In fact, the HERMES
collaboration at HERA [17] is currently taking experimental data on semi-inclusive pion
production from hydrogen and deuterium.

The organization of our paper is as follows. In Sec. II, we review the formalism for semi-
inclusive hadron production at high energies. We show that for an isoscalar nuclear target,
one can test charge symmetry violation in both the valence quark distribution functions, and
in the fragmentation function. To first order in these small effects, there is an experimental
combination for which these two quantities separate. In Sec. III, we derive predictions for
ratios of $\pi^+$ and $\pi^-$ semi-inclusive electroproduction from deuterium. We show both the
qualitative and quantitative “signatures” for charge symmetry violation in these reactions.
We present our conclusions and suggestions for future experimental work in Sec. IV.
A. General Formulas for Semi-Inclusive Reactions

In the quark/parton model, the semi-inclusive production of hadrons in deep inelastic lepton scattering from a nucleon is given by

\[
\frac{1}{\sigma_N(x)} \frac{d\sigma^h_N(x,z)}{dz} = \frac{\sum_i e_i^2 q_i^N(x) D^h_i(z)}{\sum_i e_i^2 q_i^N(x)}. \tag{1}
\]

In Eq. (1), \(q_i^N(x)\) is the distribution function for quarks of flavor \(i\), and charge \(e_i\), in the hadron \(N\) as a function of Bjorken \(x\). \(D^h_i(z)\) is the fragmentation function for a quark of flavor \(i\) into hadron \(h\). The fragmentation function depends on the quark longitudinal momentum fraction \(z = E_h/\nu\), where \(E_h\) and \(\nu\) are the energy of the hadron and the virtual photon respectively. We write the numerator in Eq. (1) as \(N_{Nh} \equiv \sum_i e_i^2 q_i^N(x) D^h_i(z)\), so the quantity \(N_{Nh}\) represents the yield of hadron \(h\) per scattering from nucleon \(N\), as a function of \(z\) and \(x\). In terms of these quantities, semi-inclusive production of a charged hadron from a proton can be described by

\[
N^{p\pm}(x,z) = \frac{4}{9} u^p(x) D^\pm_u(z) + \frac{4}{9} \bar{u}^p(x) D^\mp_u(z) + \frac{1}{9} d^p(x) D^\pm_d(z) + \frac{1}{9} \bar{d}^p(x) D^\mp_d(z) + \frac{1}{9} s^p(x) D^\pm_s(z) + \frac{1}{9} \bar{s}^p(x) D^\mp_s(z), \tag{2}
\]

where \(D^\pm_i(z)\) are the fragmentation functions for a quark (or antiquark) of flavor \(i\) into positively or negatively charged hadrons.

Charge conjugation invariance implies that \(D^\pm_u = D^{\mp}_u\) and \(D^\pm_d = D^{\mp}_d\). By making the additional assumption of charge symmetry, the fragmentation functions for pions from quarks will obey the relations

\[
D^-_d(z) = D^+_u(z)
\]

\[
D^+_d(z) = D^-_u(z)
\]

Later, we will consider the possibility of charge symmetry violation in the fragmentation functions.
We want to find an experimental signature for charge symmetry violation in valence quark distributions of the nucleon. We will therefore derive expressions for $\pi^+$ and $\pi^-$ electroproduction on an isoscalar nucleus. In this paper our expressions specifically refer to a deuteron target, although our results can be extended to any isoscalar target.

B. Experimental Extraction of Fragmentation Functions

Levelt et al. [14] derived a useful expression by which the ratio of fragmentation functions can be extracted from leptoproduction of charged pions on protons and neutrons. They proposed measuring the quantity

$$R(x,z) = \frac{N_{p\pi^+}(x,z) - N_{n\pi^+}(x,z) + N_{p\pi^-}(x,z) - N_{n\pi^-}(x,z)}{N_{p\pi^+}(x,z) - N_{n\pi^+}(x,z) - N_{p\pi^-}(x,z) + N_{n\pi^-}(x,z)}$$

(4)

In the quark/parton model, $R(x,z)$ has the form

$$R(x,z) = \frac{3}{5} \frac{\tau(x) - \bar{\tau}(x)}{\tau(x) + \bar{\tau}(x)} \frac{D_u^+(z) + D_u^-(z)}{D_u^+(z) - D_u^-(z)}$$

