A Perturbative Approach to Diffractive Deep Inelastic Scattering

J. Bartels^a, M. Wüsthoff^b

^a*II. Institut für Theoretische Physik, Universität Hamburg.*

^b*High Energy Physics Division, Argonne National Laboratory, USA.*

Abstract: We calculate the cross section for Diffractive Deep Inelastic Scattering based on perturbative QCD, including transverse and longitudinal polarization of the photon. We consider both $q\bar{q}$ and $q\bar{q}g$ production. The Pomeron which is part of the cross section is determined from data for the proton structure function. We find fair agreement with recent measurements by H1 and ZEUS. The β -spectrum is completely predicted over the full range from 0 to 1.

1 Introduction

Diffractive Deep Inelastic Scattering has gained a lot of attention, since the first Large Rapidity Gap events have been reported by H1 and ZEUS. New and more precise data have been published recently [1, 2]. The existence of a significant number of Large Rapidity Gaps events can only be explained by a colour neutral exchange, usually called 'Pomeron'. These events appear as an excess to 'standard' Monte Carlo simulations for Deep Inelastic Scattering which are based on parton showers where Large Rapidity Gaps are strongly suppressed. When the proton is considered to be in the final state without breaking up the exchange is certainly neutral. Over the time the Pomeron has been given two attributes: 'soft' following the conventional Regge phenomenology [3] and 'hard' (or 'BFKL') referring to pQCD as the underlying theory [4]. In this paper we focus on the perturbative approach, even though we also try to include nonperturbative aspects.

One could argue that due to the large scale Q^2 , the virtuality of the photon, the Pomeron should be hard. However, in [5] it has been shown that providing a large photon virtuality is not enough. One rather has to ensure the hardness of the final state, i.e. the transverse momenta of the produced partons have to be large. When allowing the transverse momenta to become small the character of the Pomeron changes from hard to soft.

The 'soft' Pomeron (Pomeranchuk Pole) has its origin in Regge phenomenology and is associated with the slow rise of the elastic or total cross section in hadron-hadron collisions at increasing energy. Single Diffractive Dissociation shows a similar soft energy-dependence. In Deep Inelastic scattering one expects the cross section for diffractive scattering to behave in the same way, provided the produced partons of the diffractive system are dominated by small transverse momenta. The transition from the hard to the soft Pomeron can qualitatively be understood in the following way: starting from final states with large transverse momenta, we expect the 'hard' BFKL Pomeron [4] to dominate with its strong rise at small x . When the momentum scale at the upper end of the hard Pomeron gets smaller, unitarity corrections become stronger and stronger, and the energy dependence becomes weaker until we reach the soft Pomeron.

The simplest QCD-model for the Pomeron is a pair of t-channel gluons with zero colour. It offers the possibility to derive the Pomeron coupling to the hadronic final state without any additional parameter except for the strong coupling constant. In perturbation theory the Pomeron coupling is fundamentally different from the soft approach, since it is dimensionless and allows a coupling to gluons as well.

The hadronic (partonic) final state that emerges within the QCD-framework exhibits a new behaviour different from the factorization assumption and the usual QCD-evolution. Most striking is the existence of a 4-gluon t-channel state [6] which builds up when the square of the invariant mass M of the hadronic final state becomes large compared to Q^2 (Triple Regge limit). This contribution is of great relevance in connection with the Leading Proton Spectrometer which is able to measure high mass Diffractive Scattering. For the present data we may restrict our analysis to the leading and next to leading order in perturbation theory, i.e. we consider as final state only a quark, anti-quark and a gluon, neglecting higher order resummation.

Before we move on to the next section we briefly mention the main kinematic variables. Commonly used are the variables $\beta = Q^2/(Q^2 + M^2)$ and $x_p = (M^2 + Q^2)/(W^2 + Q^2)$, expressed in terms of M^2 and Q^2 which we have already introduced as the square of the invariant mass of the hadronic final state (without the proton) and of the virtuality of the photon. In addition we use the total invariant mass W (including the proton). The momentum transfer is denoted by t .

2 The hadronic final state at leading and next to leading order

As we already pointed out the advantage of the perturbative approach lies in the fact that the coupling of the Pomeron to the hadronic final state is given by the QCD-coupling of the two exchange gluons. At leading order the final state consists of only a quark anti-quark pair to which the photon couples (fig.1a). When the invariant mass M is increased, however, it becomes more likely that a gluon is emitted (fig.1b) rather than the quark anti-quark pair being exclusively produced, according to the general rule that spin 1 exchanges dominate over spin 1/2 exchanges.

