ELECTRON COOLING

J. Bosser

CERN, Geneva, Switzerland

ABSTRACT
The aim of any cooling process is to reduce the emittances of an ion beam as quickly as possible. Amongst the few processes in use, the one using cold electron beams and called Electron Cooling is efficient when applied to low-energy ion beams. After a short reminder of the principles and of some essential definitions, a more descriptive than mathematical explanation of the cooling forces will be given. A full section will be devoted to the diagnostics while another will aim to emphasise the technological aspects. Finally, the secondary effects on the ion beam and a short review of the present and future coolers are presented.

1. INTRODUCTION, PRINCIPLE OF ELECTRON COOLING

Electron cooling was first proposed by Prof. G. Budker in the late sixties as a method of improving the properties of stored ion beams. Its experimental confirmation was performed at Novosibirsk and later at CERN and Fermilab. Due to the high energy of the production maximum for antiprotons, stochastic cooling, proposed by S. van der Meer, was the most appropriate choice of the cooling technique for their accumulation. In spite of that, the use of Electron Cooling in low energy ion rings, in order to improve the beam lifetime and properties, has had spectacular results and will open the way to better resolutions in nuclear and atomic physics experiments with stored beams.

Let us consider an ion beam circulating in a storage ring at average velocity \( \vec{v}_0 = \vec{\beta}_0 c \) along the theoretical trajectory (Fig. 1.1). In an orthogonal frame \( \mathcal{R}_0 \) moving at velocity \( \vec{v}_0 \), most of the ions will not be at rest. Their relative velocity can be pictured as in Fig. 1.1b).

![Diagram of Electron Cooling](image)

Fig. 1.1 Illustration of electron cooling of an ion beam:

a) Schematic of the ion velocity in the lab frame
b) The ion beam velocity in the moving frame where the electrons are at rest
More precisely the relative velocity of each individual ion can be expressed as the sum of a longitudinal and a transverse component (Fig. 1.2); such that:

\[ \vec{v}_i = v_i \cdot \hat{u}_i + v_\perp \cdot \hat{u}_\perp \]

\[ \vec{v}_i = v_x \cdot \hat{i} + v_y \cdot \hat{j} + v_z \cdot \hat{k}; \ (v_x = v_i). \]

where the symbol \( \hat{\cdot} \) means relative to the axis colinear or parallel to \( \vec{v}_0 \), \( \perp \) to the plane orthogonal to \( \vec{v}_0 \), and the vectors \( (\hat{u}_x, \hat{u}_\perp) \) and \( (\hat{i}, \hat{j}, \hat{k}) \) form an orthonormal basis of the \( \mathcal{R}_0 \) space.

![Fig. 1.2 Description of the velocity components in the moving frame \( \mathcal{R}_0 \)](image)

The rms (\( \sqrt{\langle v^2 \rangle} \)) velocities are closely related to the emittances. We will have to be more precise on that subject but at the level of this introduction we can say that the emittance \( \epsilon \) (in each plane) is proportional to \( \langle v^2 \rangle \). The cloud or gas of ions has therefore, in \( \mathcal{R}_0 \), a kinetic energy which can be compared to the definition of the temperature \( T \) of a dilute gas in thermodynamics using the classical relation

\[ \frac{3}{2} k \cdot T = \frac{1}{2} m_i \cdot \langle v_i^2 \rangle = \frac{1}{2} m_i \cdot \left[ \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \right] \]

where \( k \) is the Boltzmann constant, and \( m_i \) the ion mass.

We can thus associate for each individual rms velocity component a temperature

\[ k \cdot T_x = m_i \cdot \langle v_x^2 \rangle, \ k \cdot T_y = m_i \cdot \langle v_y^2 \rangle, \ k \cdot T_z = m_i \cdot \langle v_z^2 \rangle \]

or:

\[ k \cdot T_\parallel = m_i \cdot \langle v_\parallel^2 \rangle, \ k \cdot T_\perp = m_i \cdot \langle v_\perp^2 \rangle. \]

In fact many processes will contribute to an increase of the ion rms velocities. Amongst the most important are the collisions with the residual gas molecules within the beam pipe and the intrabeam scattering. These effects become significant for low-momentum ions. Their
emittance will continuously increase until they reach the machine acceptance and be lost. The ion-beam lifetime is then relatively short.

The goal of any cooling process is to reduce each temperature component and thus the corresponding emittances or at least to counteract the diffusion effects mentioned above. As a result the emittances will be reduced and the ion-beam lifetime increased. Figure 1.3 represents the effect of cooling on the size (1.3a) and on the momentum spread (1.3b) of a stored ion beam.

Fig. 1.3a) Schematic effect of ion beam cooling on beam size

Fig. 1.3b) Schematic effect of cooling on momentum spread

Amongst many cooling processes the one using cold electrons and named Electron Cooling or e-cool offers the possibility of obtaining relatively small emittances in short cooling times. This is mainly efficient for non-relativistic ion beams which will be the case in this lecture.
The principle consists of immersing the ion beam in a very cold (in the moving frame $\mathbb{R}_0$) electron beam over a given length. If we suppose, at first, that the electron beam has no velocity and therefore no energy in $\mathbb{R}_0$, due to Coulomb interaction the ions will undergo "collisions" with electrons (binary collision model). As a result the ions will give up some of their energy to the electrons which will therefore be heated. As a consequence the electrons must be renewed in order to obtain a very cold (in each plane) ion beam.

In practice the electrons, produced by a cathode and accelerated under a voltage $U_0$ such that $e \cdot U_0 \equiv 1/2 (m_e \cdot v_0^2)$, are not motionless in $\mathbb{R}_0$. We will see that they have a flattened velocity distribution such that $\langle v_{\parallel}^2 \rangle \ll \langle v_{\perp, e}^2 \rangle$. The goal of an efficient electron gun is to obtain transverse electron velocities (or $\langle v_{\perp, e} \rangle$) as small as possible.

According to our didactic explanations we can understand that electron cooling will proceed until the ion and the electron relative energies (always in $\mathbb{R}_0$) become equal. That means that:

$$k \cdot T_e = k \cdot T_i \quad \text{or} \quad m_i \cdot v_i^2 = m_e \cdot v_e^2$$

$$v_i = v_e \left[ \frac{m_e}{m_i} \right]^{1/2}$$

and since the ion mass $m_i$ is much greater than the electron mass $m_e$, the ion velocity will become very small. The cooling time is usually of the order of 10 ms to 10 s after which the ions are at very low temperature and therefore the ion beam has very small emittances.

We can sketch the principle as in Fig. 1.4. The electrons are produced continuously by a cathode heated to $T_C$. They are accelerated to velocity $\vec{v}_0 = \beta_0 \cdot \vec{c}$ by a gun and directed into the drift or cooling region where they overlap the ion beam over a length $\ell_e$. At the end of this section the electrons are steered away from the ions and recuperated by a collector. On their way from the gun to the collector the electrons are usually submitted to a longitudinal magnetic field which has two purposes:

![Fig. 1.4 Principle of electron cooling](image)
- First, the potential due to the electron space charge inside the beam is not constant. This will be explained later but, as a consequence, a radial electric field exists which will tend to make the e-beam diverge. The magnetic force will counteract this effect and keep the e-beam cylindrical.

- Second, we will see (section 5) that the fact that electrons are "magnetised" improves drastically, at relative low ion speed, the cooling effect. The cooling time is appreciably reduced.

We can leave electron cooling for the moment and look at two didactic examples:

a) One may take some analogy from thermodynamics where the entropy \( S = k \ln(\Omega) \) (\( \Omega \) the number of microstates) is a measure of the disorder. For a quasi-static transformation \( dS = dQ/T \) where \( dQ \) represents the quantity of heat received at absolute temperature \( T \). If a small body (representing the ions) of mass \( m \) and specific heat \( c_p \) at initial temperature \( T_i \) is immersed in a large quantity of water (representing the electrons) at constant temperature \( T_2 < T_1 \), it will cool down to \( T_2 \) and its change in entropy will be:

\[
\Delta S = mc_p \ln(T_2 / T_1) < 0.
\]

A negative change of entropy results in a reduction of the disorder (and by analogy more orderly ions).

b) Another example could be that of a ball traversing a "sand wall" or, more explicit for our purpose, that of a moving foil (Fig. 1.5). One may then consider the electrons as a foil moving with velocity \( \bar{v}_0 = \beta_0 c \). Ions moving faster than the foil (electrons) penetrate it and lose energy \( (dE / dx) \) along the direction of their momentum during each passage, until all transverse component are diminished and their longitudinal velocity is equal to the foil velocity. Slower ions traverses the foil from the opposite side. Ideally at the end all ions will have the same longitudinal velocity as the foil and no transverse component.

![Illustration of cooling as an energy loss in a moving foil](image)

The energy loss can be calculated from the Bohr equation:

\[
-dE/dx = \frac{4 \pi Z^2 n_e e^4}{(4 \pi e_0)^2 m_e v_i^2 \alpha_i},
\]

where \( Z \) is the charge on the ion, \( n_e \) the electron density and \( \alpha_i \) is a material constant.
Due to multiple scattering, the beam will blow up as it passes a foil of thickness $x_0$. This will also cause a smearing of the longitudinal energy, known as energy struggle. The rms width $\theta_{\text{rms}}$ of the angular distribution is given by:

$$
\theta_{\text{rms}}^2 = \frac{2\pi Z^2 e^4 n_e}{(4\pi e_0^2 E_i^2 x_0) \alpha_2}
$$

(1.2)

where $E_i$ is the kinetic energy of the ion and $\alpha_2$ is another material constant. We therefore distinguish two effects:

i) the energy loss (friction);

ii) the multiple scattering (diffusion).

The first effect results in cooling, the second one corresponds to heating. The diffusion is characterised by the coefficient $D$, defined as:

$$
D = \frac{d}{dt}(p_i \cdot \theta_{\text{rms}}^2)
$$

where $p_i$ is the momentum of the ion. The energy loss and the diffusion coefficient are related through:

$$
\frac{dE}{dx} = \frac{1}{2m_e} \frac{\partial D}{\partial v_i} \quad \text{(for } \alpha_i = \alpha_2). 
$$

Coming back to the electron-cooling process itself, one sees that the production of a cold electron beam by the gun is of importance. On the other hand the efficiency of the e-beam recuperation by the collector will influence the vacuum pressure and indirectly the cooling process.

2. LIST OF CONSTANTS AND TYPICAL PARAMETERS OF AN ACCELERATOR

Most of the constants which will be used in this chapter are listed below. The accelerator parameters listed will be mainly used for numerical examples to get a feeling of the electron cooling process. At the end are given some fundamental relations between the laboratory frame and the moving frame components.

2.1 Usual constants

Boltzmann constant $k$ 
$1.38066 \cdot 10^{-23}$ J $\cdot$ K$^{-1}$ $= 8.618 \cdot 10^{-5}$ eV $\cdot$ K$^{-1}$

Speed of light $c$
$3 \cdot 10^8$ ms$^{-1}$

Elementary charge $e$
$1.602 \cdot 10^{-19}$ C (or s $\cdot$ A)

Permeability of free space $\mu_0$
$4\pi \cdot 10^{-7}$ H $\cdot$ m$^{-1}$ (or m $\cdot$ kg $\cdot$ s$^{-2}$ $\cdot$ A$^{-2}$)

Permitivity constant of free space $\varepsilon_0$
$8.854 \cdot 10^{-12}$ F $\cdot$ m$^{-1}$ (or m$^{-3}$ $\cdot$ kg$^{-1}$ $\cdot$ s$^4$ $\cdot$ A$^2$)

$= \frac{1}{\mu_0 \cdot c^2} \cdot \frac{1}{4\pi \varepsilon_0} \approx 9 \cdot 10^9$

$A, Z$ ion atomic mass, charge

Proton mass $m_p$
$938.28$ MeV $/ c^2 = 1.672 \cdot 10^{-27}$ kg

$(e / m_p) = 9.579 \cdot 10^7$ C $/ \text{kg}$
Electron mass $m_e$ 

$$0.511 \text{ MeV} / c^2 = 9.109 \cdot 10^{-31} \text{ kg}$$

$$\left(\frac{e}{m_e}\right) = 1.758 \cdot 10^{11} \text{ C} / \text{ kg}$$

Plasma frequency 

$$\omega_{pl}^2 = \frac{n_e e^2}{m_e \varepsilon_0} = 4\pi n_e r_e c^2; \quad \omega_{pl} = 56.5 (n_e)^{1/2} = 2\pi f_{pl}, \text{ s}^{-1}$$

($n_e$ is the number of electrons in m$^{-3}$).

Classical radius 

$$r = \frac{e^2}{4\pi \varepsilon_0 mc^2} = \frac{e^2}{mc^2} = \begin{cases} r_e = 2.818 \cdot 10^{-15} \text{ m for electron} \\ r_p = 1.547 \cdot 10^{-15} \text{ m for proton} \end{cases}$$

$$\varepsilon^2 = \frac{e^2}{4\pi \varepsilon_0} = 2.3 \cdot 10^{-28} \text{ m}^3 \cdot \text{kg} \cdot \text{s}^{-2}.$$ 

The subscript "i" will refer to ions while "e" will refer to electrons. Cylindrical coordinates are defined in Fig. 2.1 where (0,s) shows the beam direction. An element of volume $dv$ is given by: $dv = r dr d\theta ds$.

![Fig. 2.1 Cylindrical coordinates](image)

**rms**: If the symbol $<X>$ represents the average or expectation of a random variable $X$ then the rms value of $X$ is expressed by $\sqrt{<X^2>}$. 