(5)

with the definitions

$$\tau(x) = u^p(x) - d^p(x)$$
$$\bar{\tau}(x) = -\bar{u}^p(x) + \bar{d}^p(x).$$

(6)

Integrating both the numerator and denominator of $R(x,z)$ over $x$ gave the result

$$Q(z) = \int_0^1 dx \frac{N_{p\pi^+}(x,z) - N_{n\pi^+}(x,z) + N_{p\pi^-}(x,z) - N_{n\pi^-}(x,z)}{N_{p\pi^+}(x,z) - N_{n\pi^+}(x,z) - N_{p\pi^-}(x,z) + N_{n\pi^-}(x,z)} /$$

$$\{ \int_0^1 dx [N_{p\pi^+}(x,z) - N_{n\pi^+}(x,z) - N_{p\pi^-}(x,z) + N_{n\pi^-}(x,z)] \}$$

$$= \frac{9}{5} S_G \frac{D_u^+(z) + D_u^-(z)}{D_u^+(z) - D_u^-(z)}$$

(7)

In Eq. (7), $S_G$ is the experimental value for the Gottfried Sum Rule [4].

The quantity $Q(z)$ is a function of the ratio of the “favored” and “unfavored” fragmentation functions, $D_u^+(z)$ and $D_u^-(z)$, respectively. They are so named because production of a positively (negatively) charged hadron will preferentially occur from the up (down) quark
in the nucleon. The fragmentation functions have been extracted by the EMC group \[18,19\], but an independent measurement of this can be obtained in pion leptoproduction. Feynman and Field \[20\] suggested a form for the ratio

$$\Delta(z) \equiv \frac{D_u^-(z)}{D_u^+(z)} = \frac{1 - z}{1 + z}$$

If Eq. (8) is correct, then measurement of Eq. (7) would give

$$Q(z) = \frac{9}{5z} S_G$$

The quantity $Q(z)$ of Eq. (7) requires charged hadron production from both protons and neutrons. For charged pion leptoproduction on an isoscalar target (e.g., deuterium), a useful quantity is $N_{D\pi}^+ - N_{D\pi}^-$. For an isoscalar target, the contributions from the sea will exactly cancel, and the result will depend only upon the valence quark contributions. From Eq. (2), we can show that this quantity can be written

$$N_{D\pi}^+(x, z) - N_{D\pi}^-(x, z) =$$

$$\left( \frac{4}{9} [u_p^u(x) + u_n^u(x)] - \frac{1}{9} [d_p^u(x) + d_n^u(x)] \right) \left[ D_u^+(z) - D_u^-(z) \right]$$

In Eq. (10), we assume the validity of the impulse approximation, i.e., $N_{D\pi}^+(x, z) = N_{p\pi}^+(x, z) + N_{n\pi}^+(x, z)$. Note that the valence quark distributions appearing in Eq. (10) are the quark distributions in the deuteron and not the free nucleon valence quark distributions. Integrating this quantity over all $x$, we obtain

$$N_D(z) \equiv \int_0^1 dx \left[ N_{D\pi}^+(x, z) - N_{D\pi}^-(x, z) \right]$$

$$= \left[ D_u^+(z) - D_u^-(z) \right]$$

The quantity $N_D(z)$ is proportional to the difference between the “favored” and “unfavored” fragmentation functions, $D_u^+(z)$ and $D_u^-(z)$, respectively. If we assume the Feynman–Field parameterization for the ratio of fragmentation functions, then measurement of Eq. (11) would give

$$N_D(z) = \frac{2z}{1 + z} D_u^+(z)$$
This could then be compared with the fragmentation functions extracted from the EMC group, or by the method suggested by Levelt et al. [14], which requires comparison of pion leptoproduction on both protons and neutrons separately.

The fragmentation functions have previously been measured in deep inelastic muon scattering by the EMC collaboration [18,19]. In Fig. [1a] we show the favored and unfavored fragmentation functions for quarks into charged pions, as measured by EMC. In Fig. [1b] we plot the experimental ratio of fragmentation functions $\Delta(z)$, and compare it with the Feynman-Field parameterization of Eq. (8). For moderate values of $z$, the Feynman-Field parameterization is rather accurate. For the largest values of $z$, the experimental errors are large, but the experimental results appear to be systematically larger than the Feynman-Field predictions.

