A Fresh Look at the B Semileptonic Branching Ratio and Beauty Lifetimes

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Abstract

We discuss two problems in the theory of heavy-quark decays: an understanding of the semileptonic branching ratio of $B$ mesons, and of the lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$. We also present a model-independent study of spectator contributions to the lifetimes of beauty hadrons.

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1 Introduction

Inclusive decays of heavy hadrons have two advantages from the theoretical point of view: first, bound-state effects related to the initial state can be accounted for in a systematic way using the heavy-quark expansion [1]–[3]; secondly, the fact that the final state consists of a sum over many hadronic channels eliminates bound-state effects related to the properties of individual hadrons. This last feature is based on the hypothesis of quark–hadron duality, i.e. the assumption that cross sections and decay rates are calculable in QCD after an “averaging” procedure has been applied [4]. This assumption has been tested experimentally using data on hadronic $\tau$ decays [5]. The theory of inclusive decays of heavy hadrons proved to be very successful (for a recent review, see Ref. [6]). For instance, it explains a posteriori the success of the parton model in describing inclusive semileptonic decays of heavy hadrons. However, we shall address here two potential problems of this theory: the semileptonic branching ratio of $B$ mesons, and the short lifetime of the $\Lambda_b$ baryon.

The inclusive decay width of a hadron $H_b$ containing a $b$ quark can be written as the forward matrix element of the imaginary part of the transition operator,

$$\Gamma(H_b \rightarrow X) = \text{Im} \langle H_b | T | H_b \rangle,$$

where $T$ is given by

$$T = i \int d^4x T\{ \mathcal{L}_{\text{eff}}(x), \mathcal{L}_{\text{eff}}(0) \}.$$  

(1)

For the case of semileptonic and non-leptonic decays, the effective weak Lagrangian is

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{cb} \left\{ c_1(m_b) \left[ d_L^\prime \gamma_\mu u_L \bar{c}_L \gamma^\mu b_L + s_L^\prime \gamma_\mu c_L \bar{c}_L \gamma^\mu b_L \right] 
+ c_2(m_b) \left[ \bar{c}_L \gamma_\mu u_L \bar{d}_L^\prime \gamma^\mu b_L + \bar{c}_L \gamma_\mu c_L \bar{s}_L^\prime \gamma^\mu b_L \right] 
+ \sum_{\ell=e,\mu,\tau} \bar{\ell}_L \gamma_\mu \nu_\ell \bar{\ell}_L \gamma^\mu b_L \right\} + \text{h.c.},$$  

(2)

where $q_L = \frac{1}{2}(1-\gamma_5)q$ denotes a left-handed quark field, $d'$ and $s'$ are the Cabibbo-rotated down- and strange-quark fields, and we have neglected $b \rightarrow u$ transitions. The Wilson coefficients $c_1$ and $c_2$ take into account the QCD corrections arising from the fact that the effective Lagrangian is written at a renormalization scale $\mu = m_b$ rather than $m_W$. The combinations $c_{\pm} = c_1 \pm c_2$ are renormalized multiplicatively.

In perturbation theory, some contributions to the transition operator are given by the two-loop diagrams shown on the left-hand side of Fig. 1. Since the energy release in the
decay of a $b$ quark is large, it is possible to construct an Operator Product Expansion (OPE) for the bilocal transition operator (1), in which it is expanded as a series of local operators with increasing dimension, whose coefficients contain inverse powers of the $b$-quark mass. The operator with the lowest dimension is $\bar{b}b$. It arises from integrating over the internal lines in the first diagram. There is no independent operator with dimension four, since the only candidate, $\bar{b}i\not{D}b$, can be reduced to $\bar{b}b$ by using the equations of motion. The first new operator is $\bar{b}g_{\sigma\mu\nu}G^{\mu\nu}b$ and has dimension five. It arises from diagrams in which a gluon is emitted from one of the internal lines, such as the second diagram in Fig. 1.

