X-Ray Fluctuations from
Locally Unstable Advection-Dominated Disks

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Abstract

The response of advection-dominated accretion disks to local disturbances is examined by one-dimensional numerical simulations. It is generally believed that advection-dominated disks are thermally stable. We, however, find that any disturbance added onto accretion flow at large radii does not decay so rapidly that it can move inward with roughly the free-fall velocity. Although disturbances continue to be present, the global disk structure will not be modified largely. This can account for persistent hard X-ray emission with substantial variations observed in active galactic nuclei and stellar black hole candidates during the hard state. Moreover, when the disturbance reaches the innermost parts, an acoustic wave emerges, propagating outward as a shock wave. The resultant light variation is roughly (time) symmetric and is quite reminiscent of the observed X-ray shots of Cygnus X-1.

\textbf{Key words:} Accretion, Accretion disks – black hole physics – Stars: individual (Cygnus X-1) – X-rays: stars
1. Introduction

One of the most intriguing features of stellar black-hole candidates (BHCs) is their rapid X-ray variability or flickering (see e.g., Oda 1979; Lochner, Swank, & Szymkowiak 1991; Miyamoto et al. 1991). It is known that stellar BHCs, such as Cygnus X-1, have two distinct spectral states; in the soft (or high) state the emergent spectra are roughly Planckian, whereas in the hard (or low) state, the spectra are of a power-law type. Rapid X-ray fluctuations are more pronounced during the hard state (Miyamoto et al. 1992). Similar fluctuations exist in active galactic nuclei, as well (see e.g., Lawrence et al. 1987; McHardy and Czerny 1987). Their origins need to be understood as a general phenomenon involved in accretion onto compact objects.

There have been many efforts devoted to the physical understanding of the origin and mechanism for creating such fluctuations. (e.g., Terrell 1972, Abramowicz et al. 1991, Mineshige, Takeuchi, & Nishimori 1994, Takeuchi, Mineshige, & Negoro 1995). Some of the observational features, such as the power spectra, can be reproduced by these models, but all the models are still somewhat phenomenological, and the physical cause of the variabilities remains as one of the great mysteries of X-ray astronomy.

We here stress that the noise characteristics should be understood on the basis of the disk models. It is reasonable to assume that the soft-state disk is the standard-type disk, since it predicts a Planckian spectrum with a temperature of $\sim 1$ keV, which is exactly what we observe. The standard-type disk is thermally stable, thus producing few fluctuations during the soft state. In contrast, the disk structure in the hard state is poorly understood. At least, the hard spectrum indicates the disk’s being optically thin. However, the hard-state disk cannot
be of the sort invoked by Shapiro, Lightman & Eardley (1976), since this is thermally unstable (Piran 1978).

Interestingly, there exists a distinct, optically thin solution dominated by advection (see Narayan & Yi 1995; Abramowicz et al. 1995). We demonstrate here that this type of disk can produce persistent, but fluctuating hard X-ray emission. Kato, Abramowicz, & Chen (1996), in fact, found by linear analysis that such disks are globally stable, but could be unstable for short wavelength perturbations. Since this analysis is restricted to the linear stage of a thermal instability, its observational implication are open to question. We are thus motivated to perform nonlinear, global numerical simulations of the growths of thermal perturbations in the optically thin, advection-dominated disks. The basic equations and numerical procedures are described in §2. The resultant time evolution of perturbations (or disturbances) will be presented and discussed in relation to the observed X-ray variabilities in §3.