C. Charge Symmetry Violation in Semi-Inclusive Pion Production

We want to measure charge symmetry violation in the valence quark distributions. For pion electroproduction on an isoscalar target, (such as the deuteron) this can be achieved by noting that (assuming charge symmetry) the ‘favored’ production of charged pions from valence quarks are related by

$$N_{f_{\text{fav}}}^{D\pi^+}(x, z) = 4 N_{f_{\text{fav}}}^{D\pi^-}(x, z).$$

(13)

That is, for $\pi^+$ ($\pi^-$) production, the “favored” mode of charged pion production is from the target up (down) quarks. Since the semi-inclusive reactions are proportional to the square of the quark charge, there is a relative weighting of 4 for $\pi^+$ production.

We therefore propose measuring the quantity $R^D(x, z)$, defined by

$$R^D(x, z) \equiv \frac{4 N_{f_{\text{fav}}}^{D\pi^-}(x, z) - N_{f_{\text{fav}}}^{D\pi^+}(x, z)}{N_{f_{\text{fav}}}^{D\pi^+}(x, z) - N_{f_{\text{fav}}}^{D\pi^-}(x, z)} = \frac{5\Delta(z)}{1 - \Delta(z)} - \frac{(4 + \Delta(z))\delta D(z)}{3[1 - \Delta(z)]^2} + \frac{1 + \Delta(z)}{1 - \Delta(z)} \left[ \frac{4(\delta d(x) - \delta u(x)) + 15(\bar{u}p(x) + \bar{d}p(x))}{3[u^e(x) + d^e(x)]} \right].$$

7
\[ R^D = \frac{\Delta_s(z)}{1 - \Delta(z)} \left[ s(x) + s(x) \right] \frac{[s(x) + \bar{s}(x)]}{[u(x) + d(x)]} \]  

(14)

The quantity \( R^D \) contains the charge symmetry violating quark distribution functions defined by

\[ \delta d(x) \equiv d^p(x) - u^n(x) \]
\[ \delta u(x) \equiv u^p(x) - d^n(x), \]

the strange/favored ratio of quark fragmentation functions

\[ \Delta_s(z) = \frac{D_{s^+}(z) + D_{s^-}(z)}{D_{u^+}(z)}, \]

and the charge symmetry breaking fragmentation functions,

\[ \delta D(z) \equiv \frac{D_{u^+}(z) - D_{d^-}(z)}{D_{u^+}(z)}. \]  

(15)

We can more cleanly separate the \( x \) and \( z \) dependence by multiplying \( R^D \) by a \( z \)-dependent factor, e.g.

\[ \tilde{R}^D(x, z) \equiv \frac{1 - \Delta(z)}{1 + \Delta(z)} \cdot R(x, z) \]
\[ = \frac{5\Delta(z)}{1 + \Delta(z)} - \frac{(4 + \Delta(z))\delta D(z)}{3(1 - \Delta^2(z))} + \frac{4[\delta d(x) - \delta u(x)]}{3[u(x) + d(x)]} \]
\[ + \frac{5(\bar{u}^p(x) + \bar{d}^p(x)) + \Delta_s(z)[s(x) + \bar{s}(x)]/(1 + \Delta(z))}{[u(x) + d(x)]} \]
\[ \equiv \tilde{R}_f^D(z) + \tilde{R}_{CVS}^D(x) + \tilde{R}_{sea}^D(x, z), \]  

(16)

where

\[ \tilde{R}_f^D(z) = \frac{5\Delta(z)}{1 + \Delta(z)} - \frac{(4 + \Delta(z))\delta D(z)}{3(1 - \Delta^2(z))} ; \]
\[ \tilde{R}_{CVS}^D(x) = \frac{4[\delta d(x) - \delta u(x)]}{3[u(x) + d(x)]} ; \]
\[ \tilde{R}_{sea}^D(x, z) = \frac{5(\bar{u}^p(x) + \bar{d}^p(x)) + \Delta_s(z)[s(x) + \bar{s}(x)]/(1 + \Delta(z))}{[u(x) + d(x)]} . \]  

(17)

Eq. (16) is obtained from Eq. (14) by multiplying by the experimentally measured fragmentation functions. In Eqs. (14), (16) and (17), we have expanded to first order in “small”
quantities. These are: the CSV nucleon terms, $\delta d(x)$ and $\delta u(x)$; the CSV part of the fragmentation function, $\delta D(z)$; and the sea quark distributions (Eq. (16) is only valid at large $x$ where the ratio of sea/valence quark distributions is small). We have neglected the CSV part of the “unfavored” fragmentation function. In the region of interest (moderately large $z$) the unfavored fragmentation function will be considerably smaller than the favored term, and consequently the unfavored CSV term should be proportionately smaller than the favored CSV term.