Figure 1: Leading and next to leading order diffractive final state

To ensure gauge invariance the two t-channel gluons have to couple in all possible ways to

the quark anti-quark pair (fig.1a shows only one example). To simplify the calculation we set the momentum transfer t , i.e. the sum of the transverse momenta in the t -channel, to zero. This restriction should not be too severe, since the cross section peaks at $t = 0$. Decomposing the cross section into its transverse and longitudinal component W_t and W_l

$$\left. \frac{d\sigma}{d\beta dQ^2 dx_P dt} \right|_{t=0} = \frac{\alpha_{em}}{2x_P Q^4} \left\{ -[1 + (1-y)^2] x_B W_t + 4(1-y) x_B W_l \right\} \quad (1)$$

one finds following the calculations in ref.[7, 8, 6]:

$$x_B W_t^{q\bar{q}} = - \sum_f Q_f^2 \frac{\pi}{24} \frac{Q^2}{\beta(1-\beta)} \int_0^1 d\alpha [\alpha^2 + (1-\alpha)^2] \left\{ \int \frac{dl^2}{l^2} \psi(x_P, l^2, Q_0^2) \cdot \right. \quad (2)$$

$$\left. \cdot \left[1 - 2\beta + \frac{\beta l^2/Q^2 - (1-2\beta)\alpha(1-\alpha)}{\sqrt{[\beta l^2/Q^2 + \alpha(1-\alpha)]^2 - 4\alpha(1-\alpha)\beta(1-\beta) l^2/Q^2}} \right] \right\}^2$$

and

$$x_B W_l^{q\bar{q}} = \sum_f Q_f^2 \frac{\pi}{3} Q^2 \int_0^1 d\alpha \alpha(1-\alpha) \left\{ \int \frac{dl^2}{l^2} \psi(x_P, l^2, Q_0^2) \cdot \right. \quad (3)$$

$$\left. \cdot \left[1 - \frac{\alpha(1-\alpha)}{\sqrt{[\beta l^2/Q^2 + \alpha(1-\alpha)]^2 - 4\alpha(1-\alpha)\beta(1-\beta) l^2/Q^2}} \right] \right\}^2$$

The variable α denotes the longitudinal component of the quark momentum k (see also fig.1a for notations). The virtuality k^2 of the offshell quark is given by $\alpha(M^2 + Q^2)$. Since we do not want to restrict ourselves to a particular model for the Pomeron we have introduced a general function $\psi(x_P, l^2, Q_0^2)$. It coincides with the unintegrated gluon structure function and behaves roughly like $1/l^2$ for $l^2 > Q_0^2$ and like $1/Q_0^2$ when $l^2 < Q_0^2$. Q_0 is the hadronic scale of the order of 1Gev. One can now go ahead and look for a reasonable parametrization for ψ which should be consistent with related processes involving the Pomeron, as, for example, the structure function F_2 at small x -Bjorken, vector production in DIS or J/Ψ -photoproduction. The underlying theorem which allows to exchange ψ is known as k_t -factorization theorem [9]. We also have absorbed the strong coupling constant into ψ , so that there is no parameter in eqs.(2) and (3) left to be determined.

The basic behaviour of the cross section with respect to β can be extracted from eqs.(2) and (3) by taking the limits $\beta \rightarrow 1$ and $\beta \rightarrow 0$. In the first case it is rather easy to check that eq.(2) vanishes proportional to $(1-\beta)$ whereas eq.(3) gives a finite contribution in this limit. Hence, at small masses the longitudinal contribution is enhanced compared to the transverse. A similar observation is made for vector meson production in DIS. One, however, has to be aware that the longitudinal part is a higher twist contribution, so that the ratio transverse over longitudinal part does not vanish like M^2/Q^2 at large Q^2 , but is roughly of the order of M^2/Q_0^2 . Taking the second limit, $\beta \rightarrow 0$, requires more effort. One finds that the transverse contribution is finite, i.e. in terms of the mass M it the cross section vanishes like $1/M^4$ as expected (spin-1/2 exchange). The longitudinal part (3) has an asymptotic behaviour proportional to β^2 , i.e. is negligible at large masses.

We now move on and add a gluon to the final state considering diagrams like the one shown in fig.1b. The triple gluon vertices are effective vertices summarizing elementary Feynman diagrams. The gauge has been fixed by the condition $Q' \cdot A = 0$ with the lightcone vector $Q' = q + x_B p$. In

order to be able to calculate the cross section we need to make the approximation that the square of the gluon transverse momentum k_t is much smaller than Q^2 . This assumption allows to calculate the quark loop at leading order which results in $\log(Q^2/k_t^2)$ times the well known Altarelli-Parisi-splitting function. This result remains unchanged even though a second t-channel gluon may couple to the quark anti-quark pair. The second gluon, however, is soft with a typical wave length much larger than the distance of the quark anti-quark pair, so that it couples only to the total charge without resolving the substructure.