2. Physical Assumptions and Numerical Procedures

Assuming that the disk is geometrically thin, $H < r$, with $H$ and $r$ being the half-thickness and the radius of the disk, we integrate the disk’s structure in the vertical direction. The basic equations are those of mass conservation, momentum conservation, angular momentum conservation, the energy equation (Matsumoto et al. 1984);

\[
\frac{\partial}{\partial t}(r\Sigma) + \frac{\partial}{\partial r}(r\Sigma v_r) = 0, \tag{1}
\]

\[
\frac{\partial}{\partial t}(r\Sigma v_r) + \frac{\partial}{\partial r}(r\Sigma v_r^2) = -r\frac{\partial W}{\partial r} + r^2\Sigma(\Omega^2 - \Omega_K^2) - W\frac{d\ln \Omega_K}{d\ln r}, \tag{2}
\]

\[
\frac{\partial}{\partial t}(r^2\Sigma v_\varphi) + \frac{\partial}{\partial r}(r^2\Sigma v_r v_\varphi) = -\frac{\partial}{\partial r}(r^2\alpha W), \tag{3}
\]
and
\[
\frac{\partial}{\partial t}(r\Sigma \epsilon) + \frac{\partial}{\partial r}(r\Sigma \epsilon v_r) = -\frac{\partial}{\partial r}(rWv_T) - \frac{\partial}{\partial r}(r\alpha Wv_\phi) - rQ_{\text{rad}},
\]
(4)

where \(\Sigma\) is surface density, \(W\) is integrated pressure, \(\Omega(= v_\phi/r)\) and \(\Omega_K(= (GM/r)^{1/2}/(r - r_g))\) with \(M\) the mass of the central black hole] are the angular frequency of the gas flow and Keplerian angular frequency, respectively, \(\alpha\) is the viscosity parameter, \(\epsilon\) is the internal energy of accreting gas, \(r_g \equiv 2GM/c^2 \simeq 3 \times 10^6 (M/10M_\odot)\) cm is the Schwarzschild radius. \(Q_{\text{rad}}^-\) is radiative cooling rate per unit surface area due to thermal bremsstrahlung. In the energy equation (4), the second term on the left-hand side corresponds to advective energy transport, while the last two terms on the right-hand side represent viscous heating and radiative cooling, respectively. Throughout this Letter, we assign \(M = 10M_\odot\), \(\alpha = 0.3\).

2.2. Initial and boundary conditions

We first calculate a steady disk structure by integrating the basic equations in the radial direction, omitting the terms with time derivatives. In the entire region for time-dependent calculations, cooling is dominated by advection, although we calculate the global steady solution starting integration at \(r = 1000r_g\) assuming that \(\Omega = \Omega_K\) and viscous heating is balanced with radiative cooling. Next, we add a density perturbation (or disturbance), \(\delta \Sigma\), to the steady disk structure. Its functional form is \(\delta \Sigma/\Sigma = 0.2 \exp[-(r - r_0)^2/(\lambda/2)^2]\), where \(r_0\) and \(\lambda\) are free parameters, and we set \(r_0/r_g = 50.0\) and \(\lambda/r_g = 10.0\). Small disturbances are added to all physical variables for consistency; \((\delta \Sigma/\Sigma) \approx kr(\delta W/W) \approx (kr)^{1/2}(\delta v/v) \approx (kr)^{1/2}(\delta \Omega/\Omega)\), according to the linear analysis (Kato et al. 1996), where \(k\) is the wavenumber of the disturbance, \(k = 2\pi/\lambda\).

We add a mass of \(\dot{M}_0 \Delta t\) into the disk through the outer boundary at every time step (\(\Delta t\)). We assume free boundary conditions at the inner boundary at
\( r = 2r_g \). This treatment is appropriate, since the inner boundary is located in the supersonic region. Physical quantities at the outer boundary at \( r = 100r_g \) are set to be equal to their steady values.

3. Time Evolution of Advection-Dominated Disks

3.1. Formation of X-ray shots

We display in Figure 1 the time evolution of an added disturbance in the three-dimensional \((r, t, \Sigma)\) plane. We assume a mass-input rate of \( \dot{M}_0 = 10^{-2}\dot{M}_c \) with \( \dot{M}_c \equiv \frac{32\pi c r_g}{\kappa_{es}} \) in the present study. The lowest curve represents the initial state (a steady state with a disturbance). Although the disturbance slowly damps as it accretes, the damp timescale is comparable to the accretion timescale. Once a disturbance is given at the outer portions of the disk, that disturbance will possibly persist as a blob, flowing inward with the same velocity as that of accreted matter. The accretion velocity is less than, but comparable to the free-fall velocity.