\[ \Gamma(H_b \to X_f) = \frac{G_F^2 m_b^5}{192\pi^3} \left\{ c^3_f \left( 1 - \frac{\mu^2(\not{D}_b)}{2m_b^2} \right) + c^5_f \frac{\mu^2(\not{G}_b)}{2m_b^2} + O(1/m_b^3) \right\}, \]

where $\mu^2(\not{D}_b)$ and $\mu^2(\not{G}_b)$ parametrize the matrix elements of the kinetic-energy and the chromo-magnetic operators, respectively, and $c^i_f$ are calculable coefficient functions (which also contain the relevant CKM matrix elements) depending on the quantum numbers $f$ of the final state. For semileptonic and non-leptonic decays, the coefficients $c^3_f$ have been
calculated at one-loop order \([9, 10]\), and the coefficients \(c_5^f\) at tree level \([2, 11]\). The relevant combinations of the hadronic parameters \(\mu^2(H_b)\) and \(\mu_G^2(H_b)\) for \(B\) mesons and \(\Lambda_b\) baryons can be determined from the spectrum of heavy-hadron states \([6, 8]\).

## 2 Semileptonic Branching Ratio

The semileptonic branching ratio of \(B\) mesons is defined as

\[
B_{\text{SL}} = \frac{\Gamma(B \to X e \bar{\nu})}{\sum_\ell \Gamma(B \to X \ell \bar{\nu}) + \Gamma_{\text{NL}} + \Gamma_{\text{rare}}},
\]

where \(\Gamma_{\text{NL}}\) and \(\Gamma_{\text{rare}}\) are the inclusive rates for non-leptonic and rare decays, respectively. Measurements of this quantity have been performed by various experimental groups. The status of the results is controversial, as there is a discrepancy between low-energy measurements performed at the \(\Upsilon(4s)\) resonance and high-energy measurements performed at the \(Z^0\) resonance. The average value at low energies is \(B_{\text{SL}} = (10.37 \pm 0.30)\%\) \([12]\), whereas high-energy measurements give \(B_{\text{SL}}^{(b)} = (11.11 \pm 0.23)\%\) \([13]\). The superscript \((b)\) indicates that this value refers not to \(B\) mesons, but to a mixture of \(b\) hadrons. Correcting for this fact, we find the slightly larger value \(B_{\text{SL}} = (11.30 \pm 0.26)\%\) \([6]\). The discrepancy between the low- and high-energy measurements of the semileptonic branching ratio is therefore larger than three standard deviations. If we take the average and inflate the error to account for this fact, we obtain \(B_{\text{SL}} = (10.90 \pm 0.46)\%\). An important aspect in understanding this result is charm counting, i.e. the measurement of the average number \(n_c\) of charm hadrons produced per \(B\) decay. Recently, two new (preliminary) measurements of this quantity have been performed. The CLEO Collaboration has presented the value \(n_c = 1.16 \pm 0.05\) \([12]\), and the ALEPH Collaboration has reported the result \(n_c = 1.20 \pm 0.08\) \([14]\). The average is \(n_c = 1.17 \pm 0.04\).