### 2.2 Typical accelerator and electron cooler (LEAR) parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The ions will be protons ($Z = 1$)</td>
<td>3.58 m</td>
</tr>
<tr>
<td>Dispersion in cooler $D$</td>
<td>5.3 m</td>
</tr>
<tr>
<td>Vertical betatron function in cooler $\beta_v$</td>
<td>1.9 m</td>
</tr>
<tr>
<td>Horizontal betatron function in cooler $\beta_h$</td>
<td>2.305</td>
</tr>
<tr>
<td>Horizontal tune $Q_h$</td>
<td>2.730</td>
</tr>
<tr>
<td>Vertical tune $Q_v$</td>
<td>78.59 m</td>
</tr>
<tr>
<td>Accelerator circumference</td>
<td></td>
</tr>
<tr>
<td>Off-momentum function $\eta = \frac{df}{fo} = \frac{1}{\gamma_0^2} - \frac{1}{\gamma_v^2}$</td>
<td>0.938</td>
</tr>
<tr>
<td>Proton momentum $p_0$</td>
<td>308.6 MeV/c</td>
</tr>
<tr>
<td>Proton energy $T_0$</td>
<td>44.446 MeV</td>
</tr>
</tbody>
</table>
Proton velocity $\vec{\beta}_0 = \vec{v}_0 / c$  \[ 0.3124 \ (\gamma_0 = 1.0527 \ ; \beta_0 \gamma_0 = 0.3288) \]
Nominal revolution frequency $f_0$  \[ 1.192 \text{ MHz} \]
Current for $10^9$ p circulating in the machine  \[ 0.191 \text{ mA} \]
Vacuum pressure  \[ 10^{11} \text{ Torr [N$_2$ equivalent for scattering]} \]
Electron energy $U_0$  \[ 26929 \text{ eV} \equiv (27 \text{ keV}) \]
Electron beam intensity $I_b$  \[ 2.3 \text{ A} \]
Pervance $p_e$  \[ 0.52 \times 10^{-6} \text{ A.V}^{-3/2} \]
Electron beam radius $r_0$  \[ 2.5 \text{ cm} \]
Electron beam density (in the lab. frame) $n_e^*$  \[ 8.27 \times 10^{13} \text{ m}^{-3} \]
Current density $J_e^*$  \[ 0.13 \text{ A/cm}^2 \]
Longitudinal magnetic field $B_0$  \[ 455 \text{ Gauss} \]
Toroid angle and radius  \[ \Phi_0 = 36^\circ \ R_t = 1.05 \text{ m} \]
Length of cooling section $\ell_c$  \[ 1.5 \text{ m} \]
$\eta_c = \text{length of cooling section/circumference}$  \[ 1.9 \times 10^{-2} \]
Number of protons $N$  \[ 5 \times 10^9 \]
For numerical examples

case 1, $\varepsilon_x = \varepsilon_z = 10 \pi \cdot \text{mm-mrad}, \Delta p/p_0 = 10^{-3}$, case 2, $\varepsilon_x = \varepsilon_z = 1 \pi \cdot \text{mm-mrad}, \Delta p/p_0 = 10^{-4}$.

2.3 Change of frame (Fig. 2.2)

The symbol * denotes the lab. frame. The moving frame moves at speed $\vec{v}_0 = \vec{\beta}_0 \cdot c$ along the (0,s) axis, "‖" is related to a component parallel to the (0,s) axis, for example $\vec{v}_0$ or $\vec{v}_s$ are colinear with $\vec{v}_0$, "⊥" is related to a component in the plane (0,x,z) orthogonal to the (0,s) axis.

**Force**

\[
\begin{align*}
  f^*_\parallel &= f_\parallel \\
  f^*_\perp &= f_\perp / \gamma_0
\end{align*}
\]

(Networths)

**Time**

\[ t^* = \gamma_0 \cdot t \quad (\text{s}) \]

**Electron density**

\[ n^* = \gamma_0 \cdot n \quad (\text{m}^{-3}) \]

**Velocities**

\[
\begin{align*}
  \vec{v}_\parallel &= \frac{\vec{v}_s - \vec{v}_0}{1 - v_\parallel \cdot \frac{v_0}{c^2}} \\
  \vec{v}_\perp &= \frac{\vec{v}_\perp}{\gamma_0 (1 - v_\parallel \cdot \frac{v_0}{c^2})}
\end{align*}
\]

(m·s$^{-1}$)

![Fig. 2.2 Fixed and moving coordinates](image)

3. EMITTANCES, TEMPERATURES

Emittances are important parameters of any accelerator. Since electron cooling makes use of temperatures, it is worth establishing the relations which exist between emittances and temperatures. When an electron is accelerated under a voltage $U_0$ ($U_0$ of the order of a few kV) its longitudinal initial temperature is drastically reduced. We will calculate the scale of this
effect. Finally, some approximate formula for the processes which usually increase the emittance of low energy storage rings are given.

3.1 Emittances

In Fig. 3.1, we consider the observer frame \((0^*, x^*, z^*, s^*)\) or rest frame and a moving frame \((0, x, z, s)\) moving at velocity \(\vec{v}_0 = \beta_0 \cdot c\) along the \((0, s)\) axis \(\left(\gamma_0 = 1 / (1 - \beta_0^2)^{1/2}\right)\). We will also consider almost non-relativistic particles; this is particularly the case in the moving frame. When necessary, we will use the symbol \(y\) to define either \(x\) (horizontal plane) or \(z\) (vertical plane).

![Diagram of fixed and moving frame](image)

Fig. 3.1 Fixed and moving frame

An important parameter of any particle beam is the rms emittance \(\varepsilon_r\) related to the beam width \(\sigma_x = \sqrt{\langle y^2 \rangle}\) or to its divergence \(\theta_y = \sqrt{\langle (dy/ds)^2 \rangle}\) such that (we consider \(d\beta_{h,*}(s) / ds = 0\) in the cooler section)

\[
\sigma_h = \frac{\varepsilon_h \beta_h(s)}{\pi} \left(\frac{\varepsilon_h \beta_h(s)}{\pi}\right)^{1/2}, \quad \theta_h = \frac{\varepsilon_h \beta_h(s)}{\pi\beta_h(s)}^{1/2} = \theta_h
\]

\[
\sigma_v = \frac{\varepsilon_v \beta_v(s)}{\pi} \left(\frac{\varepsilon_v \beta_v(s)}{\pi}\right)^{1/2}, \quad \theta_v = \frac{\varepsilon_v \beta_v(s)}{\pi\beta_v(s)}^{1/2} = \theta_v
\]

\(\beta_{h,*}(s)\) being the betatron amplitude function (in meters), while \(\varepsilon_r, \sigma_r\) and \(\theta_r\) are usually given in \(\pi \text{ mm-mrad}\), \(\text{mm}\) and \(\text{mrad}\) respectively. The normalized emittance \(\varepsilon_{n,r}\), when expressing the emittance as \([\pi \varepsilon] = \pi \cdot \text{mm} \cdot \text{mrad}\),

\[
\varepsilon_{n,r} = \sigma_r \cdot \theta_r \cdot \beta_0 \cdot \gamma_0 = \varepsilon_r \cdot \beta_0 \cdot \gamma_0
\]

remains constant in the absence of cooling and heating.

For circular accelerators the mean value of the betatron function is approximately given by:

\[
\langle \beta_h \rangle = \frac{R}{Q_h}, \quad \langle \beta_v \rangle = \frac{R}{Q_v}
\]
where $Q_x, Q_y$ are the tunes in the two planes and $R$ the accelerator radius.

### 3.2 Temperatures

It is well known, in thermodynamics, that the internal energy $U$ of a gas is related to its absolute temperature $T$ by:

$$ U = \frac{3}{2} k \cdot T. $$

For a dilute gas its internal energy is related to the rms velocity $v$ by:

$$ U = \frac{3}{2} k \cdot T = \frac{1}{2} m \cdot v^2 = \frac{1}{2} m \left[ < v_x^2 > + < v_y^2 > + < v_z^2 > \right] $$

or, by definition: $k \cdot T_p = m \cdot v_p^2 = 2 \cdot E c_p$ ($E c_p$ is the kinetic energy, and the subscript $p$ refers to each plane).

Note: $v_p$ is the rms velocity spread in each plane, $\sqrt{3/2} \cdot v_p$ is the total rms velocity spread when one assumes an isotropic distribution of the velocities. $1/2 \cdot m v^2$ is the kinetic energy spread per plane. It is sometimes convenient to express $T$ in eV, (the reader must remember that $1 \text{ eV} \leq 11000 \text{ K}$) and later omit $k$.

#### 3.2.1 Ion transverse temperature

We can apply these considerations to a beam by working in the particle rest frame $\mathcal{R}_0$ which moves around with the nominal particle velocity. The beam temperatures are then given by the velocity spreads in this frame. Under these conditions it is easy to show that the temperature is, in the moving frame, given by:

$$ k \cdot T_h = m_i \cdot c^2 \cdot \beta_0^2 \cdot \gamma_0^2 \cdot \theta_h^2 \text{ horizontally and } k \cdot T_v = m_i \cdot c^2 \cdot \beta_0^2 \cdot \gamma_0^2 \cdot \theta_v^2 \text{ vertically.} $$

The transverse temperature $k \cdot T_{\perp} = k(T_h + T_v)$ will be, according to our simplifications:

$$ k \cdot T_{\perp} = m_i \cdot c^2 \cdot \beta_0^2 \cdot \gamma_0^2 \cdot [Q_x + Q_y] \frac{\epsilon}{R} \text{ when } \epsilon_x = \epsilon, = \epsilon. $$

Thus the reduction of $T_{\perp}$ corresponds to a reduction of the transverse emittance and vice versa.

#### 3.2.2 Ion longitudinal temperature

The particle may have a small velocity spread difference $dv^*$ around $v_0 = \beta_0 \cdot c$. We know that $p_0 = m_i \cdot \gamma_0 \cdot v_i^*$ so that $\Delta p = m_i (\gamma_0 \Delta v^* + v_0 \Delta \gamma)$. In the moving frame then:

$$ k \cdot T_i = m_i \cdot c^2 \cdot \beta_0^2 \left( \frac{\Delta p}{p_0} \right)^2. $$

A reduction of the momentum spread $\Delta p$ results in a reduction of the longitudinal temperature $T_i$ and vice versa.

#### 3.2.3 Electron temperatures

The temperature $T_e$ of the electrons at the output of a cathode is defined as:
\[ k \cdot T_e = k(T_{el} + T_{e\perp}) \]

where \(T_{el}\) and \(T_{e\perp}\) are respectively the longitudinal and transverse rms electron temperature spreads and \(k \cdot T_e\) is about 0.1 eV. Let us apply to the electron beam emitted by the cathode, an accelerating voltage \(U_0\) such that \(e \cdot U_0 = (1/2)m_e v_0^2\), \(v_0\) being the velocity of the moving frame.

In the fixed, or laboratory frame, the transverse electron temperature remains unchanged after acceleration: \(k \cdot T_{e\perp} = k \cdot T_e\) (with \(k \cdot T_e\) of the order of 0.1 to 0.5 eV). Therefore in the moving frame:

\[ k \cdot T_{e\perp} = \gamma_0^2 (k \cdot T_{e\perp}) = k \cdot T_e. \]

The longitudinal temperature is, however, drastically reduced after acceleration to the nominal kinetic energy \(E_e = (1/2)m_e v_e^2\). Let us look carefully, in the non relativistic case, at this phenomenon in order to get an estimate of the longitudinal temperature rms spread \(T_{el}\) in the moving frame. We can write, in the laboratory frame, that the kinetic energy \(E_e\) after the electron beam has been accelerated under a voltage \(U_0\) is:

\[ E_e = \frac{(m_e v_{el}^*)^2}{2} = e \cdot U_0 + k \cdot T_{el} = W_0 + k \cdot T_{el}. \]

Let us write: \(v_{el}^* = v_0 + \Delta v_e^*\) (with \(\Delta v_e^* \ll v_0\)) and \(\frac{m_e v_0^2}{2} = W_0\);

\(\Delta v_e^*\) is then the rms electron beam longitudinal velocity spread we are looking for. Thus:

\[ \frac{m_e}{2} \left(v_0^2 + 2v_0 \cdot \Delta v_e^* + (\Delta v_e^*)^2\right) = \frac{m_e}{2} v_0^2 + k \cdot T_{el}, \]

\[ m_e \cdot v_0 \cdot \Delta v_e^* = k \cdot T_{el}, \]

\[ \Delta v_e^* = \frac{k \cdot T_{el}}{m_e v_0} = \Delta v_{el}. \]

If we use the equality \(k \cdot T_{el} = m_e (\Delta v_{el})^2\) then:

\[ k \cdot T_{el} = \frac{(k \cdot T_{el})^2}{2(e \cdot U_0)}. \]

If for example we take \(k \cdot T_{el} = 0.1\) eV and since according to our typical accelerator table \(U_0 = 27\) kV:

\[ k \cdot T_{el} = 1.7 \cdot 10^{-7} \text{ eV} \ll k \cdot T_e \approx 2k \cdot T_{e\perp}. \]

This is called the kinematic contraction or flattened distribution effect of the electron beam. It shows that in the next sections of this lecture, we can use the inequality \(k \cdot T_{el} \ll k \cdot T_{e\perp}\).
On the other hand, if the accelerating voltage $U_0$ has a ripple of about 1 V, this can be considered as an effective cathode longitudinal temperature spread $k \cdot T_{e\parallel} = 1$ eV such that:

$$k \cdot T_{e\parallel} = 1.7 \cdot 10^{-5} \text{ eV}$$

and again, in the moving frame, the equivalent electron temperature spread is much smaller in the longitudinal plane than in the transverse one.

**Numerical example**

It is left as an exercise to compute the ion and electron transverse and longitudinal velocities in $\Re$. For that purpose cases 1 and 2 of section 2.2 have to be used and for the electron one should take $k \cdot T_c = 0.5$ eV and 1 eV. Then compare $v_{e\parallel}, v_{i\perp}, v_{e\perp}$.

**Exercise**

Show that in the relativistic case:

$$k \cdot T_{e\parallel} = (k \cdot T_{e\parallel})^2 / \beta_0^2 \gamma_0^2 m_e c^2.$$

### 3.3 Natural heating of the ion beam

Secondary effects will run counter to cooling and, without cooling, will lead to an emittance growth so that the circulating ion beam will be lost after several minutes. Among all these heating phenomena, we will only mention the effect of the residual gas and the intrabeam scattering.

#### 3.3.1 Residual gas disturbance

Repeated small-angle scattering of stored ions by residual gas molecules leads to:

**a)** Emittance growth. The angular spread can be calculated with Eq. (1.2):

$$\hat{\theta}_{e\parallel}^2 = \frac{8 \pi Z^2 \cdot Z^2 \cdot n_{gas} \cdot r_i^2 \cdot c \cdot L_{res}}{\beta_0^3 \gamma_0^2},$$

where we replaced $\alpha_2$ by the Coulomb logarithm responsible for this scattering ($L_{res} \equiv 10$), $x_0$ by $\beta_0 c \cdot t$ and $E_i = (\gamma / 2) m_i \cdot v_i^2$.

The transverse emittance growth is then:

$$\frac{\dot{\varepsilon}_{e\perp}}{\pi} = \frac{8 \pi Z^2 \cdot Z^2 \cdot n_{gas} \cdot r_i^2 \cdot c \cdot \beta_j \cdot L_{res}}{\beta_0^3 \gamma_0^2} \cdot \frac{1}{2},$$

$$n_{gas} = 3.5 \times 10^{16} \text{ Torr}^{-1} \text{ cm}^{-3} \cdot P_{gas}.$$

where the factor 1/2 comes from averaging over the betatron phase.