According to the linear analysis by Kato et al. (1996), a thermal instability tends to grow for any perturbation. The growth timescale is roughly the same order as the accretion timescale, \( \sim (kr)^{-1/2}(\alpha \Omega)^{-1} \). The usual viscous diffusion process, on the other hand, rather suppresses the growth of the perturbations as global effects. Therefore, whether the disturbance will grow or damp depends on the wavelength and nature of the disturbance. It is important to note that the disk material will reach the inner edge before the disturbance grows to significantly alter the global disk structure. We thus expect persistent X-ray radiation, although it could be fluctuating at every time. This is exactly what is observed in X-ray binaries during the hard (low) state and in active galactic nuclei.
It is interesting to note that when the disturbance reaches the innermost parts, it gives rise to an acoustic wave, which propagates outward and evolves to become a shock wave. Figure 2 displays the time variations in the radial profile of Mach number (top) and that of integrated pressure (W, bottom). We have calculated several models with different $\lambda$-values, finding that the wave reflection occurs only when $\lambda > r_g$. We have also calculated several models with different $\dot{M}$-values ($\dot{M}_0 = 10^{-2}, 10^{-4}, 10^{-5} \dot{M}_c$) and found that the time evolution of the disks is basically similar.

The plausible reason for the wave reflection is as follows: The incident wave is a thermal mode (Kato et al. 1996) derived by using the short wavelength approximation, the essential point of which is that the ambient gas can be regarded homogeneous to the perturbation. When the thermal mode reaches the innermost region, where the ambient gas is no longer homogeneous, it cannot exist as a pure thermal mode and evolves into a mixture of thermal and acoustic modes. The thermal mode and the acoustic mode propagating inward are swallowed by central black hole and only the acoustic mode propagating outward can escape, which is the reflected wave itself.

Finally, we calculated the total disk luminosity, $L_d = \int 2\pi r Q_{rad}^- dr$, and depict the light curve in Figure 3. We see a broadly peaked, (time) symmetric profile in the light curve that will be observed as an X-ray shot. Using Ginga data, Negoro, Miyamoto, & Kitamoto (1994) directly obtained a mean time profile of the shots by superposing a number of shots aligning their peaks; it was sharply peaked and is rather (time) symmetric. We here demonstrate that such a symmetric light variation can be reproduced by the advection-dominated disk model. Although the initial perturbation amplitude was only $\sim 20\%$, the disk luminosity changed by a factor of $\sim 60\%$. Perturbations with amplitudes of $20 – 30\%$ are sufficient to reproduce observed light variations. We note here that our
prime result of symmetric light curve is a result of wave reflection itself rather than growth of the wave to the shock wave, since the most of all luminosity comes from the region inside the radius of $10r_g$ while the wave grows to the shock wave at the radius larger than $20r_g$.

Another noteworthy feature of X-ray shots is spectral hardening. The spectrum is softer than the source’s average before the peaks of individual X-ray shots. It then suddenly becomes harder just after the peak. (Negoro et al. 1994) The sudden temperature rise at the innermost region due to the accoustic wave reflection can explain this spectral hardening. Note also that temperature of the incident thermal mode is lower than the steady state because of efficient cooling, which will explain the softer spectrum before the peak.

In conclusion, the observed X-ray variability can be explained, if the disk is optically thin and advection-dominated. Note that the optically thick, advection-dominated disks are likely to undergo relaxation oscillations (Honma, Matsumoto, & Kato 1991).

Although we have assumed the disk to be geometrically thin, this assumption may not be perfectly satisfied. Even so, however, Narayan & Yi (1995) have shown that the height integrated solutions are good representation of nearly spherical flows, if one interprets the height integration as a spherical average. Hence it seems reasonable to believe that our results show (at least qualitatively) the properties of time evolution of optically thin advection dominated accretion flows. However, two-dimensional simulations (in the radial and vertical directions) are urgently needed to confirm our present findings. When two-dimensional flow is allowed, circulating flow or turbulence may appear, since the flow speeds in the equatorial plane and in the disk surface layer may differ.