In the parton model, \(B_{\text{SL}} \simeq 13\%\) and \(n_c \simeq 1.15\) \([15]\). Whereas \(n_c\) is in agreement with experiment, the semileptonic branching ratio is predicted to be too large. With the establishment of the \(1/m_Q\) expansion the non-perturbative corrections to the parton model could be computed, and they turned out to be too small to improve the prediction \([16]\). The situation has changed recently, when it was found that higher-order perturbative corrections lower the value of \(B_{\text{SL}}\) significantly \([10]\). The exact order-\(\alpha_s\) corrections to the non-leptonic width have been computed for \(m_c \neq 0\), and an analysis of the renormalization scale and scheme dependence has been performed. In particular, it turns out that radiative corrections increase the partial width \(\Gamma(B \to X_{c\bar{s}})\) by a large amount. This has two effects: it lowers the semileptonic branching ratio, but at the price of a higher value of \(n_c\).
The original analysis of Bagan et al. has recently been corrected in an erratum [10]. Here we shall present the results of an independent numerical analysis using the same theoretical input (for a detailed discussion, see Ref. [17]). The semileptonic branching ratio and \( n_c \) depend on the quark-mass ratio \( m_c/m_b \) and on the ratio \( \mu/m_b \), where \( \mu \) is the scale used to renormalize the coupling constant \( \alpha_s(\mu) \) and the Wilson coefficients \( c_{\pm}(\mu) \) appearing in the non-leptonic decay rate. Below we shall consider several choices for the renormalization scale. We allow the (one-loop) pole masses of the heavy quarks to vary in the range \( m_b = (4.8 \pm 0.2) \text{ GeV} \) and \( m_b - m_c = (3.40 \pm 0.06) \text{ GeV} \), corresponding to \( 0.25 < m_c/m_b < 0.33 \). Non-perturbative effects appearing at order \( 1/m_b^2 \) in the heavy-quark expansion are described by the single parameter \( \mu_\pi^2(B) \); the dependence on the parameter \( \mu_\pi^2(B) \) is the same for all inclusive decay rates and cancels out in \( B_{\text{SL}} \) and \( n_c \). For the two choices \( \mu = m_b \) and \( \mu = m_b/2 \), we obtain

\[
B_{\text{SL}} = \begin{cases} 
12.0 \pm 1.0\%; & \mu = m_b, \\
10.9 \pm 0.9\%; & \mu = m_b/2, 
\end{cases}
\]

\[
n_c = \begin{cases} 
1.21 \mp 0.06; & \mu = m_b, \\
1.22 \mp 0.06; & \mu = m_b/2. 
\end{cases}
\]  

(5)

The uncertainties in the two quantities, which result from the variation of \( m_c/m_b \) in the range given above, are anticorrelated. Notice that the semileptonic branching ratio has a stronger scale dependence than \( n_c \). This is illustrated in Fig. 2, where we show the two quantities as a function of \( \mu \). By choosing a low renormalization scale, values \( B_{\text{SL}} < 12\% \) can easily be accommodated. The experimental data prefer a scale \( \mu/m_b \sim 0.5 \), which is indeed not unnatural; it has been estimated that \( \mu \gtrsim 0.32m_b \) is an appropriate scale to use.
The combined theoretical predictions for the semileptonic branching ratio and charm counting are shown in Fig. 3. They are compared with the experimental results obtained from low- and high-energy measurements. It was argued that the combination of a low semileptonic branching ratio and a low value of $n_c$ would constitute a potential problem for the Standard Model [16, 19]. However, with the new experimental and theoretical numbers, it is only for the low-energy measurements that a small discrepancy remains between theory and experiment.

![Fig. 3. Combined theoretical predictions for the semileptonic branching ratio and charm counting as a function of the quark-mass ratio $m_c/m_b$ and the renormalization scale $\mu$. The data points show the average experimental values for $B_{SL}$ and $n_c$ obtained in low-energy (LE) and high-energy (HE) measurements, as discussed in the text.]

3 Lifetime Ratios of Beauty Hadrons

The heavy-quark expansion predicts that the lifetimes of all beauty hadrons agree up to non-perturbative corrections suppressed by at least two powers of $1/m_b$. By explicit evaluation of the general result (3) for semi- and non-leptonic decays, one finds [17]

$$\frac{\tau(B^-)}{\tau(B^0)} = 1 + O(1/m_b^0), \quad \frac{\tau(\Lambda_b)}{\tau(B^0)} = 0.98 + O(1/m_b^3).$$

(6)

These theoretical predictions may be compared with the average experimental values for the lifetime ratios [20]: $\tau(B^-)/\tau(B^0) = 1.02 \pm 0.04$ and $\tau(\Lambda_b)/\tau(B^0) = 0.78 \pm 0.05$. 