When the emittance equals the machine acceptance $A_x$ or $A_\perp$, the ions begin to be lost.

**b)** Energy loss which can be calculated from Eq. (1.1) after including the effects of the residual gas molecules:

$$\frac{dE}{dx} = \frac{4 \pi Z^2 \cdot Z^2 \cdot n_{gas} \cdot r_i^2 \cdot m_e \cdot c^3}{\beta_0^2} \cdot L_{res}$$
where we put $\alpha_1 = \alpha_2 = L_{\alpha_i}$. Again, when the particles have been decelerated down to the inner side of the machine longitudinal acceptance the beam is lost.

We can compare the emittance growth rate with the energy loss:

$$- \frac{dE}{dt} = \frac{\dot{E}_{\alpha_i}}{E_i} = \frac{\dot{\varepsilon}_{\alpha_i}}{\varepsilon_i} \frac{m_i}{\langle \beta \rangle m_e}.$$

More detailed explanations are given in the lecture on "Beam Interactions with Residual Gas" in this course [4].

3.3.2 Intrabeam scattering

An elaborate description of this process is given in the lecture on "Intrabeam scattering" in these proceedings [5], from where the reader can find out the emittance growth time $\tau_{\alpha_i}$ and how it evolves when the ion emittance or temperature decreases. Other phenomena have to be taken into account. Some of them will be explained later but, of course, physics experiments using internal targets [6] will contribute to a fast degradation of the emittances. This motivates the use of cooling techniques (stochastic, electronic or other) which aim to counteract these diffusion processes and consequently improve the ion beam quality and lifetime.

4. DEFINITIONS OF THE COOLING FORCES AND TIMES

The cooling process can be explained in terms of plasma physics (Refs. [1 to 3]). Explanations, however, become more difficult in the case of magnetised electrons. We will therefore use a classical explanation, using electromagnetic laws, after a brief reminder of a few plasma definitions. Except for an elementary, and in consequence, approximate demonstration, electron cooling requires tedious and difficult computations which are out of the scope of this lecture. We give here just a few parameters and definitions which will be of some use in the section on cooling forces.

4.1 Reminder of plasma physics

Imagine a positive ion embedded in the electron cloud. The neighbouring electrons will rearrange themselves to shield the ion Coulomb field. The electric field decays exponentially and the shielding radius is called the Debye length $\lambda_d$ (see Fig. 4.1).

![Debye radius](image)

**Fig. 4.1 Debye radius**

$$\frac{1}{\lambda_d^2} = \frac{e^2}{\varepsilon_0 \cdot k} \left[ \frac{n_e}{T_e} + \frac{n_i \cdot Z^2}{T_i} \right]$$

$$\lambda_d = \left[ \frac{\varepsilon_0 \cdot k \cdot T_e}{n_e \cdot e^2} \right]^{1/2}$$
when \( n_e \gg n_i \) and where \( T_e \) is the electron beam temperature in K.

The Debye length can be compared to the average radius between the electrons:

\[
\frac{4}{3} \pi r_{av}^3 = \frac{1}{n_e} \quad \text{or} \quad r_{av} = \left( \frac{3}{4 \cdot \pi \cdot n_e} \right)^{\frac{1}{3}}.
\]

The time it takes for the perturbed electrons to rearrange themselves around the ion is about one period of the plasma frequency:

\[
f_{pl}^2 = \frac{n_e \cdot r_e \cdot c^2}{\pi} = \frac{1}{(t_{pl})^2}
\]

If the electron gas has a very low temperature and if the ion moves at relative velocity \( v_i \) the screening length \( \ell_s \) is about (Fig. 4.2):

\[
\ell_s = v_i \cdot t_{pl}.
\]

![Fig. 4.2 Screening length](image)

One can thus, empirically, understand that the ion while moving will create an electric field in its wake. The force due to this electric field will try to slow down the ion itself. The energy consumed in the electron disturbance will be lost by the ion.

### 4.2 Relation between cooling force and time

We have seen that the objective of electron cooling is to decrease the ion velocity spread \( v_i \) in the moving frame. This implies a reduction of the ion energy and therefore the existence of a friction or cooling force. The cooling time will depend on the cooling force intensity.

Let us consider a single particle at velocity \( v_i \). For an exponential decrease of the ion velocity we can write:

\[
v_i(t) = v_i(t = 0) \cdot \exp(-t / \tau_c).
\]

Then the cooling time variable \( \tau_c \) (in s) is given by:

\[
\frac{1}{\tau_c} = \frac{1}{v_i} \frac{d v_i}{dt} = \frac{1}{m_i \cdot v_i} \left( m_i \frac{d v_i}{dt} \right) = - \frac{F(v_i)}{p_i}
\]
where $F$ is the cooling force and $p_i$ the ion momentum.

The friction rate is therefore given by:

$$\frac{1}{\tau_i} = \frac{F}{p_i}.$$

It shows that an understanding of the cooling (or friction) force is essential for determining the cooling time. Section 5 will be devoted to this subject where it will be shown that $F$ depends on $v_i$ and $v_e$ and so $\tau_i = (v_i, v_e)$. The computation of the cooling $\Delta t = t[v_i = v_i \text{ final}] - t[v_i = v_i \text{ initial}]$ is thus not straightforward.

In the laboratory frame, the cooling time variable will then be (see Section 2.3):

$$\tau_i^* = \gamma_0 \frac{1}{\eta_c} \tau_i,$$

where $\eta_c$ is the ratio of the cooling section length to the accelerator circumference.

What is observed by an experimentalist is the average cooling time $\Delta t = \langle \Delta t_i \rangle$.

5. ESTIMATION OF COOLING FORCES

Although most electron coolers make use of a longitudinal magnetic field, it is worth considering the case of non-magnetic cooling forces. Firstly, it is an elegant didactic approach to the cooling process and secondly, for large relative ion velocities $\tilde{v}_i$, the magnetised forces are relatively weak with respect to the non-magnetised forces. The exact explanations are out of the scope of this lecture (consult Bibliography). All the computations take place in the moving frame.

5.1 Without magnetic field

![Fig. 5.1 Geometry of the collision](image-url)
As a first, but illustrative approach, we use the single electron-ion interaction represented by Fig. 5.1. The ion initially at position \((\infty, b)\) with a velocity \(\vec{v}_i = v_i \cdot \vec{u}_i\) will interact with an electron at rest at \((0, 0)\). Due to the Coulomb force exerted between the two particles the ion will be deviated by an angle which is supposed to be small. The deviation \(\psi\) is given by:

\[
\cot \left( \frac{\psi}{2} \right) = -\frac{b m_e (v_i)^2}{Z e^2} 4 \pi \varepsilon_0.
\]

Let us estimate the exchange in energy, due to this interaction, when the time runs from \(-\infty\) to \(+\infty\). The Coulomb force \(\vec{f}\) will induce a change in momentum \(\frac{dp}{dt}\) given by:

\[
\vec{f} = \frac{Ze^2}{4\pi \varepsilon_0} \cdot \frac{1}{d^2} \vec{u} = \frac{Ze^2}{d^2} \frac{d\vec{u}}{dt} = \frac{e^2}{4\pi \varepsilon_0}.
\]

Approximately:

\[
\Delta \vec{p} = \int_{-\infty}^{\infty} \vec{f} \cdot dt \equiv Z e^2 \int_{-\infty}^{\infty} \vec{u} \cdot \vec{u} \cdot dt = \Delta p_l \vec{u}_l + \Delta p_\perp \vec{u}_\perp
\]

where

\[
\vec{u} = \frac{\vec{f}}{f} = -\cos \theta \cdot \vec{u}_l - \sin \theta \cdot \vec{u}_\perp = \frac{-s}{\sqrt{s^2 + b^2}} \vec{u}_l + \frac{-b}{\sqrt{s^2 + b^2}} \vec{u}_\perp.
\]

For small-angle scattering the longitudinal component of the force, being the integral of an odd function, \(\Delta p_l\) is zero, while for the transverse component we get:

\[
\Delta p_\perp = -Z e^2 b \int_{-\infty}^{\infty} \frac{dt}{(s^2 + b^2)^{3/2}}
\]

and since \(s = v_i \cdot t\),

\[
\Delta p_\perp = -Z e^2 b \lim_{\lambda \to \infty} \left[ \frac{s}{\sqrt{s^2 + b^2}} \left| \frac{A}{-A} \right| \right] = -2Z e^2 \frac{b}{v_i b}
\]

where, as usual, \(b\) is called the impact parameter.

Due to the electrical field compression, the effective time of interaction or collision time is rather short and is of the order of:

\[
\Delta t = \frac{b}{\gamma' v_i} \quad \text{(Fig. 5.2)}
\]

\((\gamma' \equiv 1\text{ since we work in } R_0)\).
The energy lost by the ion, which is that gained by the electron is then:

$$\Delta E(b) = \frac{(\Delta p)^2}{2m_e} = \frac{2Z^2e^4}{m\nu_i^2b^2}. \quad (5.1)$$

It is useful to notice the dependence on $v_i$.

Up to now we have considered a single collision. Of course we must take into account multiple collisions with all possible impact parameters $b$. If $n_e$ is the electron beam density then the number of electrons in the volume $\pi b^2 ds$ will be $n = \pi b^2 n_e ds$ while (Fig. 5.3) $dn = 2\pi b n_e ds db$ is the number of electrons between $b$ and $b + db$ over the length $ds$. The energy lost by the ion per unit of length is then:

$$\frac{dE}{ds} = 2\pi \int_{b_{\text{min}}}^{b_{\text{max}}} b n_e \Delta E(b) db$$

$$\frac{dE}{ds} = \frac{4\pi Z^2e^4}{m\nu_i^2} n_e \ell n \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right).$$

An estimation of the Coulomb Logarithm $L_e(v_i) = \ell n \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right)$ has to be found. Concerning $b_{\text{min}}$, it can be estimated by the maximum momentum transfer to the electron (classical head-on collision):
\[ \frac{2Z e^2}{v_i b_{\text{min}}} = \Delta p_{\text{max}} = 2m_e v_i \rightarrow b_{\text{min}} = \frac{Z e^2}{m_e v_i^2}. \]

For \( b_{\text{max}} \) we take \( b_{\text{max}} = \min(\lambda_d, r_0) \). Of course \( b_{\text{max}} \gg b_{\text{min}} \). Usually \( L_e \), of the order of 10, is logarithmically dependent on \( v_i \) and is taken as constant.

In fact, the electrons are not quite mono-energetic and therefore the friction force must be weighted by the electron speed distribution \( f(v_e) \) which can be expressed in a Gaussian form:

\[ f(v_e) = \frac{e^{-\left(\frac{x^2}{2\Delta_{\parallel}^2} + \frac{y^2}{2\Delta_{\perp}^2}\right)}}{(2\pi)^{3/2} \Delta_{\parallel} \Delta_{\perp}} \quad \text{and} \quad 1 = \int f(v_e) \, dv_e \]

\[ \Delta_{\parallel}^2 = \frac{k \cdot T_{\parallel}}{m_e} ; \quad \Delta_{\perp}^2 = \frac{k \cdot T_{\perp}}{m_e} \]

We have shown in section 3.2 that \( T_{\parallel} \ll T_{\perp} \). The friction force is then:

\[ \vec{F} = -\frac{4\pi Z^2 e^4 n_e}{m_e} L_e \int \frac{\vec{v}_e - \vec{v}_i}{|\vec{v}_e - \vec{v}_i|^3} f(v_e) \, dv_e. \]

(5.2)

where (see Fig. 5.4):

\[ \vec{v}_i = v_{i\parallel} \cdot \hat{u}_{\parallel} + v_{i\perp} \cdot \hat{u}_{\perp} \]

\[ \vec{v}_e = v_{e\parallel} \cdot \hat{u}_{\parallel} + v_{e\perp} \cdot \hat{u}_{\perp} \]

![Diagram showing electron and ion relative velocities in \( \mathcal{R}_0 \)](image)

Fig. 5.4 Electron and ion relative velocities in \( \mathcal{R}_0 \)

\( L_e \) is considered to be independent of the velocity. This has to be numerically integrated.

At this level it is important to notice the analogy between the expression of the force \( \vec{F} \) and that of an electrical field \( \vec{E} \), at a point \( P \) due to a distribution of charge \( \rho \) (Fig. 5.5).
Fig. 5.5 Analogy with an electrical field

\[ \vec{E} = \int \frac{\vec{F}}{|\vec{F}|^3} \rho \, d^3 r, \quad (d^3 r \equiv d^3 v) \]

where \( \vec{F} = \vec{F}_p - \vec{F}_v \)

\[ \rho = -\frac{4\pi n_e Z^2 \epsilon^4}{m_e} L_c \cdot f(\vec{r}) \]

\( \vec{F} \) is similar to \( \vec{E} \) in the velocity space.

In the real case of a Maxwell distribution of the electron speed for which \( \Delta_{e\|} \ll \Delta_{e\perp} \) computations have been made [1]. The following gives the asymptotic expressions of the forces.

*In the longitudinal direction* \( (v_{\perp} = 0) \) (Fig. 5.6)

\[
F_i(v_{\parallel}) = -\frac{4\pi Z^2 \epsilon^4}{m_e} n_e \cdot L_c \left[ \begin{array}{c}
\frac{1}{v_{\parallel}} \\frac{1}{\Delta_{e\|}} \\
\frac{1}{\Delta_{e\perp}} \\frac{v_{\parallel}}{(2\pi)^{3/2}} \end{array} \right]
\]

where:

\[
L_c = \ell n \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right) \quad \text{and} \quad b_{\text{max}} = \min \left\{ \frac{<\vec{v}_i - \vec{v}_e}>}{\omega_{pl}}, \tau <\vec{v}_i - \vec{v}_e>, r_0 \right\}
\]

\[
b_{\text{min}} = \frac{Z \epsilon^2}{m_e <\vec{v}_i - \vec{v}_e>}
\]

\( \omega_{pl} \) the plasma pulsation, \( \tau \) the ion time in the electron beam.
Fig. 5.6 Shape of the "non-magnetised" longitudinal cooling force

\[ F_{\parallel}(V_{\parallel}) \]

In the transverse direction \((v_{\parallel} = 0)\) (Fig. 5.7)

\[
F(v_{\perp}) = -\frac{4\pi Z^2 e^4}{m_e} n_e L_c \left\{ \begin{array}{l}
\frac{1}{v_{\perp}^2} ; \quad |v_{\perp}| > > \Delta_{e\perp} \\
\frac{\sqrt{\pi} v_{\perp}^4}{8 \Delta_{e\perp}^3} ; \quad |v_{\perp}| < < \Delta_{e\perp}
\end{array} \right. \tag{5.4a}
\]

The transverse force is maximum when the ion speed is the same as the electron transversal rms speed.