The quantity $\tilde{R}^D(x, z)$ separates into three pieces. The first piece, $\tilde{R}^D_f(z)$, depends only on $z$, as is shown in Eq. (17). It contains a small part which is proportional to the CSV part of the fragmentation function. The dominant piece of $\tilde{R}^D_f(z)$ has the form

$$\frac{5\Delta(z)}{1 + \Delta(z)} \approx \frac{5(1 - z)}{2},$$

(18)

where the relation in Eq. (18) follows if we adopt the Feynman-Field parameterization. For most values of $z$, this term should decrease monotonically as $z$ increases (although, at the largest values of $z$, it may increase, as suggested in Fig. [1b]). The second term, $\tilde{R}^D_{CSV}(x)$ depends only on $x$, and is proportional to the nucleon CSV fraction (relative to the valence quark distributions). The term $\tilde{R}^D_{sea}(x, z)$ is proportional to the sea quark contributions.

Experimentally, one needs to measure accurately the $x$-dependence of $\tilde{R}^D(x, z)$ for fixed $z$; in this case the $z$-dependent term will be large (of order one) and constant. The sea quark contribution will be large at small $x$, but should fall off monotonically and rapidly with $x$. So, if one goes to sufficiently large $x$, the sea quark contribution will be negligible relative to the CSV term. One then has to extract the small, $x$-dependent term in Eq. (16) from the large term independent of $x$. As a general rule, the larger the values of $x$ and $z$ at which data can be taken, the larger the CSV term will be relative to the $z$-dependent term.

Note that for a nuclear target, the quark distributions which appear in Eq. (14) are the nuclear quark distributions, and not the free nucleon distributions. However, for the nonstrange quarks a common assumption is that the quark distributions for flavor $i$, in a nucleus with $A$ nucleons, are related to the free ones by $q^A_i(x) = \epsilon(x)q^N_i(x)$. If $\epsilon(x)$ is
independent of quark flavor (most models of nuclear effects on quark distributions make this assumption), then this quantity cancels in the ratio in Eq. (16), which would then be identical to the ratio for free nucleons.

III. PREDICTIONS OF CHARGE SYMMETRY VIOLATION IN PION LEPTOPRODUCTION

A. A Simple Model for the Fragmentation Function into Charged Pions

We want an estimate of the fragmentation function for a quark into a charged pion. Our main interest will be to use this simple model to estimate the magnitude of charge symmetry violation in this fragmentation function, as this quantity enters into the ratio $\tilde{R}^D_D(z)$ of Eq. (17). Our calculation is based on a method used recently by Nzar and Hoodbhoy [21]. They calculated the fragmentation function for a quark into a nucleon plus diquark. The fragmentation function can be expressed in terms of the quark field operators and the light cone momentum fraction $z = P^+/k^+$

$$4 \frac{D^{\pi^j}_q(z)}{z} |_{P^+} = \sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \text{Tr} \left< 0 | \gamma^+ \psi(0) | P; X \right> \left< P; X | \bar{\psi}(\lambda n) | 0 \right>$$

In Eq. (19), $D^{\pi^j}_q(z)$ is the fragmentation function for a quark of flavor $i$ and four-momentum $k^\mu$ fragmenting into a meson of charge $j$ and four-momentum $P^\mu$, and the unmeasured state $X$, as is shown in Figure 2(a). We also define null vectors $p^\mu$ and $n^\mu$ such that $p^2 = n^2 = 0$, $p \cdot n = 1$, and $n \cdot A = A^+$ for any four vector $A$.

To calculate the fragmentation function of Eq. (19), we need to evaluate the contribution for every state $X$ and sum over the complete set of states. The simplest approximation is that for charged pions the fragmentation function is dominated by a quark which fragments into a pion with momentum $P^\mu$ and a single quark of momentum $(k - P)^\mu$; this is shown schematically in Figure 2(b). We further assume that we can approximate the amplitude for this process as
\[ \langle P; (k - P)|\bar{\psi}|0 \rangle \approx \bar{u}(k - P)\Phi(k^2) \frac{i}{k - m} \]  

(20)

In Eq. (20), a quark with current quark mass \( m \) fragments into a pion with mass \( M \) and a quark with constituent mass \( m_q \). The matrix \( \Phi(k^2) \) is given by \( \Phi(k^2) = \phi(k^2)\gamma_5 \).