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Whereas the lifetime ratio of charged and neutral $B$ mesons is in good agreement with the theoretical prediction, the low value of the lifetime of the $\Lambda_b$ baryon is surprising.

To understand the structure of the lifetime differences requires to go further in the $1/m_b$ expansion. “Spectator effects” [21], i.e. contributions from decays in which a light constituent quark participates in the weak process, contribute directly to the differences in the decay widths of different beauty hadrons. They are suppressed because the $b$ quark and a light quark in the heavy hadron need to be close together. Since the portion of the volume that the $b$ quark occupies inside the hadron is of order $(\Lambda_{QCD}/m_b)^3$, such effects appear only at third order in the heavy-quark expansion, and it might seem safe to neglect them altogether. However, as a result of the difference in the phase space for $2 \to 2$-body reactions as compared to $1 \to 3$-body decays, spectator effects are enhanced by a factor of order $16\pi^2$. This can be seen from Fig. 4, which shows that the corresponding contributions to the transition operator $\mathbf{T}$ arise from one-loop rather than two-loop diagrams. The situation is different from gluonic dimension-six operators of the type $\bar{b}\gamma_\mu(iD_\nu G^{\mu\nu})b$, which appear at the same order in the heavy-quark expansion. Such operators arise from decays in which the light spectators interact only softly. Since their matrix elements are flavour blind and not enhanced by phase space, they can be safely neglected.

If one cuts the internal lines in Fig. 4, one obtains the spectator contributions to the decay operator $\Gamma = 2 \text{Im} \mathbf{T}$. The corresponding spectator effects are referred to as Pauli interference and $W$ exchange [21]. The spectator contributions to the non-leptonic width
of beauty mesons and baryons are given by the matrix elements of the local operator [17]

$$
\Gamma_{\text{spec}} = \frac{2G_F^2m_b^2}{3\pi} |V_{cb}|^2 (1 - z)^2 \left\{ (2c_+^2 - c_-^2) O_{V-A}^q + 3(c_+^2 + c_-^2) T_{V-A}^q \right\} - \frac{2G_F^2m_b^2}{9\pi} |V_{cb}|^2 (1 - z)^2 \left\{ (2c_+ - c_-)^2 \left[ (1 + \frac{z}{2}) O_{Y-A}^q - (1 + 2z) O_{S-P}^q \right] + \frac{3}{2}(c_+ + c_-)^2 \left[ (1 + \frac{z}{2}) T_{V-A}^q - (1 + 2z) T_{S-P}^q \right] \right\} - \frac{2G_F^2m_b^2}{9\pi} |V_{cb}|^2 \sqrt{1 - 4z} \left\{ (2c_+ - c_-)^2 \left[ (1 - z) O_{V-A}^q - (1 + 2z) O_{S-P}^q \right] + \frac{3}{2}(c_+ + c_-)^2 \left[ (1 - z) T_{V-A}^q - (1 + 2z) T_{S-P}^q \right] \right\}, \tag{7}
$$

where \( z = m_c^2/m_b^2 \). The local four-quark operators appearing in this expression are defined by

$$
\begin{align*}
O_{V-A}^q &= \bar{b}_L \gamma_\mu q_L \bar{q}_L \gamma^\mu b_L, \\
T_{V-A}^q &= \bar{b}_L \gamma_\mu t_a q_L \bar{q}_L \gamma^\mu t_a b_L, \\
O_{S-P}^q &= \bar{b}_R q_L \bar{q}_L b_R, \\
T_{S-P}^q &= \bar{b}_R t_a q_L \bar{q}_L t_a b_R, \\
\end{align*}

\tag{8}
$$

where \( t_a \) are the generators of colour SU(3). These operators are renormalized at the scale \( m_b \), which will be implicit in our discussion below. We note that in the limit \( z = 0 \) our result agrees with Ref. [8], and with the corresponding calculations for charm decays [21].