Fig. 5.7 Shape of the "non-magnetised" transverse cooling force

\[ F_{\perp}(V_{\perp}) \]

We observe that:

- the forces are not independent of the ion relative velocities
- for large ion velocities the forces scale as \(1/(v_i^2)\), suggesting that a beam with a relatively large emittance will have a large cooling time
- for small velocities the forces are proportional to \(v_i\)
Numerical example

Let us simplify and take a mono-energetic electron beam

\[ f(v_x) = \delta(v_{\|}, v_{\perp}) \, . \]

Then, using Eq. (5.2) and considering one single particle at speed \( v_i \):

\[ \vec{F} = -F_i \frac{\vec{v}_i}{|v_i|^3} \quad ; \quad F_\| = -F_i \frac{v_{\|i}}{|v_i|^3} \quad ; \quad F_\perp = \frac{-F_i \, v_{\perp i}}{|v_i|^3} \]

with:

\[ F_i = \frac{4\pi \, Z^2 \, e^4 \, n_e \, L_e}{m_e} = \frac{4\pi \, Z^2 \, n_e' \, L_e \cdot m_p \cdot r_e \cdot r_e' \, c^4}{\gamma_0} = \frac{m_p \, Z^2}{\gamma_0} \, F_0 \quad ; \quad m^3 \cdot \text{kg} \cdot \text{s}^{-4} \]

since:

\[ n_e = \frac{n_e'}{\gamma_0} \, . \]

With \( v_{\|i} = v_{\perp i} \), the cooling time variable for that particular velocities will be (see section 4.2)

\[ \tau_x = \frac{A \cdot m_p \cdot v_i \, |v_i|^3}{F_i \, v_{\perp i}} = \frac{\gamma_0 \, A \, (v_i)^3}{Z^2 \, F_0 \, v_{\perp i}} \, . \]

In the laboratory frame (\( Z = 1 = A \)) and taking into account the cooling length

\[ \tau_c = \frac{\gamma_0^2 \, (v_i)^4}{n_e \cdot F_i \cdot v_{\perp i}} = \frac{\gamma_0^2 \, |v_i|^3}{n_e \, \sqrt{2} \, F_i} \, . \]

Using the numbers of our typical accelerator (section 2.2) with

\[ \frac{v_{\perp i}}{c} = \beta_0^4 = 10^{-3} \quad = \sqrt{2} \, \frac{v_{\perp i}}{c} \, . \]

gives \( \tau_c = 2.726 \, \text{s} \).

When expressed in practical units \( \tau^* = \beta_0^4 \, \gamma_0^4 \theta_y^3 / l_s = \beta_0^4 \, \gamma_0^4 \left( \varepsilon_y / \beta_{hs} \right)^{3/2} / l_s \).

We see that as \( \tau^* = \beta_0^4 \, \gamma_0^4 \), electron cooling for relativistic particles appears to be less efficient.

Exercise
Starting from Eq. (5.2), derive Eqs. (5.3a) to (5.3c). Use the Gauss theorem and Eq. (5.4a)

5.2 Friction force with magnetised electrons

The magnetic field limits the transverse motion of the electrons which appear to be frozen in this plane. The electrons rotate around their axis at the cyclotron frequency \( f_t \) with a radius equal to \( r_t \). If the impact parameter \( b \) is much larger than \( r_t \) and if the collision time \( \Delta t = b / v_i \) is larger than \( 1 / f_t \), then, since the transverse motion is "frozen", the electron cooling efficiency is determined by the electron longitudinal temperature which is a few orders of magnitude
lower than the transverse one. Cooling is highly improved. This type of collision is often called adiabatic collision ($r_t < b$).

The case where $b_{\text{min}} < b < r_t$, is called a fast collision and looks like the case previously studied (without magnetisation of electrons). We can explain it in the following way (see Fig. 5.8). If the ion is motionless and the electron rotates at angular frequency $\omega$, on a circle of radius $r_t$ the force exerted on the electron is

$$\vec{f} = \text{Cst} \cdot \vec{\ell} / \ell^3$$
(Cst is a constant).

![Diagram](image_url)

Fig. 5.8  Illustration of the Coulomb force exerted between a fixed ion and an electron moving on a circle of cyclotron radius over one full cyclotron period

$$\vec{\ell} = -r_t \sin \theta \vec{i} + (b - r_t \cos \theta) \vec{k}$$

$$\ell^2 = b^2 + r_t^2 - 2 \cdot r_t \cdot b \cos \theta$$

$$d\vec{\ell} = \left[ -r_t \cos \theta \vec{i} + r_t \sin \theta \vec{k} \right] d\theta.$$

The work of the force $dW = \vec{f} \cdot d\vec{\ell}$ over one turn, which is equivalent to the exchange in energy between the ion and the electron, is zero:

$$W = \int_{\theta=0}^{\theta=2\pi} \vec{f} \cdot d\vec{\ell} = 0.$$

There is no exchange of energy in the transverse plane (in the adiabatic case).

If now the ion moves very quickly with respect to the electron then the interaction time is shorter than the cyclotron period. The electron will move on a part of the circle during this time and (Fig. 5.9):

$$W = \int_{\theta=0}^{\theta=\theta_e} \vec{f} \cdot d\vec{\ell} \neq 0.$$

We are now in the previous situation (no magnetic field) where the electron is considered as "free".
Fig. 5.9 Illustration of the Coulomb force exerted between a fixed ion and an electron moving on a circle of cyclotron radius over part of one period.

We can also make a comparison with the energy transfer of an ion to a harmonically bounded electron (see Classical Electrodynamics: J.D. Jackson; chapter on collision between charged particles [Bibliography]). In this case the electron is considered to be bounded in an atom and executes an orbital trajectory at angular frequency \( \omega_t \). The collision time is again given by:

\[
\Delta t = \frac{b}{\gamma' v_i}.
\]

If the collision is shorter than \( 1/\omega_t \), the electron is assumed to be free and the previous results apply. If on the other hand the collision time is very long compared to the orbital period, the electron will make many cycles of motion as the incident particle passes slowly by and will be influenced adiabatically by the field. We can define a threshold impact parameter \( b_t \) given, since:

\[
\Delta t = \frac{1}{\omega_t}, \quad b_t = \frac{\gamma' v_i}{\omega_t} \Delta t.
\]

The exact energy transfer to harmonically bound charge is given by:

\[
\Delta E(b) = \frac{2Z^2 e^4}{m_v v_i^2} \frac{1}{b^2} \left[ \xi^2 K_2(\xi) + \frac{1}{\gamma'^2} \xi^2 K_0(\xi) \right]
\]

with

\[
\xi = \frac{\omega_t b}{\gamma' v_i}
\]

and \( K_{1,0} \) the modified Bessel functions. The term between square brackets behaves asymptotically as:

\[
\left[ \right] = \begin{cases} 
1 & \text{for } \xi = \frac{b}{b_t} \ll 1 \\
\left(1 + \frac{1}{\gamma'^2}\right) \frac{\pi}{2} \xi e^{-2\xi} & \text{for } \xi = \frac{b}{b_t} \gg 1
\end{cases}
\]
We see that for \( b \ll b \), the energy transfer is essentially the result given for a free electron, since then (5.5) equals (5.1), while for \( b \gg b \) (and therefore small ion velocity spread) it falls exponentially to zero thus showing that there is no transfer of energy.

The same type of argument can be applied to the cooling process with magnetised electrons where, for low velocity ions, there is no energy transfer with the electrons' transverse movement whereas the exchange in energy occurs mainly with the electrons' longitudinal degree of liberty. Since the electron longitudinal velocity spread is very small the magnetisation will significantly enhance the cooling force in the case of low velocity ions.

A third explanation (I. Meshkov, private communication) can be given with the help of Fig. 5.10. Here again we have to take into account only the longitudinal displacement of an electron along the magnetic field lines \( \vec{B}_0 \) which are crossed by the ion at angle \( \theta \).

![Diagram](image)

**Fig. 5.10** Description of the third explanation for magnetised cooling

Starting from position (1), the ion and electron will interact. After a time \( t \), when in position (2), the electron will have executed a small displacement \( \Delta \ll b \) such that:

\[
\Delta = \frac{\text{acceleration} \cdot t^2}{2} = \frac{f}{m_e} \sin \theta \cdot \frac{t^2}{2} = \frac{Ze^2}{2m_e b^2} \sin \theta \left( \frac{b}{v_i} \right)^2 = \frac{Ze^2}{2m_e v_i^2} \sin \theta.
\]

This gives a small change in the impact parameter and it is easily shown that \( \Delta b \equiv \Delta \).

This small difference will result in a change in the drag force depending on whether the ion is on the first (1) or second (2) half of its trajectory. The momentum transfer induced by the collision is:

\[
\Delta \vec{p}_{\text{imp}} = \Delta p_x \vec{u}_x + \Delta p_\perp \vec{u}_\perp = \Delta p_x \vec{u}_x + \Delta p_\perp \vec{u}_\perp.
\]
with $\Delta p_{\parallel} << \Delta p_{\perp}$ since the phenomenon is the same as for a collision of a gas molecule with a hard wall.

The result is:

$$\Delta p_{\parallel} = \Delta p_{\perp} \sin \theta + \Delta p_{\parallel} \cos \theta \equiv \Delta p_{\perp} \sin \theta$$

$$\Delta p_{\parallel} = \Delta p_{\perp} \cos \theta - \Delta p_{\parallel} \sin \theta \equiv \Delta p_{\perp} \cos \theta.$$  

To get an estimate of $\Delta p_{\perp}$ we can write (see section 5.1):

$$\Delta p_{\perp} \equiv \Delta p_{\parallel} \frac{2}{\Delta b} \left( \frac{Z \varepsilon^2}{b^2} - \frac{Z \varepsilon^2}{(b + \Delta b)^2} \right) \frac{ds}{v_i^3} \frac{\Delta b}{b^2} \equiv \frac{2 \Delta p_{\parallel} \sin \theta}{m_e v_i^3} \frac{b^2}{b^2}$$

since the integration is effective over the distance $b$ only.

We then come to the simplified form:

$$\Delta p_{\parallel} = \frac{2 \Delta \varepsilon}{m_e v_i^3} \sin^2 \theta; \Delta p_{\parallel} \equiv \frac{2 \Delta \varepsilon}{m_e v_i^3} \sin \theta \cos \theta.$$  

The friction force resulting from the electron cloud is:

$$F_{\parallel} = - \int b_{max}^{b_{min}} \Delta p_{\parallel} n_e \frac{2 \pi b}{db}$$

such that:

$$F_{\parallel} = \frac{-4 \pi \varepsilon}{m_e} \frac{\varepsilon}{v_i^2} n_e L_c \frac{\sin^2 \theta}{v_i^2} \sin \theta \cos \theta; F_{\parallel} \equiv \frac{-4 \pi \varepsilon}{m_e} n_e L_c \frac{\sin \theta \cos \theta}{v_i^2}.$$  

When comparing $F_{\parallel}$ to (5.3) or (5.4), it is important to notice that there is no influence of the electron transverse velocity spread $\Delta v_{\parallel,e}$, keeping in mind that our explanation is valid for an ion velocity which scales to about $v_i \geq 100 \Delta v_{\parallel,i}$.

In consequence, to the non-magnetic case determined before, a magnetic force $F_{\parallel}$ has to be added to the previous force. Again numerical computation must be used [1]. We give the asymptotic expressions of the forces and define:

a) A relative velocity:

$$\vec{u}_{\parallel} = \vec{v}_i - \vec{v}_e = \begin{cases} \vec{v}_{\parallel,i} - \vec{v}_{\parallel,e} \\ \vec{v}_{\perp,i} \end{cases}.$$  

b) A threshold impact parameter $b_i = v_i / \omega$ to distinguish between the two impact parameter regions:

- inner impact, or fast collision region $b_{\text{min}} < b < b_i$ where the formula given in (5.1) applies,
- outer impact, or adiabatic region \( b_i < b < b_{\text{max}} \), where the following formula apply and for which the Coulomb logarithm:

\[
L_i^a = \ln(\lambda_i / r_i) .
\]

For \( |v_i| \gg \Delta_{e_i} \), we first set:

\[
F_i = \frac{4\pi Z^2 e^2}{m_i} n_e \cdot L_c = 4\pi Z^2 \frac{n_e \cdot L_c \cdot m_p \cdot r_p \cdot c^4}{\gamma_0} \cdot \text{m}^3 \cdot \text{kg} \cdot \text{s}^{-4}.
\]

Then:

\[
\mathcal{F}^{ad}(\vec{v}_i) = -\frac{1}{2} F_i \cdot L_c \cdot v_{i\perp} \int \frac{u_{ad}}{u_{ad}} \cdot \frac{\partial f(v_e)}{\partial v_{el}} \cdot d^3 v_e
\]

where \( \equiv \) means \( \parallel \) or \( \perp \). If \( f(v_e) = \delta(v_{el}) \) and \( |v_i| \gg \Delta_{e_i} \), the evaluation of the integral leads to:

\[
F^{ad \parallel}(v_i) = -\frac{1}{2} F_i \cdot L_c^a(u_{ad}) \frac{v_{i\perp}^2}{v_i^2} \cdot \frac{v_{i\parallel}}{v_i^2}
\]

\[
F^{ad \perp}(v_i) = -\frac{1}{2} F_i \cdot L_c^a(u_{ad}) \frac{v_{i\perp}^2 - 2v_{i\parallel}^2}{v_i^2} \cdot \frac{v_{i\perp}}{v_i^2}.
\]

Using a similar procedure when \( |v_i| \ll \Delta_{e_i} \), one obtains:

\[
F^{ad \parallel}(v_i) = -\frac{1}{\sqrt{2\pi}} F_i \cdot L_c \cdot \ln \left( \frac{\Delta_{e_i}}{v_{i\perp}} \right) \cdot \frac{v_{i\parallel}}{\Delta_{e_i}^3}
\]

\[
F^{ad \perp}(v_i) = -\frac{1}{\sqrt{2\pi}} F_i \cdot L_c^a(v_{i\parallel}) \cdot \frac{v_{i\parallel}}{\Delta_{e_i}^3}.
\]

The longitudinal force component for both regions (\( v_i \ll \Delta_{e_i} \) and \( v_i \gg \Delta_{e_i} \)) can be approximately expressed by:

\[
F_i^{ad \parallel} \equiv -9 \cdot F_i \cdot L_c^a(u_{ad}) \cdot \frac{v_{i\parallel}^2}{6v_i^2} \cdot \frac{v_{i\parallel}}{v_{i\perp}^2} .
\]

As mentioned above the magnetic effect appears mainly when the relative ion velocity \( v_i \) is small. This is of course doubly true, since, the transverse rms speed of the magnetised electron, with respect to the ion, is rather small. On the other hand, once the cooling process has started, and therefore the relative ion speed is lowered, the cooling effect is enhanced more and more resulting in much shorter cooling times.