Inserting Eq. (20) into Eq. (19), summing over the intermediate states and using the kinematic conditions gives

\[
D^\pi_q(z) = \frac{z}{4(1-z)^2} \int_0^\infty \frac{dk^2_{\perp}}{(2\pi)^2} \frac{\widetilde{m}_q^2 + z^2k^2_{\perp}}{(k^2 - m^2)^2} |\phi|^2 ,
\]

\[
\widetilde{m}_q^2 \equiv m_q^2 + (1-z)^2m^2 - 2mm_q(1-z) .
\]

(21)

The kinematic conditions for the fragmenting quark imply that

\[
k^2 = \frac{M^2}{z} + \frac{m_q^2 + zk^2_{\perp}}{1-z} ,
\]

(22)

while for the form factor, \( \phi \), we choose the damping factor

\[
\phi(k^2) = N \left( \frac{k^2 - m^2}{(k^2 - \Lambda^2)^3/2} \right) .
\]

(23)

With these assumptions the integral in Eq. (21) can be done analytically, with the result

\[
D^\pi_q(z) = \frac{N^2}{32\pi^2} \left( \frac{\widetilde{m}_q^2}{(1-z)\beta^2} + \frac{z}{\beta} \right) ,
\]

\[
\beta = \frac{M^2}{z} + \frac{m_q^2}{1-z} - \Lambda^2 .
\]

(24)

For our calculations we chose \( \Lambda = 0.4 \text{ GeV} \). For the fragmentation function \( D^\pi_u \) (i.e., an up quark fragmenting into a \( \pi^+ \) and a down quark), we chose \( m_u = 5 \text{ MeV} \) and \( m_q = 353 \text{ MeV} \). The resulting theoretical fragmentation functions are shown as the solid curve in Figure 3(a). We have arbitrarily normalized our fragmentation functions to the experimental measurements of the EMC collaboration [18,19], at large \( z \). As can be seen, for \( z < 0.4 \) there is a significant deviation between the calculated and measured fragmentation functions. This is not surprising as we expect that the truncation of the complete set of states, \( X \), at a single, constituent quark should only be reasonable at large \( z \).
Using these approximations for the fragmentation function, we can estimate the magnitude of charge symmetry violation in the favored fragmentation function, by recalculating $D_d^{-}(z)$ and taking into account light quark mass differences. For this fragmentation function we used $m_d = 8$ MeV and $m_q = 350$ MeV. In Figure 3(b) we plot the charge symmetry violating fraction for the fragmentation function, $\delta D(z)$, given by Eq. (15). The predicted CSV term is of the order of a few percent. $\delta D(z) < 0$ as expected, since $D_d^{-}$ involves creation of a relatively light $u\bar{u}$ pair compared to the heavier $d\bar{d}$ pair created in the case of the fragmentation function $D_u^{+}$. Our simple model predicts a rather large CSV term. This is because the pion mass is unusually small, and the light quark mass differences are small in magnitude but large as a fraction of the current quark masses. We emphasize that this calculation should give only a crude estimate of the size of CSV terms in the quark fragmentation function.

B. Calculations of CSV in Pion Leptoproduction

In the previous section we proposed measuring the quantity $\tilde{R}^{D}(x, z)$, a ratio of charged pion leptoproduction cross sections on the deuteron. As seen from Eqs. (16) and (17), this quantity separates into three parts: one piece which depends only on $z$, a second which depends only on $x$ and is proportional to the quark CSV, and a third part depending on the contribution from sea quarks.

In Fig. [4] we plot the $z$-dependent contribution to $\tilde{R}^{D}(x, z)$, i.e. $\tilde{R}^{D}_f(z)$ of Eqs. (16) and (17). The solid dots are the values for this quantity using the EMC fragmentation functions [18,19]; they do not include a CSV contribution from the fragmentation function, and therefore depend only on the ratio of favored to unfavored fragmentation functions. The open dots include our estimate of the charge symmetry violating term from the fragmentation function, taken from Eqs. (15) and (24). The CSV term is roughly 1% of the total $z$-dependent term. The overall $z$-dependent term is of order unity; it falls off monotonically and smoothly with increasing $z$, until $z \approx 0.6$, after which it remains constant and may
increase somewhat. The solid curve shows this quantity as approximated by the Feynman-Field parameterization [20].