3.2. Origins of X-ray fluctuations
Several independent mechanisms seem to be involved for producing the observed X-ray variabilities: (1) generation of disturbances in smooth accretion flow, (2) formation of X-ray emitting blobs from the disturbance, and (3) production of complex variability light curves, as are observed, by superpositions of X-ray shots. What we have discussed so far concerns the second step; the thermal instability provides a mechanism to create an X-ray shot from a disturbance, once the advection-dominated disk is perturbed somehow.

As to the third step above, we note that the observed X-ray fluctuations are made up of plenty of individual peaks or shots with different peak intensities. The peak intensities are smoothly distributed from large ones to smaller ones, roughly showing a power-law or exponential distribution (Negoro et al. 1995). This can be understood if the disk is in a self-organized critical (SOC) state (see Takeuchi et al. 1996).

The smooth distribution in the released energy of one shot suggests that the ignition radii, at which a disturbance is given, and initial perturbation amplitudes are also distributed smoothly from the smaller one to the larger. The maximum radius can be determined from the time duration of the long X-ray shots. The accretion timescale is roughly given by the free-fall timescale, which is

$$\tau_{\text{ff}} \sim \left( \frac{r^3}{GM} \right)^{1/2} \simeq 4 \left( \frac{r}{10^3 r_g} \right)^{3/2} \left( \frac{M}{10 M_\odot} \right) s. \quad (6)$$

The duration of $\sim 10$ s for large X-ray shots corresponds to the free fall timescale at $\sim 10^3 r_g$ for $M \sim 10 M_\odot$. We thus understand that the maximum radius, at which shots are created, is of the order of $\sim (1 - 2) \times 10^3 r_g$. This number is a good agreement with the radius separating the outer, standard-type and inner, advection-dominated portions of the disk, which was estimated from the fitting to the optical – soft X-ray spectra of black-hole X-ray transients during the quiescence (Narayan, McClintock, & Yi 1996).
Finally, we discuss the possible seeds for assumed disturbances (the first step above). The most promising possibility at present is magnetic flares (Galeev, Rosener, & Vaiana 1979; Mineshige, Kusunose, & Matsumoto 1995). Because of strong differential rotation and rapid accretion, magnetic fields are at any times being amplified. Magnetic reconnection is then a way to release magnetic energy out of the disk when magnetic pressure becomes comparable to gas pressure (Shakura & Sunyaev 1973). We expect that magnetic flares can occur when magnetic energy is stored up to a certain level, regardless of the main radiation mechanisms of the disk. Once a flare occurs, then a disk material at that part will fall onto the black hole. We need further theoretical studies on the detailed flare mechanism to confirm the above scenario.

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Fig. 1. Time evolution of the optically thin, advection-dominated disks under a density disturbance applied at the outer parts. The disturbance in the surface density distribution propagates inward and is reflected at the innermost parts as an outgoing acoustic wave. The surface density $\Sigma$ is shown in unit of $g\text{cm}^{-2}$.

Fig. 2. The time variations in the radial Mach-number profile (top) and radial integrated-pressure profile ($W$, bottom) in units of $g/s^2$ after the wave reflection. The elapsed times are 600 (indicated by “i”), 800, 900, 1000, 1100, and 1200 (“f”) in units of $c/r_g = 10^{-4}s$. The dotted line represents the initial steady state (without disturbances). The acoustic wave emerges and evolves to become a shock wave.

Fig. 3. The light variation from the advection-dominated disk under the density disturbance. The units of time and the disk luminosity are $c/r_g = 10^{-4}s$ for $M = 10M_\odot$. The rather symmetric time profile is due to the wave reflection (cf. Fig. 1). The time duration of one X-ray shot is of the order of the accretion timescale.