The complete set of Feynman diagrams gives the following result [10]:

$$x_B W_t^g = - \sum_f Q_f^2 \frac{\pi}{2} \int_0^{Q^2} dk^2 \frac{\alpha_s}{8\pi} \ln \left(\frac{Q^2}{k^2} \right) \int_\beta^1 \frac{dz}{z^2} \left[\left(1 - \frac{\beta}{z} \right)^2 + \left(\frac{\beta}{z} \right)^2 \right] \frac{9}{4} \frac{1}{(1-z)^2} \cdot \quad (4)$$

$$\cdot \left\{ \int \frac{dl^2}{l^2} \psi(x_P, l^2, Q_0^2) \left[z^2 + (1-z)^2 + \frac{l^2}{k^2} - \frac{[(1-2z)k^2 - l^2]^2 + 2z(1-z)k^4}{k^2 \sqrt{(k^2 + l^2)^2 - 4(1-z)l^2 k^2}} \right] \right\}^2.$$

Two new variables appear in this equation, z which denotes the momentum fraction of the t-channel gluon with respect to the Pomeron momentum x_P and k^2 , the virtuality of the same gluon. The virtuality is related to the transverse momentum by $k^2 = k_t^2/(1-z)$. The longitudinal contribution is negligible in this case. We, again, take the two limits $\beta \rightarrow 0$ and $\beta \rightarrow 1$ in order to reveal the basic properties of this contribution. For small β the integrand is dominated by the singularity $1/z^2$ enhancing the region of small z , so that the second line in eq.(4) may be approximated by setting z to zero. Integrating over z then results in $1/\beta$, i.e. the cross section is divergent when β approaches zero. The small β or triple Regge limit has already been considered before in refs.[7, 8, 11, 12, 13] and we found our results to be consistent with those previous calculations.

Taking the other limit $\beta \rightarrow 1$ which was not covered by previous calculations one finds that eq.(4) vanishes like $(1-\beta)^3$. This result emerges by taking the limit $z \rightarrow 1$ resulting in $(1-z)^2$ and integrating over z .

It remains to remark that the function ψ is assumed to be universal and should be the same for all three eqs.(2-4). The only parameter that enters in eq.(4) is the strong coupling constant α_s . It depends on the scale somewhere between Q and Q_0 . A reasonable value for α_s is 0.25.

3 Numerical results

The function ψ remains to be determined. This is done by taking the data for the proton structure function F_2 which is related to ψ by the following equation:

$$F_2(x_B, Q^2) = \sum_f Q_f^2 \frac{Q^2}{4\pi} \int dl^2 \psi(x_B, l^2, Q_0^2) \cdot \quad (5)$$

$$\cdot \int_0^1 dx \int_0^1 d\alpha \frac{[1 - 2x(1-x)][1 - 2\alpha(1-\alpha)] + 8x(1-x)\alpha(1-\alpha)}{x(1-x)l^2 + \alpha(1-\alpha)Q^2}$$

In principle any reasonable parametrization that gives a good fit to the data in the small x_B -region, i.e. x_B smaller than approximately 0.05, could be used. It is important to notice that ψ should be defined down to zero with respect to l^2 and should be integrable there. A good starting point is the following ansatz:

$$\psi(x_B, l^2, Q_0^2) \sim \frac{1}{l^2 + Q_0^2} x_B^{1-\alpha_P(Q^2)} \quad (6)$$

Figure 2: F_2^D -data from ZEUS and the theoretical prediction (solid line)

with the effective Pomeron-intercept

$$\alpha_P(Q^2) = 0.08 + \begin{cases} 0.1 \ln[\ln(Q^2/1\text{GeV}^2) + 3] & \text{if } Q^2 > 0.05\text{GeV}^2 \\ 0 & \text{else} \end{cases} \quad (7)$$

The ansatz (6) has to be optimized to give a good fit to the data. A crucial point is the scale-

Figure 3: ZEUS β -spectrum. Separately displayed are the longitudinal (dashed line) and the transverse part of the $q\bar{q}$ -pair production (dotted line), the gluon radiation (dot-dashed line) and their sum (solid line).