In Fig. [5] we plot our predictions for the $x$-dependent terms in $\tilde{R}^D(x, z)$, at $Q^2 = 10$ GeV$^2$. The long dashed curve is the contribution from nonstrange sea quarks to $\tilde{R}^D_{sea}(x, z)$. This depends only on $x$, and is calculated using the CTEQ3M parton distributions from the CTEQ group [22]. The short dashed curve is our prediction for the parton charge symmetry violating term, $\tilde{R}^D_{CSV}(x)$; this uses the CTEQ3M parton distributions, plus the bag model prediction for valence quark CSV from Londergan et al. [13]. The dot-dashed curve is the contribution from strange quarks. This last term is proportional to the strange quark fragmentation function. As we do not know this, we have removed the $z$ dependence of this term by approximating $\Delta_s(z)/(1 + \Delta(z)) \approx 1$. This will substantially overestimate the strange quark contribution, but even with this approximation the strange quarks contribute only a small fraction of the total. The solid curve is the sum of the three terms.

From Fig. [5] we see that for $x \approx 0.5$, the CSV term is as large as the sea quark contribution, and with increasing $x$ the CSV term dominates the $x$ dependence of this ratio. We predict the maximum CSV contribution will be of order $0.02 - 0.04$. In the small-$x$ region dominated by the sea quarks, $\tilde{R}^D_{sea}(x, z)$ (at constant $z$) should decrease rapidly with increasing $x$. However, for $x \geq 0.55$ the CSV term is predicted to dominate this ratio, and at the largest values of $x$ the terms $\tilde{R}^D_{CSV}(x) + \tilde{R}^D_{sea}(x, z)$ should be dominated by the CSV term.

If we measure the $x$ dependence of $\tilde{R}^D(x, z)$ at constant $z$, then the $x$ dependent contribution shown in Fig. [5] will sit on a large and constant $z$ dependent term, estimated in Fig. [4]. At small $x$ the sea quark term should be rapidly and smoothly falling with $x$. The value of the constant $z$-dependent background can be obtained by extrapolating the sea quark contribution to zero at large $x$; the CSV contribution would then be the difference between this extrapolated value and the measured value at large $x$.

From our previous work on parton charge symmetry violation [11,13] we predict that
the two CSV terms in Eq. (16) (i.e., $\delta d(x)$ and $\delta u(x)$) will be roughly equal in absolute value (each of them should be of the order 1-2% of the average up + down valence quark distribution), and they should have opposite signs. As a result, in Eq. (16) the two CSV contributions would add constructively, and should produce a term whose value is of the order of $0.02 - 0.04$. Sather’s CSV parton distributions [10] have the same qualitative behavior as ours.

We predict that the $z$-dependent term will be much larger then the $x$-dependent terms, as can be seen from comparing Figs. [4] and [5]. Using the EMC measured values for the fragmentation functions, as $z$ goes from 0.4 to 0.8 the $z$-dependent term in Eq. (16) should vary between approximately 1.5 and 1. So the CSV term is expected to be between 1-4% of the $z$-dependent term.

One major result of our calculations is the prediction that the $x$ and $z$-dependent parts of $\tilde{R}^D(x, z)$ will separate. Our current calculation has been carried out in the “naive parton model;” we have not included things like higher-order contributions to leptoproduction or scaling violations. Since we predict that CSV terms will contribute at the few percent level, it will be necessary to investigate whether these higher-order contributions to charged pion electroproduction are negligible compared with our lowest-order contributions. Another question is whether these additional contributions will preserve the separation of variables $x$ and $z$ in the quantity $\tilde{R}^D(x, z)$ of Eq. (16). These will require more sophisticated calculations, which we are presently undertaking.

In the HERMES experiment at HERA, the goal is to make precision measurements of the spin structure functions. For this one must know very accurately the spin dependence of high energy electron scattering from deuterium. This requires precise knowledge of the sources of systematic error, so the prospect for obtaining very accurate spin-averaged charged pion leptoproduction data is excellent. Only data from deuterium targets is required; efficient detection of both signs of charged pions is important, but absolute yields are not required as overall normalizations cancel out in the ratio of Eq. (16).
IV. CONCLUSIONS AND FUTURE EXPERIMENTS

In conclusion, we have shown that one can use charged pion electroproduction on an unpolarized deuteron target to measure the charge symmetry violating [CSV] contribution in the nucleon’s valence parton distributions. These are expected to be of the order of a few percent, so one has to extract a small term in the presence of a large background. Since one of the terms depends only on $z$, and the other only on $x$, one expects to exploit this feature experimentally. This would constitute the first direct measurement of charge symmetry violation in the valence quark distribution functions.