If we analyse the practical case where \( v_i \gg \Delta_{e_i} \), we can deduce from the formula that:

- The individual forces \( F_i^{ad \parallel} \) or \( F_i^{ad \perp} \) depend both on the longitudinal and transverse components of the ion velocity. This can provoke a coupling of the motion in the different planes,

- The transverse force cancels when \( v_{i\perp} = \sqrt{2} \cdot v_{i\parallel} \) corresponding to an angle \( \phi = 54.7^o \). For larger angles the ion beam is transversally heated,
- Even when \( v_i < 10 \Delta_{e||} \), the forces go approximately as \( 1/v_i^2 \) which is quite an improvement when compared to the non-magnetic case.

Figure 5.11 illustrates the shape of the two transverse cooling forces (remember that \( \Delta_{e||} < \Delta_{e\perp} \) ) where \( F^0 \) refers to the non-magnetic case.

Fig. 5.11 Shape of the transverse cooling force with and without magnetisation

**Equilibrium distribution of the ion beam**

Theoretical equilibrium is reached when the ion temperature equals that of the electrons:

\[
T_i = T_e \quad \text{or} \quad v_i = v_e \left( \frac{m_e}{m_i} \right)^{1/2}
\]

which is quite favourable for the ion due to the small mass ratio \( (m_i / m_e) \). However, several other diffusion processes mentioned before such as interaction with residual gas molecules, intra-beam scattering etc., lead to much larger values. Typically one can say that transverse emittances of the order of \( 1 \pi \text{ mm} \cdot \text{mrad} \) and \((\Delta p / p_0) \approx 10^{-5}\) are currently obtained in cooling times of the order of a second. However, the longitudinal cooling time is about five times shorter than the transverse cooling time.

It is not very useful to analyse how these limit values are distributed in the longitudinal and transverse planes. For that purpose, it is left as an exercise to compute, for a very cooled proton beam, the longitudinal proton temperature when \( \Delta p / p_0 = 10^{-5} \) and the transverse proton temperature when \( \epsilon = 1 \pi \text{mm} \cdot \text{mrad} \); and then to compare these values with that of the electron beam when \( kT_{el} = 10^{-6} \text{ eV} \) and \( kT_{e\perp} = 0.5 \text{ eV} \).

It is easily understood that the electron beam must be well aligned with the axis defined by the theoretical closed ion orbit. Any inclination with respect to this axis will induce a "transverse electron velocity" as seen by the ions and therefore come to larger final emittances.
6. DIAGNOSTICS

Many parameters have to be measured amongst which are:

- the longitudinal and transverse emittances. The rate at which the emittances are reduced gives the cooling time. Emittances can be determined in an indirect way by the observation of the so-called Schottky signals [7] and in a more straightforward way with scrapers and ionisation beam profile monitors.

- the beam position. Since the ion and electron beams must be perfectly aligned it is important to have an accurate measurement of their position in the drift space.

- the cooling times and forces.

6.1 Longitudinal and transverse emittance measurement

6.1.1. Schottky signals

One of the easiest ways to measure the cooling effects on the emittances, in a relative manner at least, is to make use of the so-called Schottky pickups. These are mainly wide-band electromagnetic detectors which measure the statistical properties of the ion beam. We will give a brief explanation of the principle applied to unbunched beams.

a) Longitudinal Schottky signal

For a single ion circulating in a circular machine (charge $Z \cdot e$, revolution period $T_p = 1/f_p$; $f_p$ is near $f_0$ but not necessarily equal to $f_0$) the beam current seen by a sum pickup (Fig. 6.1a) at a given location in the ring is composed of an infinite train of Dirac pulses separated by time $T_p$ (Fig. 6.1b):

$$I(t) = Z \cdot e \cdot f_p \sum_{n=-\infty}^{\infty} \delta(2 \cdot \pi \cdot f_p \cdot t + \theta_i - 2 \cdot \pi \cdot n)$$

where $\theta_i$ is referred to time $t = 0$.

![Fig. 6.1a) Longitudinal Schottky signal: explanation of symbols](image)
Fig. 6.1b) Time domain Dirac pulses

In the frequency domain:

\[ I(t) = Z \cdot e \cdot f_p \sum_{k=-\infty}^{\infty} \exp j \cdot k \cdot (\theta_i + \omega_p \cdot t). \]

Looking at the positive frequencies only (Fig. 6.1c)

\[ I(t) = Z \cdot e \cdot f_p \left[ 1 + 2 \sum_{n=1}^{\infty} \cos \left( n \cdot (2 \cdot \pi f_p \cdot t + \theta_i) \right) \right]. \]

Fig. 6.1c) Positive frequency domain

For \( N_p \) particles moving with the same frequency \( f_p \), when looking at the nth harmonic, the average will be 0 except for a DC term if the \( \theta_i \) (\( i = 1, \ldots, N_p \)) are randomly distributed. The rms current however

\[ \langle f^2 \rangle = \left[ 2 \cdot Z \cdot e \cdot f_p (\cos n\theta_i + \cos n\theta_2 + \ldots + \cos n\theta_{N_p}) \right]^2 \]

does not vanish because \( \langle \cos \theta^2 \rangle = 1/2 \). We thus obtain:

\[ I_{\text{rms}} = \sqrt{\langle I^2 \rangle} = 2Z \cdot e \cdot f_p \sqrt{\frac{N_p}{2}}. \]

In fact particles have slightly different frequencies resulting from the relative momentum spread:

\[ \Delta f = n\Delta f_p = n \cdot f_0 \eta \frac{\Delta p}{p_0}; \eta \text{ the off-momentum function} \]
\[ \eta = \left( \frac{1}{\gamma^2} - \frac{1}{\gamma_p^2} \right). \]

Displaying the power spectrum density \( \langle I^2 \rangle / \Delta f \) on a spectrum analyser, at harmonic \( n \), gives a measurement of \( \Delta p/p_0 \). During longitudinal cooling, the decrease in \( \Delta f \) can be observed to deduce the ratio \( \Delta p/p_0 \) and therefore the longitudinal emittance. This is represented in Fig. 6.2a) and in practice by Fig. 6.2b).

![Fig. 6.2a) Evolution of the power spectrum during cooling](image)

![Fig. 6.2b) Display of the power spectrum for 10^9 0^8](image)

In the case of strong cooling the phases \( \theta_i \) are no longer random, the Schottky signals are strongly enhanced and their shape is modified (see Refs. [8, 11]). Then a different interpretation is necessary to deduce \( \Delta p/p_0 \) and the emittances. The same applies to the next paragraph b) on transverse Schottky signals.

b) Transverse Schottky signal

For a single particle the beam current \( I(t) \) must be replaced by the dipole moment (Fig. 6.3a):

\[ d(t) = a(t) \cdot I(t) \]
where \( a(t) \) is the transverse displacement. This displacement can be measured by taking the difference of the signals measured on the two plates of the pickup. The \( p \)th particle executes a sinusoidal betatron oscillation of amplitude \( a_p \) which can be written:

\[
a(t) = a_p \cos(q_p \cdot \omega_p \cdot t + \varphi_i).
\]

Here, \( q_p \cdot f_p \) is the observed frequency at a fixed location in the ring, \( q_p \) being the non-integer part of the betatron tune.

![Fig. 6.3a) Transverse oscillation versus time of a single particle](image)

![Fig. 6.3b) Frequency domain representation of the single-particle transverse oscillation](image)

In the frequency domain (Fig. 6.3b)

\[
d(t) = \frac{a_p \cos(q_p \omega_p t + \varphi_i)}{a(t)} Z e f_p \sum_{n=-\infty}^{\infty} \exp(j n \omega_p t) = a_p Z e f_p \Re \left\{ \sum_{n=-\infty}^{\infty} \exp \left[ (n + q_p) \omega_p t + \varphi_i \right] \right\}.
\]
The spectrum is again a series of lines spaced by the revolution frequency of the particle but shifted by $q_p \cdot f_p$. Looking at positive frequencies only (Fig. 6.3b) one obtains two betatron lines per revolution frequency band.

For $N_p$ particles in the beam, at the same $f_p$ and $q_p$, but randomly distributed in azimuth and in betatron phase, averaging gives $<d^2> = 0$ but:

$$<a^2> = (Z \cdot e)^2 \cdot f_p^2 \cdot \frac{N_p}{2}$$

$$d_{rms} = Z \cdot e \cdot f_p \cdot a_{rms} \cdot \sqrt{\frac{N_p}{2}}$$

where $a_{rms}$ is the rms oscillation amplitude, which is closely related to the emittance.

Each Schottky band has a finite width which results from the spread of the revolution frequency:

$$\frac{\Delta f}{f_0} = \eta \cdot \frac{\Delta p}{p_0}$$

but now, to first order, we must also consider the spread of the betatron frequency related to the chromaticity $\xi$ since $\Delta q = Q_o \cdot \xi \cdot \Delta p / p_0$. Consequently

$$\Delta f = f_0 \cdot \frac{\Delta p}{p_0} \left[ (n \pm q) \eta \pm Q_o \xi \right].$$

A difference Schottky pickup (Fig. 6.4a) will display the two side bands (Fig. 6.4b). As the beam is cooled, the amplitude observed on the spectrum analyser firstly increases (curve 2) since the longitudinal cooling is faster ($\Delta p$ decreases) and secondly the amplitude is reduced (curve 3) owing to the transverse cooling. The real case is illustrated by Fig. 6.4c).

Fig. 6.4a) Implementation of a transverse Schottky pickup
Fig. 6.4b) Evolution in time of the transverse Schottky power spectrum

Fig. 6.4c) Practical measurement on a spectrum analyser with $5 \cdot 10^9 \bar{p}$

6.1.2. The scraper (Fig. 6.5)

Suppose that the transverse ion-beam distribution is Gaussian (though the procedure applies to any distribution) of variance $\sigma$ and that we have a means to measure the beam intensity. We introduce a metallic block progressively into the vacuum chamber until the beam intensity is reduced by 2.5%. Then a second block opposite to the first one is introduced until another 2.5% of the beam intensity is removed. The distance $d$ between the two blocks is equal to $4\sigma$ or more precisely 95% of the initial particles are inside $4\sigma$. Since $\sigma_h=\sqrt{\epsilon_h\beta_h(s)/\pi}$, knowledge of $\beta_h$ allows us to determine the horizontal $\epsilon_h$ or vertical $\epsilon_v$ emittances depending on the movement of the blocks.
6.1.3 Ionisation beam profile monitor (Fig. 6.6)

The ion beam when passing through the residual molecular gas, or an artificial molecular gas curtain, will ionise the molecules. The positive ions are then accelerated toward a micro-channel plate (MCP). Each ion will produce $10^4$ to $10^5$ electrons at the MCP output. These secondary electrons are collected on metallic strips or on a phosphor screen. The beam profile (H or V) is thus obtained. If the profile can be measured in a few ms the cooling rate can also be estimated.

6.2 Beam position monitors

Beam position monitors are usually of the electrostatic type. Since this type of detector cannot measure unbunched beams:

- the ions must be bunched by the RF cavities. The bunch duration is of the order of a micro second.

- the electron beam is modulated in density. Its current intensity is related to the acceleration voltage by $I = p_i U_0^{5/2}$ (see section 7.1); one can therefore modulate the high voltage power supply at frequencies of the order of a few kHz.
In consequence the PU electronic systems must have a large bandwidth (1 kHz to several MHz). Synchronous demodulation is sometimes used and the absolute accuracy is about 0.1 mm.

6.3 Measurement of the longitudinal cooling time and force

6.3.1 Longitudinal cooling time

Two methods have been used though both are quite difficult and marred by significant errors.

The first method consists in stepping away the electron beam energy via the high voltage power supply and then resetting the supply to the operational value. By observing on a spectrum analyser which is triggered at the same time as the voltage step, the longitudinal spectral Schottky density in a narrow bandwidth around a harmonic of the revolution frequency, the cooling time can be determined. Figure 6.7 illustrates this method.

Center frequency: 24.236 MHz; Span: 0 Hz; Sweep time: 10 s

Fig. 6.7 Longitudinal cooling time measurement using first method. The curve gives the noise density (which is proportional to the square root of the particle density) at the nominal momentum. Horizontal scale is 1 s/div. At $t = 0$ the cooling is stepped away and the beam density decreases due to the various diffusion mechanisms. At $t \approx 3$ s, the cooling is reset and the beam recooled to equilibrium in about 0.7 s.

The second method uses the spectrum analyser in the same way, but instead of stepping away from the operational voltage, radio-frequency noise at a different harmonic of the revolution frequency is put on a longitudinal gap with bandwidth and power adjusted to blow up the beam's momentum spread by about $10^3$. When this noise is switched off the spectrum analyser is triggered and the spectral density evolution is observed. An example of the signal observed is shown in Fig. 6.8.
Center frequency: 24.236 MHz; Span: 0 Hz; Sweep time: 1 s

Fig. 6.8 Longitudinal cooling time measurement using second method. The curve shows the increase in noise density as the heating noise is switched off. Horizontal scale is 0.1 s/div. At \( t = 0.3 \) s, the heating is switched off and the beam reaches a new equilibrium density after about 0.4 s.