The hadronic matrix elements of the four-quark operators in (8) contain the non-perturbative physics of the spectator contributions to inclusive decays of beauty hadrons. In most previous analyses of spectator effects, these matrix elements have been estimated using simplifying assumptions. For the matrix elements between \( B \)-meson states the vacuum saturation approximation [22] was assumed, i.e. the matrix elements of the four-quark operators were evaluated by inserting the vacuum inside the current products, in which case they are determined by the square of the decay constant \( f_B \) of the \( B \) meson. In order to avoid such model-dependent assumptions, we define without loss of generality [17]:

$$
\begin{align*}
\frac{1}{2m_B} \langle B | O_{V-A}^q | B \rangle &= B_1 \frac{f_B^2m_B}{8}, \\
\frac{1}{2m_B} \langle B | O_{S-P}^q | B \rangle &= B_2 \frac{f_B^2m_B}{8}, \\
\frac{1}{2m_B} \langle B | T_{V-A}^q | B \rangle &= \varepsilon_1 \frac{f_B^2m_B}{8}, \\
\frac{1}{2m_B} \langle B | T_{S-P}^q | B \rangle &= \varepsilon_2 \frac{f_B^2m_B}{8}. \\
\end{align*}

\tag{9}
$$

The values of the dimensionless hadronic parameters \( B_i \) and \( \varepsilon_i \) are currently not known; ultimately, they may be calculated using some field-theoretic approach such as lattice
gauge theory or QCD sum rules. The vacuum saturation approximation corresponds to setting $B_i = 1$ and $\varepsilon_i = 0$ (at some scale $\mu$, where the approximation is believed to be valid\(^1\)). For real QCD, however, it is known that $B_i = O(1)$ and $\varepsilon_i = O(1/N_c)$, where $N_c$ is the number of colours.

In the case of $\Lambda_b$ baryons, we find it convenient to introduce the operators ($i, j$ are colour indices) $\tilde{O}_{V-A} = \bar{b}_L^i \gamma_\mu q^j \bar{q}_L^\gamma \mu b_L^i$ and $\tilde{O}_{S-P} = \bar{b}_R^i q^j \bar{q}_L^i b_R^j$ instead of $T_{V-A}$ and $T_{S-P}$. They are related by the colour Fierz identity $T = -\frac{1}{6} O + \frac{1}{2} \tilde{O}$. The heavy-quark spin symmetry, i.e. the fact that interactions with the spin of the heavy quark decouple as the heavy-quark mass tends to infinity, implies the relations $\langle \Lambda_b | O_{V-A} | \Lambda_b \rangle = -\frac{1}{2} \langle \Lambda_b | O_{S-P} | \Lambda_b \rangle$ and $\langle \Lambda_b | \tilde{O}_{S-P} | \Lambda_b \rangle = -\frac{1}{2} \langle \Lambda_b | \tilde{O}_{V-A} | \Lambda_b \rangle$ [17]. This leaves us with two independent matrix elements of the operators $O_{V-A}$ and $\tilde{O}_{V-A}$. The analogue of the vacuum insertion approximation in the case of baryons is the valence-quark assumption, in which the colour of the quark fields in the operators is identified with the colour of the quarks inside the baryon. Since the colour wave-function for a baryon is totally antisymmetric, the matrix elements of $O_{V-A}$ and $\tilde{O}_{V-A}$ differ in this approximation only by a sign. Hence, we define a parameter $\tilde{B}$ by

$$
\langle \Lambda_b | \tilde{O}_{V-A} | \Lambda_b \rangle \equiv -\tilde{B} \langle \Lambda_b | O_{V-A} | \Lambda_b \rangle ,
$$

with $\tilde{B} = 1$ in the valence-quark approximation. For the baryon matrix element of $O_{V-A}$ itself, our parametrization is guided by the quark model. We write

$$
\frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | O_{V-A} | \Lambda_b \rangle \equiv -\frac{f_{B}m_{B}}{48} r ,
$$