In defining parton distribution functions, it is routinely assumed that charge symmetry is valid. Surprisingly enough, there are very few tests of this assumption. A reasonable estimate of charge symmetry violation would be at roughly the 1% level, but it is conceivable that charge symmetry violation could be a few times this value and would not have been observed to date. An experimental upper limit for charge symmetry in valence quark distributions would be very useful. Measurement of a nonzero effect would be extremely interesting. In this paper we have explained why the measurement of charged pion leptoproduction from an isoscalar target, for example charged pion electroproduction from deuterium, would constitute an excellent test of charge symmetry in the valence quark parton distributions. We also estimated the charge symmetry violation in the pion fragmentation functions for the first time and showed that it would not interfere with the extraction of information about the nucleon valence distributions. As measurements of semi-inclusive charged pion production on the deuteron are currently in progress in the HERMES experiment at HERA, we are hopeful that suitable data might soon be available.

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1. Fragmentation function for quarks into pions, vs. momentum fraction \( z \). (a) Circles: favored fragmentation function \( D_{u}^{\pi^{+}}(z) \); triangles: unfavored fragmentation \( D_{u}^{\pi^{-}}(z) \), vs. \( z \). Data is from the EMC collaboration, Ref. [18] for open points and Ref. [19] for solid points. (b) Unfavored/favored ratio \( \Delta(z) = D_{u}^{\pi^{-}}(z)/D_{u}^{\pi^{+}}(z) \). Data from the EMC collaboration. Solid curve: theoretical prediction of Feynman and Field, Ref. [20].

2. (a) Fragmentation of quark with momentum \( k \) into a pion with momentum \( P \), and any remaining state \( X \). (b) Approximation to fragmentation function where the fragmentation is assumed to be dominated by a quark-pion-quark vertex.

3. (a) Estimate of quark-pion favored fragmentation function \( D_{u}^{\pi^{+}}(z) \). Solid curve: model result given by Eq. (24), assuming \( m_u = 5 \, \text{MeV} \), \( m_q = 353 \, \text{MeV} \), \( \Lambda = 400 \, \text{MeV} \); experimental points are those of the EMC group, Ref. [18,19]. Dashed curve: fragmentation function \( D_{d}^{\pi^{-}}(z) \), from Eq. (24), with \( m_d = 8 \, \text{MeV} \), \( m_q = 350 \, \text{MeV} \). (b) Estimate of charge symmetry violation in fragmentation function. The quantity \( \delta D(z) \) of Eq. (15) and the model fragmentation functions of Fig. (3a).

4. The quantity \( \tilde{R}_{D}^{P}(z) \) of Eq. (17). Fragmentation functions taken from the EMC experiment, Ref. [18,19]. Solid dots: does not include charge symmetry violation in fragmentation function; open dots: includes estimate of CSV in fragmentation function, from Eqs. (15) and 24 (and shown in Fig. [3]). Solid curve: the quantity \( \tilde{R}_{D}^{P}(z) \) using the Feynman-Field approximation to the fragmentation functions, Ref. [20].

5. \( x \)-dependent contributions to \( \tilde{R}_{D}^{P}(x,z) \) of Eq. (16). Long-dashed curve: contribution from nonstrange sea quarks; dashed curve: \( \tilde{R}_{CSV}^{P}(x) \) of Eq. (17), the contribution from CSV in nucleon parton distributions; dot-dashed curve: estimate of contribution to \( \tilde{R}_{sea}^{P}(x,z) \) from strange quarks; solid curve: total contribution including both sea quark and CSV contributions. Parton distributions (CTEQ3M) are from the CTEQ group, Ref. [22], at \( Q^2 = 10 \, \text{GeV}^2 \); CSV term is from bag model calculation of Londergan et al., Ref. [13].
Fig. 1 (a)

Fig. 1 (b)

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Fig. 2
Fig. 3 (a)

Fig. 3 (b)
Fig. 4
Fig. 5