6.3.2 Longitudinal friction force

Two methods can be used to measure this force:

a) First method

The frictional force at low relative velocities is determined by analysing the distribution in equilibrium between a constant stochastic heating power, as mentioned in section 6.3.1 and the cooling force itself (Fig. 6.9). In order to obtain the velocity dependence of the frictional force \( F(v) \) from the equilibrium distribution \( \rho(v) \), one has to solve the one-dimensional Fokker-Planck equation for a diffusion constant \( D(v) \):

\[
\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial v} \left( -F(v)\rho(v) + D(v) \frac{\partial \rho}{\partial v} \right).
\]

In the equilibrium case \( \partial \rho/\partial t = 0 \), the shape of the frictional force is determined by the normalised slope of the distribution function:

\[
F(v) = D(v) \frac{\partial \rho/\partial v}{\rho(v)}.
\]

The diffusion constant \( D \) is derived experimentally from an independent measurement. Figure 6.10 shows the results of two measurements made with 50 MeV \( \overline{p} \) (on our standard machine), one with aligned beams and the other with misaligned beams (angle of about 1 mrad).
Fig. 6.9 Principle of the longitudinal frictional force measurement

Fig. 6.10 The longitudinal frictional force $F(v)$ as a function of $v$ for 50 MeV $\bar{p}$. The filled triangles correspond to the measurement made with aligned beams, the unfilled triangles to misaligned beams

b) Second method [9]

This uses the principle of the Betatron. A large coil is powered by a variable current and since

$$\vec{r} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

a longitudinal electric field is induced which results in a force on the ions. Equilibrium occurs when the electric force is equal to that of the electron cooling drag force.
7. COMPONENTS OF THE ELECTRON COOLER

All the components of an e-cooler must be designed with care. The cathode together with its gun has to provide a quasi-monokinetic electron beam whereas the collector efficiency must be almost equal to unity. On the other hand, the guiding solenoidal magnetic field must be free of any error since it may influence the electron rms transverse temperature. Firstly, we will consider the electrical arrangement of Fig. 7.1 which concerns only the e-beam.

![Electron beam electrical arrangement](image)

Fig. 7.1 Electron beam electrical arrangement

The electron beam, having a diameter of a few cm, emitted from a cathode is accelerated by a set of anodes (A1, A2, ...) forming the gun. When the beam enters the drift tube it has an energy of about $eU_o$ eV. At the output of the drift tube the e-beam $I_b$ is decelerated by an electrode at a potential $U_r$ with respect to the cathode. $U_r$ is of the order of 1.5 kV, relative to the cathode so that, in a very simplified approach, the e-beam energy is about $eU_r$. The beam enters the collector at a potential $U_c > U_r$ ($U_c$ is also relative to the cathode) and therefore it reaches the collector wall at energy $eU_c$. The dissipated power on the collector is thus $P_c = I_b U_c$ (5 to 10 kW) necessitating a cooling of the collector walls (usually by water).

Except for the collector, the cooler is surrounded by a solenoid giving a longitudinal magnetic field. This magnetic field ends near the collector entrance since one wants the e-beam to diverge in order to be spread over a large area of the collector. When optimising the trajectories, the presence of at least one coil at the collector entrance is of some help. Some of the electrons may be lost. They are represented on Fig. 7.1 by $I_t$.

7.1 Electron gun

7.1.1 Cathode

The current density $J$ emitted by a cathode at temperature $T_e$ is given by the Richardson-Dushman equation:
\[ J = \text{Constant} \cdot T_e^2 \cdot e^{-\frac{W_e}{kT_e}} \]

where \( W_e \) is the extraction work. More interesting in our case is the energetic distribution of the emerging electrons. It follows a Maxwell-Boltzmann distribution. The mean energy of the electrons is equal to \( 2kT_e \). The distribution is, however, not isotropic. The energy carried by the longitudinal component (orthogonal to the cathode surface) is \( kT_e \) the velocity component in the two other orthogonal directions has a mean energy \( 0.5kT_e \). Therefore \( T_e = 1000 \text{ K} \) and \( kT_e = 0.0862 \text{ eV} \), which is equal to the longitudinal temperature \( T_l \) and transverse temperature \( T_\perp = T_z + T_y \).

One sees the importance of having a thermo emissive cathode which operates at low temperature while having a long life time. The use of cold cathodes is of course interesting.

7.1.2 Gun

The gun itself consist of a Pierce electrode surrounding the cathode and several (two or more) accelerating electrodes all operating in the space-charge regime which means that a virtual cathode is formed near the cathode. If \(-\Phi m\) is the potential-minimum plane, only those electrons with velocity normal to the cathode greater than \((2q \cdot \Phi m / m)^{1/2}\) pass through the potential and proceed to the anodes. Electrons which are emitted with a velocity normal to the cathode less than this value return to the cathode.

The current \( I_e \) at the gun output is given by:

\[ I_e = p_e \cdot U_0^{3/2} \]

where \( p_e \) is the so-called gun permeance, of the order of \( 10^{-6} \text{ A.V}^{-3/2} \), and \( U_0 \) the accelerating voltage.

The overall device (cathode + gun) is usually embedded in a longitudinal magnetic field. The goal of the gun is to provide an essentially monokinetic cylindrical electron beam having a diameter of a few cm. Since one has to take into account the space charge effects, these are usually simulated by computer. The accelerating electrodes will induce a transverse electric field and even with careful adjustment of the magnetic field (resonant optics) the transverse energy dispersion may rise to \( 0.5 \text{ eV} \) or indeed \( 1 \text{ eV} \). That is an important deviation from the ideal case mentioned previously and needed in order to produce efficient cooling. Presently studies are underway which aim to get very "cold" electron guns and cathodes.

Two types of gun are mainly used: the resonant or the adiabatic type.

a) Resonant gun in a simplified form (Fig. 7.2)

Submitted to a longitudinal magnetic field \( B_0 \), the electron will describe in the transverse plane a circle \( C_t \) of radius \( r_t = p_{e,1} / |e|B_0 \) and move longitudinally by \( \lambda_t = 2\pi p_{e,1} / |e|B_0 \) each cyclotron period. When the electron passes at time \( t_1 \) the first anode \( A_1 \) at potential \( V_1 \), then, due to the radial electrical field \( E_{r,t} \), it will be submitted to a transverse kick and follow, transversally, the path \( C_2 \). At time \( t_2 \) when it passes the anode \( A_2 \) at potential \( V_2 \) it will receive a second kick due to the transverse electrical field \( E_{t,2} \). If an appropriate relation or resonance is founded between \( (V_2 - V_1) \), \( (t_2 - t_1) \) and \( B_0 \) the electron may come, at the output of \( A_2 \), to its initial trajectory \( C_t \). In this case the initial transverse energy remains unchanged.
b) Adiabatic gun

If the transverse electric field varies slowly so that:

$$\frac{dE_x}{ds} \leq \frac{E_x}{\lambda_t}$$

the condition of adiabatic motion is satisfied which means that the perturbations due to transverse fields are negligible. This, usually, needs higher magnetic fields (small $\lambda_t$) than for a resonant gun.

7.2 Drift space

7.2.1 Transversal and longitudinal temperature spread

During acceleration the transverse energy spread should theoretically remain constant, of the order of 0.2 to 0.5 eV as mentioned before. The longitudinal velocity spread, with respect to the moving frame is subject to an important reduction as seen in section 3. The electron velocity distribution is thus flattened.

7.2.2 Effect of the space charge on the longitudinal velocity

The electron beam is assumed to be circular with a radius $r_0$, and to travel along the axis of a circular tube of radius $r_e$, at ground potential. The electron beam density is then (Fig. 7.3):

$$n_e^* = \frac{p U_e^{3/2}}{e \cdot \pi \cdot r_0^2 \cdot \beta_0 \cdot c} \text{ m}^{-3}.$$
Due to the electron space charge a transverse radial electrical field $E_r$ will exist within the e-beam. It can be computed using the Gauss theorem. The expressions of the electric field and potential are:

For $0 \leq r \leq r_0$

$$U_f = \frac{en_e^* (r^2 - r_0^2)}{4\varepsilon_0} - \frac{en_e^* r_0^2}{2\varepsilon_0} \ln \left( \frac{r}{r_0} \right)$$

$$E_r = -\frac{en_e^*}{2\varepsilon_0} r$$

For $r_0 \leq r \leq r_c$

$$U_f = -\frac{en_e^*}{2\varepsilon_0} r_0^2 \ln (r_c / r)$$

$$E_r = -\frac{en_e^*}{2\varepsilon_0} \frac{r_0^2}{r}$$

Fig. 7.4 Potential distribution versus radius in the drift tube
This means that even at the centre of the beam \((r = 0)\) there exists a negative potential (Fig. 7.4):

\[
U_f = -\frac{e n_e^* r_0^2}{4\varepsilon_0} \left[ 1 + 2\ln \left( \frac{r_e}{r_0} \right) \right]
\]

\[
U_f = -n_e^* \pi r_0^2 \left( \frac{m_e c^2}{e} \right) \left[ 1 + 2\ln \left( \frac{r_e}{r_0} \right) \right]
\]

so that the kinetic energy of the electrons at the centre of the beam is:

\[
Ec = e U_0 - n_e^* \pi r_0^2 \left( \frac{m_e c^2}{e} \right) \left[ 1 + 2\ln (r_e / r_0) \right]
\]

\[
Ec = e U_0 \left[ 1 - \frac{p_e U_0^{1/2}}{\beta_0} \right] \cdot 91.849.
\]

**Example:**
At 309 MeV/c, \(U_0 = 27.10\) kV, \(\beta_0 = 0.312\), \(p_e = 0.52 \times 10^{-6}\), \(r_e = 70\) mm gives \(U_f\) \((r = 0)\) \(- 676\) V. Therefore the theoretical accelerating voltage must be corrected by this amount in order to obtain the requested velocity \(v_0\).

More important is the fact that electrons having a radius \(0 < r \leq r_0\) will undergo different acceleration voltages and will not move at the same longitudinal velocity as the electron on axis. The electron beam is far from being monokinetic. The longitudinal rms velocity spread remains however unchanged.

### 7.2.3 Effect of the space charge on the transverse velocity

The Ampere theorem can be used to get the magnetic field due to the electron beam itself at radius \(0 < r < r_0\):

\[
\vec{B} = \frac{\mu_0}{2} (n_e^* e \beta_0 c) r \vec{u}_r.
\]

The total force which acts on an electron of the beam with no initial transverse velocity, due to the space charge effect of the beam alone will be:

\[
\vec{f} = -e \left[ \vec{E}(r) \cdot \vec{u}_r + \vec{v} \times \vec{B} \right] = \left[ \frac{e^2 n_e^* r}{2\varepsilon_0} - \frac{\mu_0}{2} e^2 n_e^* r \beta_0^2 c^2 \right] \vec{u}_r = \frac{e^2 n_e^* r}{2\varepsilon_0} (1 - \beta_0^2) \vec{u}_r.
\]

The ratio of the electrical force to the magnetic force is \(1/\beta_0^2\). Hence the electron would acquire a radial velocity and therefore increase its transverse temperature.

The effects of the space charge explain the necessity for an additional guiding field, given by the main solenoid and toroids \(\vec{B}_0 = B_0 \vec{k}\) so that at least \(\vec{f}\) will cancel (Brillouin flow). The guiding field will exert a force \(\vec{f}_g = -e \cdot r \cdot B_0 \cdot \vec{\theta} \cdot \vec{u}_r\). The movement will become rather complex but we see that, even with no transverse energy at the cathode output, an angular velocity \(\vec{v} = r \cdot \vec{\theta}\) will exist which introduces an additional transverse temperature (Numerical example: 0.1 eV). In order to reduce these space-charge effects one may trap the ions, resulting from the collisions between the e-beam and the residual gas molecules in the drift region, so as to neutralise the electron space charge and therefore cancel the electric field. In this respect it is clear that the uncontrolled occurrence of such ions will modify the above expression.
7.2.4 Effect of the space charge on the cooling process

We consider,

a) The parabolic potential shape mentioned before which gives a parabolic electron longitudinal velocity

b) The longitudinal cooling force component on ions

c) The influence of the dispersion function $D(s)$ which gives the ion horizontal deviation $r_i$ from the nominal orbit:

$$r_i = D(s) \cdot \frac{\Delta \beta_i}{\beta_0} = D(s) \cdot \frac{\Delta \beta}{\beta_0} = \frac{D(s)}{\zeta} \cdot \frac{\Delta f}{f_0}.$$ 

Therefore the ion horizontal position is:

$$x_i(s) = d_i(s) + r_i(s).$$

![Diagram showing ion and electron velocity distribution](image)

**Fig. 7.5** Ion and electron velocity distribution, versus radius, in the drift space

We have plotted on Fig. 7.5 the velocity distributions as a function of the radius. If we neglect the betatron oscillation $d_i$, we see that:

- An ion at $B_1$ will be submitted to a strong cooling force and will therefore rapidly converge to the centre $O$,

- An ion initially at $B_2$ will also converge to $O$, but more slowly since the cooling force (proportional to $1/|\vec{v}_i - \vec{v}_e|^2$) is rather weak,

- An ion at $B_3$ will diverge and be lost.
At this point, it is important to see that the two beams have to be well aligned and the average velocities of the two beams well equalised. In the case of a misalignment (dashed curve of Fig. 7.5) a large part of the ion beam may be lost.

In the vertical plane such a process does not occur.

7.3 The collector

The task of this device is to collect the incoming electrons with a maximum efficiency. Here again many types of collector have been proposed. The simplest one consists of a Faraday cup. The electrons impinging on the collector surface will:

- dissipate power on the collector walls which have to be cooled
- create secondary electrons.

The primary electrons which do not enter the collector, or the secondary electrons which escape from the collector volume, are reaccelerated toward the drift tube and the cathode. Nearby the cathode they are reflected at the virtual cathode level. They bounce back and forth in the cooler, are heated transversally, and eventually hit the vacuum wall. This current loss \( I_e \) imposes a load for the high voltage power supply and upon impact with the vacuum chamber surface the electrons liberate gas and deteriorate the vacuum. The ratio:

\[
\frac{I_e}{I_{e-beam}} = \frac{I_e}{I_b}
\]

determines the collector inefficiency. Good collectors have an inefficiency of the order of \( 10^{-4} \).

In order to reduce the dissipated power and the maximum energy of the secondary electrons it is essential to reduce \( U_c \). According to the negative space-charge potential due to the e-beam, \( U_c \) must be of the order of a few kV. An important characteristic of the collector is its perveance \( p_e = I_e/U_c^{3/2} \). It must be such that:

\[
I_b = p_x \cdot U_0^{3/2} < p_e \cdot U_c^{3/2}
\]

\( p_x \) being the gun perveance. Therefore in order to have small dissipation (\( U_c \) small) \( p_e \) must be large.

Computer codes have to be used to calculate the correct dimensions, potentials and magnetic field. However the influence of the secondary electrons and of the ionised ions is very difficult to simulate.