dependence of the Pomeron-intercept α_P . A comparison of the slow rise of the photoproduction cross section and the much stronger increase in Deep Inelastic Scattering suggests the reduction of this parameter with a decreasing scale Q^2 . In the cross section for Diffractive Deep Inelastic Scattering Q^2 has to be substituted by the virtuality k^2 which is the relevant scale in that case. The data in the fit that we have used include the HERA-data [14, 15] below $Q^2 = 50\text{GeV}^2$ and the E665 [16] data down to the smallest Q^2 -values. In the limit $Q^2 \rightarrow 0$ we take α_P to be the soft Pomeron-intercept of 0.08. If available one can also use the unintegrated gluon structure function directly. The relation to ψ is

$$x_B g(x_B, Q^2) = \int_0^{Q^2} dl^2 \frac{1}{\alpha_s} \psi(x_B, l^2, Q_0^2) . \quad (8)$$

So far we have considered the forward direction, i.e $t = 0$. Since we know only very little about the t -dependence in Diffractive Deep Inelastic Scattering, we will simply assume the same dependence as for the soft Pomeron including the soft Pomeron slope α'_P . In fig.2 we present the prediction for the diffractive structure function F_2^D assuming three active flavours. F_2^D is given by

$$F_2^D = \frac{\beta}{x_P} \left(-\frac{x_B(W_t^{q\bar{q}} + W_t^g)}{4\pi} + \frac{x_B W_t^{q\bar{q}}}{2\pi} \right). \quad (9)$$

The data points show the recent ZEUS results [2]. A similar analysis can also be done for the H1-data. The theoretical x_P -slope turns out to be approximately 1.2, i.e. rather low, since the dominant contribution is governed by low virtualities k^2 (or transverse momenta k_t). It is also nearly constant, i.e. 'Regge factorization' is only weakly broken (we have explicitly broken Regge factorization by introducing the scale dependent Pomeron intercept). The x_P -slope and even the absolute normalization are in fair agreement with the data.

In fig.3, finally, we show the β -spectrum (F_2^D) after integrating F_2^D over x_P from 0.00063 up to 0.01. It is illustrated how the three contributions eqs.(2), (3) and (4) cover three rather distinct regions in β , the longitudinal contribution at large β , the transverse contribution with only the quark anti-quark pair in the final state at medium β and the third contribution which includes the gluon in the final state at small β .

4 Summary

Our analysis has shown that the perturbative approach for Diffractive Deep Inelastic Scattering gives a fairly good description of the present data. We have introduced a parametrization for the Pomeron based on the small- x_B data for the proton structure function F_2 taking into account the transition from the hard to the soft region by making the Pomeron intercept scale dependent. Since the process of Diffractive Deep Inelastic Scattering is dominated by low transverse momenta the x_P -slope turns out to be rather soft (1.2). The coupling of the Pomeron to the hadronic final state is calculated from QCD and does not require any additional parameters. We also find a characteristic β -spectrum (fig.3) showing that the emission of an additional gluon is bound to the small β region (triple Regge limit) whereas the other end of the spectrum (large β) is dominated by the longitudinal part of the cross section.

In a further step the present calculation has to be extended by including high order corrections (Q^2 evolution) and by allowing t to be nonzero.

References

- [1] H1 Collab., Phys. Lett. B348: 681 (1995).
- [2] ZEUS Collab., Z. Phys. C68: 569 (1995).
- [3] A.Donnachie, P.V.Landshoff, Nucl. Phys. B303: 634 (1988),
Nucl. Phys. B244:322 (1984)
- [4] V.S.Fadin, E.A.Kuraev, L.N.Lipatov: Sov. Phys JETP 44: 433 (1976),
Sov. Phys. JETP 45: 199 (1977);
Ya.Ya.Balitskii, L.N.Lipatov Sov. J. Nucl. Phys. 28: 822 (1978).

- [5] J.Bartels, H.Lotter, M.Wüsthoff, Z. Phys. C68: 121 (1995).
- [6] J.Bartels, M.Wüsthoff, Z. Phys. C66: 157 (1995).
- [7] A.H.Mueller, Nucl. Phys. B335: 115 (1990).
- [8] N.Nikolaev, B.G.Zakharov, Z. Phys. C53: 331 (1992).
- [9] S.Catani, F.Hautmann, Nucl. Phys. B427: 475 (1994).
- [10] M.Wüsthoff, Ph.D. thesis, preprint DESY-95-166.
- [11] L.V.Gribov, E.M.Levin, M.G.Ryskin, Phys Rep. 189: 267 (1990).
- [12] M.G.Ryskin, M.Besancon, Proceedings of the HERA workshop 'Physics at HERA', Vol. 1 (edited by W.Buchmüller, G.Ingelman).
- [13] J.Bartels, Phys. Lett. B298: 204 (1993), Z. Phys. C60: 471 (1993).
- [14] H1-Collab., Nucl. Phys. B439: 471 (1995).
- [15] ZEUS Collab., Z. Phys. C65: 379 (1995).
- [16] E665 Collab., FERMILAB-Conf-95/046-Expt (Mar. 1995).