7.4 Magnetic field

We have seen that the presence of a longitudinal field enhances the cooling forces and therefore reduces the cooling time (the field is of the order of \( 3 \cdot 10^{-2} \) to \( 1.5 \cdot 10^{-1} \) Tesla). We will see (section 8) that the vertical component of the magnetic field in the toroids creates a horizontal kick where the ion beam enters and leaves the toroids. On the other hand, for resonant type electron guns, the magnetic field must be set at well defined values to get very small transverse velocities.
Since the electron and ion beams must be well aligned it is very important that the magnetic field lines are strictly colinear with the theoretical electron trajectory. If the solenoid field makes an angle (Fig. 7.6) with respect to the theoretical trajectory over a length $\ell$ which is larger than

$$\beta_0 c \frac{1}{\omega_e},$$

the electron trajectory will follow the magnetic lines and will therefore acquire a transverse velocity

$$v_\perp \equiv \alpha \beta_0 c.$$

Its transverse temperature will in consequence be increased. Therefore all the magnetic system must be built within well specified tolerances. Additional coils are used to correct the eventual imperfections. We must also mention that some small horizontal and vertical dipoles are needed to perform the e-beam steering.

### 7.5 Vacuum system

Storage rings in general operate under ultra-high vacuum conditions ($<10^{-10}$ Torr). This is of course valid for the cooler itself which must be bakeable in situ at 300°C and make use of high vacuum materials and technology. A carefully estimated vacuum budget has to be established in order to evaluate the necessary pumping speeds. Pressure bumps mainly occur at the level of the hot cathode, in the drift tube where some high energy electrons are lost, and at the collector.

Suitable pumps are non-evaporable getters (NEG). Titanium sublimation pumps are sometimes used for special applications and for short periods. The NEG-pumps are activated during the system bake-out.

### 7.6 Controls

Operation of a cooler requires quite an elaborate control system with some of the electronics placed at the terminal potential. Figure 7.7 gives a synoptic diagram.

Communication with the system at ground potential is made via a fibre-optic link. For data acquisition, standard electronic components (ADC, DAC, etc...) are used in conjunction with systems such as CAMAC or VME. With the advent of workstations (WS) many processes may run in parallel for the control of the apparatus. Programs to control the cooler can be run from different WS simultaneously accessing parameters that may be on different CAMAC loops via an Ethernet link. The J11 microprocessor situated in the system crate of each CAMAC loop acts as the CAMAC/ETHERNET interface.

If energy ramping of the cooler is needed, the use of function generators is imperative in order to perturb the machine as little as possible.
8. EFFECTS ON THE ION BEAM

The interaction between the electrons and the ions is not without some disturbances. The ions will be submitted to a tune shift due either to their own space charge, which increases when cooling proceeds, or to the electron beam which acts as a lens. The solenoid will also twist the ion-beam trajectory and thereby induce a coupling between the transverse planes. When passing through the toroids, the ions will be influenced by the vertical component of the e-beam guiding field and will therefore receive a horizontal kick. As in many other accelerators, when reaching a given density threshold, the ion beam becomes unstable transversely. These coherent instabilities have to be damped. Lastly when the ion velocity comes close to that of the electron beam (the goal of any e-cooler) one can foresee nuclear interactions, which may deteriorate the ion beam lifetime. Let us now look at some details of all these phenomena.

8.1 Tune shifts

A tune shift is introduced on the ion beam due to the ion-beam space charge and to the electron beam. The ion-space charge effect is given by:

$$\Delta Q_s = \frac{Z \cdot N_i}{2 \cdot \pi \cdot \varepsilon \cdot \beta_0^2 \cdot \gamma^2 \cdot \frac{1}{B_f}}$$

where $B_f$ is the bunching factor ($B_f = 1$ for coasting beam). It will influence the stability at low energies when $\beta_0$ is small and when the beam is strongly cooled, $\varepsilon$ small. The electron beam acts as a lens giving a tune shift:

$$\Delta Q_e = \frac{Z}{2 \cdot \beta_0^2 \cdot \gamma} \cdot \frac{n_e^* \cdot \beta_e}{\varepsilon \cdot <\beta_y>}.$$ 

For a fixed perveance electron gun, since $n_e^* \propto \beta_e^2 \cdot \Delta Q_e$, should remain constant:
Numerical example $\Delta Q_{sp} = 1.08 \cdot 10^{-3}$ for case 1, $1.08 \cdot 10^{-2}$ for case 2, and $\Delta Q_{e} = 4.2 \cdot 10^{-3}$.

In experiments $\Delta Q = 0.01$ was achieved.

This effect must be carefully taken into consideration mainly for variable energy storage rings and even at fixed energy due to the change in ion density. This is true since for any triplet $(m, n, k)$ of integer the inequality $n \cdot Q_z + m \cdot Q_v \neq k$ must be fulfilled to keep the ion beam stable.

8.2 Coupling of transverse phase-space plane

The solenoid longitudinal magnetic field causes the ion beam to execute a cyclotron rotation around the longitudinal axis at frequency

$$\omega_i = \frac{Z \cdot e \cdot B_0}{m_i}.$$ 

Therefore, at each passage, the solenoid magnetic field will twist the ion beam by

$$\delta \theta = \frac{Z \cdot e \cdot B_0}{m_i} \frac{\ell_c}{v_0} = \frac{Z \cdot e \cdot B_0}{p_0} \frac{\ell_c}{v_0} \quad \text{(Numerical example } 6.63 \cdot 10^{-2} \text{ rad).}$$

This will induce a coupling between the horizontal and vertical planes and eventually depolarise polarised beams. It may also affect the cooling if there are different betatron functions in the two directions. To compensate this drawback, a solenoid producing a field in the opposite sense must be installed in the accelerator (usually near the cooler itself). Skew quadrupoles may also be used.

8.3 Effects on the closed orbit

The transverse component (vertical) of the guiding magnetic field in the toroids of the cooler gives a horizontal kick to the ion beam. The deflection angle is

$$\theta[rad] = \int \frac{B_i \, dl}{(p_0/e)} = \frac{B_i R_i}{p_0/e} \ln(\cos \Phi_0),$$

and the displacement of the ion beam is

$$\Delta x = \frac{Ze B_0 R_i^2}{m_i \beta_0 c \gamma_0} \left| \Phi_0 - \tan \Phi_0 + \frac{1}{2} \tan \Phi_0 \ln(1 + \tan^2 \Phi_0) \right|,$$

where $\Phi_0$ and $R_i$ are the bending angle and radius of the toroid. Numerically $\Delta x = 2.72 \cdot 10^{-3}$ m. Two vertical dipoles, positioned at each end of the cooler, correct to a first approximation for this displacement. For a fixed magnetic field the displacement increases as the ion energy is decreased.

8.4 Transverse instabilities of the ion beam

One may refer to the lecture [10] and [11] on this subject.
8.4.1 Observation of instabilities

If as a result of the cooling, the density $N / e_{e}e_{e}$ becomes larger than a given threshold, the ion beam becomes unstable. Roughly speaking, for our standard machine this happens when $N$, the number of circulating particles, exceeds $10^8$. This phenomenon is observed with:

- **Schottky transverse pickups.** Figure 8.1 shows this measurement with a spectrum analyser set at a fixed frequency (frequency span = 0) at one of the betatron sidebands $(n \pm Q) f_{0}$. More explicitly, at the time where the threshold occurs one sees an abrupt change of the transverse Schottky signal followed by a cooling period of about 10s until the density threshold is again reached. In Fig. 8.1a) the height of the transverse Schottky signal near 40 MHz is displayed. The horizontal scale is 1 s/div. The instability occurring at this frequency causes a large coherent oscillation which smears out and leads to an emittance growth during 0.2 s. This blow-up is then compensated during 0.8 s by the cooling before the next burst of instabilities occurs.

![Figure 8.1](image)

a) Center frequency: 49.82 MHz, Span: 0 Hz, Sweep time: 10 s
b) Center frequency: 159.60 kHz, Span: 20 kHz, Sweep time: 1 s

Fig. 8.1 Observation of transverse instabilities on (a) the Schottky pickup and (b) on the position pickup

- **Transverse position pickups.** It is well known that in the case of instability the beam centre is represented by a travelling wave pattern described by:

$$y(s,t) = y_{0} e^{i[(n \pm Q) \omega_{0} \cdot t - \frac{nx}{k}]}$$

where $n$, an integer, is named the mode of oscillation. At a fixed azimuth $s$ the pickup will see the mode frequency:

$$\omega_{n} = (n \pm Q) \omega_{0}. \quad (8.2)$$

In most cases the instabilities are observed for lower order ($Q_{e} = 2.3, Q_{a} = 2.7$, modes are $n = 3, 4, 5$) slow waves corresponding to the minus sign in Eq. (8.2). In Fig. 8.1b) the spectral density at low frequency near one of the bands where the instability occurs is
displayed. The spike representing the beam oscillation jumps up with each burst of the instability. Usually, when the instability occurs, part of the beam is lost until $N < 10^9$ ions. These instabilities are also observed in many other situations.

### 8.4.2 Causes of instabilities

Normal beams tend to resist coherent instabilities by virtue of Landau damping. A small difference between the oscillation frequency prevents the ensemble responding coherently to the driving force exerted by the beam induced fields. On the other hand, cooled beams become susceptible responding coherently for at least two reasons:

- The tune spread due to non-linearities and the momentum dependence decreases.
- Some of the induced fields, such as the direct space-charge field, increase as the beam cools down.

When differentiating formula (8.2) one can define

- a mode frequency shift: $\Delta \omega_* = \pm \Delta Q \cdot \omega_0$
- a mode frequency spread: $\delta \omega_* = (n \pm Q) \cdot \delta \omega_0 - \omega_0 \cdot \delta Q$.

If the mode frequency shift has an imaginary part, see section 8.4.1, we can see that the oscillation pattern can self amplify.

As a rule of thumb stabilization of transverse instabilities by Landau damping requires that

$$|\Delta \omega_*| \leq \frac{1}{\pi} |(n \pm Q)\delta w_0 - w_0 \cdot \delta Q|.$$ 

To stay within the scope of this lecture we can not go deeper into this subject. One can just mention the Keil-Schnell criteria which states that at low energy the space-charge contribution to the longitudinal stability is obtained when (see lecture on impedances and Landau damping these proceedings [6-7]):

$$\left( \frac{\Delta p}{\rho_0} \right)^2 > \left| \frac{Z_{sc}}{n} \right| \frac{e}{m_i c^2 \beta_i^2 \gamma_i \eta_i}$$

where $I_i$ is the ion-beam intensity, and

$$\frac{Z_{sc}}{n} = \frac{377}{2 \beta_0^2 \gamma_0^2} g$$

is the longitudinal impedance in $\Omega$, and

$$g = 1 + 2 \ln \left( \frac{\text{chamber height}}{\text{beam height}} \right).$$

(Numerically $g$ is 5.9 $\Omega$ for cooled beams, 3.5 $\Omega$ for normal beams). Other contributions to the impedance come from the changes in the wall resistivities of chamber sizes, RF cavities, ferrites and dielectric structures seen by the beam. So, to a first approximation:
\[
\frac{\Delta p}{p_0} > 8.783 \cdot 10^{-9} \frac{[N' \cdot f_0]^{1/2}}{[\beta_0 \gamma_0]^{3/2} |\eta|^{1/2}}
\]

where \( N' \) is the number of particles expressed in units of \( 10^9 \). For \( N' = 1 \) one obtains:

<table>
<thead>
<tr>
<th>Momentum MeV/c</th>
<th>309</th>
<th>200</th>
<th>100</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \left( \frac{\Delta p}{p_0} \right)_{\text{min}} )</td>
<td>( 5 \cdot 10^{-5} )</td>
<td>( 8 \cdot 10^{-5} )</td>
<td>( 1.6 \cdot 10^{-4} )</td>
<td>( 2.63 \cdot 10^{-3} )</td>
</tr>
</tbody>
</table>

On the other hand, the transverse stability criterion is given by the following inequality:

\[
\frac{Z_\perp}{|n - Q|} \leq \frac{F m_e c^2}{G e} \frac{2Qb^2}{R^2} \beta_0 \gamma_0 \frac{\Delta p}{p_0} |(n \pm Q) \eta - \xi Q|
\]

where \( F, G \) are form factors of the order of 1, \( b \) is the vacuum chamber height (35 mm for our standard machine), \( R \) is the mean radius of the accelerator (\( \approx 12.5 \) m), \( Q \) the tune (\( Q_\perp = 2.305 \), \( Q_\| = 2.73 \)),

\[
\eta = \frac{1}{\gamma_0^2} - \frac{1}{\gamma_\perp^2} \approx 1 \text{ at our standard machine}
\]

and \( \xi \) is the chromaticity. \( Z_\perp / |n - Q| \) is the transverse impedance converted into longitudinal units via:

\[
\frac{Z_\perp}{n} = \frac{b^2}{R} \frac{Z_\perp}{n - Q}.
\]

This is valid for simple structures but not for the space-charge contribution to the transverse impedance, which in the transverse coasting beam case is:

\[
\frac{Z_\perp}{n - Q} = \frac{377}{2 \beta_0^2 \gamma_0} \left( \frac{b^2}{a^2} - 1 \right) \Omega
\]

\( a \) being the beam height. In consequence transverse stability requires that:

\[
\frac{\Delta p}{p_0} > \left( \frac{Z_\perp}{n} \right) \left( \frac{e}{m_e c^2} \right) \frac{I_R^2}{2 \cdot Q \cdot b^2 \cdot \beta_0 \gamma_0} \frac{1}{|(n \pm Q) \eta - \xi Q|}.
\]

If one takes:

\[
(n - Q) \eta - \xi Q = 0.3
\]

we come to:
\[
\frac{\Delta p}{p_0} > 7.13 \cdot 10^{-12} \frac{N' f_0}{(\beta_0 \gamma_0)^{2}}.
\]

For \(N' = 1\) one obtains:

<table>
<thead>
<tr>
<th>Momentum MeV/c</th>
<th>309</th>
<th>200</th>
<th>100</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{\Delta p}{p_0})_{\text{min}}</td>
<td>(7.85 \cdot 10^{-5})</td>
<td>(1.25 \cdot 10^{-4})</td>
<td>(2.57 \cdot 10^{-4})</td>
<td>(4.18 \cdot 10^{-4})</td>
</tr>
</tbody>
</table>

The above tables show that for a beam of a few \(10^6\) particles the loss of Landau damping occurs (with our standard machine) approximately when the longitudinal cooling reaches the ratio \(\Delta p / p_0 = 10^{-4}\). This is directly linked to the double-peaked spectrum signal observed with strong cooling. Of course, many other phenomena may induce instabilities, such as the interaction of electrons from the residual gas trapped in the proton beam. By the way, the impedances \((Z/n)\) have to be measured rather than estimated.

It is worth mentioning that coherent motion gives signals, on a spectrum analyser, which are proportional to \(N\), the number of particles, while for Schottky (random) signals the amplitudes are proportional to \(\sqrt{N}\).

8.4.3 Partial cure of transverse instabilities

In order to suppress the transverse coherent instability a damper is an important electronic device for stabilising dense beams. A simple damper (Fig. 8.2) consists mainly of:

- a horizontal and vertical electrostatic pickup,
- a horizontal and vertical kicker placed at a given distance \(S_0\) from the pickups.

![Fig. 8.2 Principle of the damper](image)

The transverse position signal, taken from the position pickup is linearly amplified, appropriately delayed then applied to the kicker plates. If we take the phase from Eq. (8.1):
\[ \theta = (n - Q) \cdot \omega_b \cdot \tau \cdot \frac{n_{si}}{R}. \]

$s = S_0$ is chosen such that the phase of the particle has increased by an odd multiple of $\pi/2$ between the pickup and the kicker (usually named $\lambda/4$ advance). The amplifier bandwidth is directly related to the mode $n$ of oscillations to be covered and to its time response to the growth rate of the instability. With such a damper high intensity beams with large densities can be obtained which are very useful for physics experiments.

### 8.5 Other electron-ion interactions

At the beginning of this lecture, we mentioned that in order to explain the cooling process the electron beam may be considered as a target. There are some other reactions that should be considered. The most dominant are:

- $A^{z+} + e^- \rightarrow A^{(z-1)^+} + h\nu$ 
  spontaneous radiative recombination
- $A^{z+} + e^- \rightarrow [A^{(z-1)^+}] \rightarrow A^{(z-1)^+} + h\nu$ 
  dielectronic recombination
- $A^{z+} + e^- \rightarrow (A^{z^+})^+ + e^-$ 
  ion excitation
- $A^{z+} + e^- \rightarrow A^{(z+1)^+} + e^- + e^-$ 
  ionisation.

These are processes which should be taken into account near thermal equilibrium and therefore may influence the cold ion-beam lifetime.

#### Spontaneous recombination

When positive ions are cooled by electrons, occasionally cooling electrons are radiatively captured by beam ions into atomic states with main quantum number $n$. If $N$ is the number of stored ions, we can define

\[ \frac{1}{N} \frac{dN}{dt} = \frac{1}{\tau} = \frac{R_s}{N}. \]

Therefore the number of lost ions per unit of time is

\[ R_s = \frac{N \cdot \eta_e \cdot \alpha_r \cdot n_i^*}{\gamma_0^2}; \quad \alpha_r = 9.3 \cdot 10^{-19} Z^2 (m^3 s^{-1} eV^{1/2}) (kT)^{-1/2}. \]

For $kT = 1$ eV; $\alpha_r = 9.3 \cdot 10^{-19}$, $R_s = 2537$.

In the case of protons (ions) the detection of these neutral atoms (recharged ions) is an indirect way to adjust the cooler parameters, until one obtains the maximum $H_0$ rate, and to measure the ion beam transverse emittance.

#### Dielectronic recombination

This reaction will come into play if the circulating ions are not fully stripped. An electron is captured in an auto-ionising state, and the energy gained is used to lift one of the core electrons to an excited state. The auto-ionising states decay either back into the original channel or into a lower state.
The phenomenon is of resonant type and so will depend on the ion particular state. When the resonant condition is fulfilled, the cross-section attains very high values of the order of 10^6 mBarn.

We will not comment on the other two phenomena which are more relevant to nuclear physics (see lecture [4]).

9. PRESENT AND FUTURE ELECTRON COOLER RINGS

Table 9.1 shows the design parameters of the present and near future electron-cooling rings. Most of them are dedicated to atomic physics experiments. The pioneer coolers built at INP Novosibirsk, Fermilab and at CERN (ICE) are no longer operational. The CERN-LEAR cooler is not represented in this list since it is already mentioned in section 2 as a typical accelerator. The numerical applications refer to data and numbers which have been physically measured. COSY will be the next electron cooler to go into operation while classical cooler projects are underway in USSR (Dubna, Kiev). A study of a 6 MeV electron energy cooler is underway at Bloomington (U.S.A.) and the results are awaited with interest.

Figure 9.1 shows the layout of five of the machines mentioned in Table 9.1. It is easy to recognise the fundamental components mentioned in this lecture. Also worth mentioning is the fact that, except for COSY and Dubna K4 - K10, they have reached their design parameters.

Fig. 9.1a) Cross-sectional view of the ESR electron cooler
Fig. 9.1b) Layout of the TSR electron cooler

Fig. 9.1c) Layout of the TARN II electron cooler device
Fig. 9.1d) Longitudinal section of the COSY electron cooler

Fig. 9.1e) Electron cooling system of the ring K10: 1 and 10 = electron gun and collector (respectively) in their tanks; 2 and 11 = solenoids of the electron gun and collector (respectively); 3, 5 and 9 = vacuum pumps; 4, 8 = toroid magnets, 6, 7 = the winding and the body of the main solenoid; 12, 13 = power supplies; 14 = platform
<table>
<thead>
<tr>
<th>Name of Ring</th>
<th>IUCF COOLER</th>
<th>TSR</th>
<th>CEIUS</th>
<th>TARN II</th>
<th>ESR</th>
<th>DUBNA</th>
<th>CRYRING</th>
<th>COSY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bloomington</td>
<td>Heidelberg</td>
<td>Uppsala</td>
<td>Tokyo</td>
<td>Darmstadt</td>
<td>K4</td>
<td>K10</td>
<td>Stockholm</td>
</tr>
<tr>
<td>Circumference (m)</td>
<td>86.8</td>
<td>78.8</td>
<td>77.8</td>
<td>108.4</td>
<td>83.12</td>
<td>146.24</td>
<td>51.6</td>
<td>184</td>
</tr>
<tr>
<td>Magnetic rigidity (T-m)</td>
<td>3.6</td>
<td>7</td>
<td>6.1</td>
<td>10</td>
<td>4.0/0.7</td>
<td>10.0/0.7</td>
<td>1.44</td>
<td>11</td>
</tr>
<tr>
<td>Energy range (MeV/A)</td>
<td>500</td>
<td>5-30</td>
<td>1360 (for p)</td>
<td>100,400 (for p)</td>
<td>30,560</td>
<td>170</td>
<td>830</td>
<td>0.3-24</td>
</tr>
<tr>
<td>Type of ions:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- highest charge</td>
<td>3</td>
<td>47</td>
<td>18</td>
<td>10</td>
<td>92</td>
<td>92</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>- highest A</td>
<td>7</td>
<td>127</td>
<td>40</td>
<td>20</td>
<td>238</td>
<td>238</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>- lowest Q/M</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>0.2</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>Expected ion intensity</td>
<td>10 mA</td>
<td>10 mA</td>
<td>$10^{11}$ (p)</td>
<td>$10^8$ up to $10^{11}$</td>
<td>$3 \times 10^9$ - $8 \times 10^{11}$</td>
<td>$4 \times 10^9$</td>
<td>$2 \times 10^8$</td>
<td>$1 \times 10^{11}$</td>
</tr>
<tr>
<td>Average ring vacuum (Torr)</td>
<td>$&lt; 3 \times 10^{-9}$</td>
<td>$&lt; 10^{-11}$</td>
<td>$10^{-11}$</td>
<td>$10^{-11}$</td>
<td>$10^{-11}$</td>
<td>$10^{-11}$</td>
<td>$10^{-11}$</td>
<td>$1 \times 10^{-10}$</td>
</tr>
<tr>
<td>Horizontal acceptance (π mm-rad)</td>
<td>25</td>
<td>500</td>
<td>120</td>
<td>250</td>
<td>450</td>
<td>50</td>
<td>25</td>
<td>200</td>
</tr>
<tr>
<td>Vertical acceptance (π mm-rad)</td>
<td>25</td>
<td>120</td>
<td>120</td>
<td>15</td>
<td>150</td>
<td>50</td>
<td>25</td>
<td>100</td>
</tr>
<tr>
<td>Longitudinal acceptance for $\varepsilon = 0$ (%)</td>
<td>$\pm 0.2$</td>
<td>$\pm 3$</td>
<td>$\pm 0.3$</td>
<td>$\pm 0.1$</td>
<td>$\pm 2$</td>
<td>1.0</td>
<td>2.0</td>
<td>$\pm 0.7$</td>
</tr>
<tr>
<td>Length of cooling section (m)</td>
<td>2.8</td>
<td>1.5</td>
<td>2.5</td>
<td>1.5</td>
<td>2.5</td>
<td>3</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>Typical working point:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- $Q_B$</td>
<td>4.15</td>
<td>2.75</td>
<td>1.68</td>
<td>1.75</td>
<td>2.1-2.45</td>
<td>2.4</td>
<td>2.8</td>
<td>2.3</td>
</tr>
<tr>
<td>- $Q_A$</td>
<td>5.15</td>
<td>2.825</td>
<td>1.9</td>
<td>1.8</td>
<td>2.1-2.45</td>
<td>2.8</td>
<td>3.3</td>
<td>2.27</td>
</tr>
<tr>
<td>Average horizontal $\beta$ function (m)</td>
<td>2.7</td>
<td>3.15</td>
<td>7.75</td>
<td>8</td>
<td>8-10</td>
<td>5.5</td>
<td>8.3</td>
<td>4</td>
</tr>
<tr>
<td>$\beta_B$ in cooling section (m)</td>
<td>2.3</td>
<td>5.5</td>
<td>8.14</td>
<td>10</td>
<td>10</td>
<td>10.9</td>
<td>5.6</td>
<td>2.3</td>
</tr>
<tr>
<td>$\beta_A$ in cooling section (m)</td>
<td>4</td>
<td>5.7</td>
<td>5.04</td>
<td>4</td>
<td>4-8</td>
<td>6.3</td>
<td>5.8</td>
<td>2.9</td>
</tr>
<tr>
<td>Dispersion in cooling section (m)</td>
<td>0</td>
<td>0-1</td>
<td>1.48</td>
<td>4.7</td>
<td>4.7</td>
<td>0-6</td>
<td>0</td>
<td>1.6</td>
</tr>
<tr>
<td>Transition $\gamma$</td>
<td>4.85</td>
<td>2.96</td>
<td>2.05</td>
<td>1.88</td>
<td>2.7</td>
<td>5.26</td>
<td>2.26</td>
<td>2.23</td>
</tr>
<tr>
<td>Internal target type: Jet/dust fibre</td>
<td>Storage cell</td>
<td>Cluster/C fibre</td>
<td>Jet</td>
<td>Jet/cluster</td>
<td>microclusters'</td>
<td>Cluster</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- density (atoms/cm²)</td>
<td>1-100 ng/cm²</td>
<td>10⁹⁴</td>
<td>10⁹⁴</td>
<td>10⁹⁴</td>
<td>10⁹⁴</td>
<td>10⁹⁴</td>
<td>10⁹⁴</td>
<td></td>
</tr>
<tr>
<td>- nuclear mass</td>
<td>H</td>
<td>1-129</td>
<td>1-238</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cathode/beam diameter (cm)</td>
<td>2.5</td>
<td>2.5</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Electron energy range (keV)</td>
<td>10-270</td>
<td>3-20</td>
<td>10-30</td>
<td>≤ 120</td>
<td>10-320</td>
<td>15-100</td>
<td>100-250</td>
<td>2-20</td>
</tr>
<tr>
<td>Electron current (A)</td>
<td>0.4-8</td>
<td>1</td>
<td>0.2-6</td>
<td>≤ 10</td>
<td>0.5-5</td>
<td>0.5-5</td>
<td>5</td>
<td>0.01-0.3</td>
</tr>
<tr>
<td>Nominal gun permeance (μP)</td>
<td>0.7</td>
<td>1.7</td>
<td>0.36 (40 kV)</td>
<td>1</td>
<td>1.85</td>
<td>0.44</td>
<td>0.16</td>
<td>0.16-0.04</td>
</tr>
<tr>
<td>Magnetic field (KG)</td>
<td>1-1.5</td>
<td>0.55</td>
<td>0.5-2</td>
<td>1.2</td>
<td>0.1-2.5</td>
<td>1.2</td>
<td>2</td>
<td>2.5</td>
</tr>
<tr>
<td>- at electron energy (keV)</td>
<td>60</td>
<td>45</td>
<td>90</td>
<td>45</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>50</td>
</tr>
<tr>
<td>Toroidal angle (°)</td>
<td>2-7</td>
<td>1.2-2</td>
<td>2-7</td>
<td>2-7</td>
<td>310</td>
<td>310</td>
<td>13</td>
<td>22</td>
</tr>
<tr>
<td>Gun-collector voltage (kV):</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- at electron-energy (kV)</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>10</td>
<td>10</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>- at current (A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vacuum in cooler (Torr)</td>
<td>$1 \times 10^{-9}$</td>
<td>$&lt; 10^{-10}$</td>
<td>$10^{-11}$</td>
<td>$&lt; 10^{-10}$</td>
<td>$10^{-11}$</td>
<td>$10^{-11}$</td>
<td>$10^{-11}$</td>
<td>$1 \times 10^{-10}$</td>
</tr>
<tr>
<td>Electron losses (collector)</td>
<td>$10^{-6}$-$10^{-3}$</td>
<td>$&lt; 10^{-4}$</td>
<td>$10^{-4}$</td>
<td>$&lt; 10^{-3}$</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
<td>$1 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
10. CONCLUSION

Though we have chosen an approach which is more didactic than rigorous, we hope that the usefulness of electron cooling, for low-energy storage rings, has been sufficiently explained.

Amongst all the cooling processes (at least for accelerators operating at momenta lower than about 600 MeV/c) electron cooling is presently the only one which makes it possible to reach very small emittances in very short times. However, it involves a broad knowledge of many physical domains and needs good engineering of almost all the components in order to reach the required performances.

The cooler itself is only part of the process. A careful study must be made, and many precautions must be taken, concerning the insertion of this device into the storage ring. This is particularly true for variable energy accelerators, where the effects on the ion beam (Q shifts, closed orbit, instabilities) must be minimised.

REFERENCES

Almost all the references which refer to electron cooling can be found in the basic report [1] (which has been taken as a reference for this lecture):


Written in the frame of a lecture are:


More elaborate reports are:


BIBLIOGRAPHY


Y. Debernev, I. Meshkov et al., CERN 77-08 yellow report.


Y. Debernev and A.N. Skrinsky, Particle Accelerator 8 (1977) 1.


Proceedings of the Workshop on ECOOL-1984, Karlsruhe, Editor: H. Poth, KfK 3846,