where in the quark model $r$ is the ratio of the squares of the wave functions determining the probability to find a light quark at the location of the $b$ quark inside the $\Lambda_b$ baryon and the $B$ meson, i.e. $r = |\psi_{bq}^\Lambda(0)|^2 / |\psi_{bq}^B(0)|^2$ [21]. Assuming that the wave functions of the $\Lambda_b$ and $\Sigma_b$ baryons are the same, the ratio $r$ can be estimated from the ratio of the spin splittings between $\Sigma_b$ and $\Sigma_b^*$ baryons and $B$ and $B^*$ mesons [23]. This leads to

$$
r = \frac{4}{3} \frac{m_{\Sigma_b}^2 - m_{\Sigma_b^*}^2}{m_B^* - m_B^2} \approx 0.9 \pm 0.1 ,
$$

where we have taken the baryon mass-splitting to be $m_{\Sigma_b}^2 - m_{\Sigma_b^*}^2 \approx m_{\Sigma_b}^2 - m_{\Sigma_b^*}^2$.

\(^1\)Usually, the vacuum saturation approximation is applied at a typical hadronic scale $\mu_{\text{had}} \ll m_b$. The values of $B_i$ and $\varepsilon_i$ at the scale $m_b$ are then affected by renormalization effects. Taking, for instance, $\alpha_s(\mu_{\text{had}}) = 0.5$ (corresponding to $\mu_{\text{had}} \sim 0.75$ GeV), we find $B_1(m_b) = B_2(m_b) \approx 1.01$ and $\varepsilon_1(m_b) = \varepsilon_2(m_b) \approx -0.05$. 

8
3.1 Lifetime ratio $\tau(B^-)/\tau(B^0)$

Because of isospin symmetry, the lifetimes of the charged and neutral $B$ mesons are the same at order $1/m_b^2$ in the heavy-quark expansion, and differences arise only from spectator effects. The explicit calculation of these effects leads to a contribution to the decay width given by [17]

$$\Gamma_{\text{spec}}(B) = 16\pi^2 \frac{f_B^2 m_B}{m_b^3} \zeta_B \Gamma_0; \quad \Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2,$$

where $\zeta_B \simeq -0.4B_1 + 6.6\varepsilon_1$ and $\zeta_{B^0} \simeq -2.2\varepsilon_1 + 2.4\varepsilon_2$. Note the factor of $16\pi^2$, which arises from the phase-space enhancement of spectator effects. Since the parton-model result for the total decay width is $\Gamma(B)_{\text{tot}} \simeq 3.7\Gamma_0$, the characteristic scale of spectator contributions is $(2\pi f_B/m_B)^2 \simeq 5\%$. Thus, it is natural that the lifetimes of different beauty hadrons differ by a few per cent.

The precise value of the lifetime ratio crucially depends on the size of the hadronic matrix elements. Taking $f_B = 200$ MeV for the decay constant of the $B$ meson, i.e. absorbing the uncertainty in this parameter into the definition of $B_i$ and $\varepsilon_i$, leads to [17]

$$\frac{\tau(B^-)}{\tau(B^0)} \simeq 1 + 0.03B_1 - 0.7\varepsilon_1 + 0.2\varepsilon_2.$$

The most striking feature of this result is that the coefficients of the colour-octet operators $T_{V-A}$ and $T_{S-P}$ are orders of magnitude larger than those of the colour-singlet operator $O_{V-A}$. As a consequence, the vacuum insertion approximation, which was adopted in Ref. [8] to predict that $\tau(B^-)/\tau(B^0)$ is larger than unity by an amount of order $5\%$, cannot be trusted. With $\varepsilon_i$ of order $1/N_c$, it is conceivable that the non-factorizable contributions dominate the result, and without a detailed calculation of the parameters $\varepsilon_i$ no reliable prediction can be obtained. Given our present ignorance about the true values of the hadronic matrix elements, we must conclude that even the sign of the sum of the spectator contributions cannot be predicted. A lifetime ratio in the range $0.8 < \tau(B^-)/\tau(B^0) < 1.2$ could be easily accommodated by theory. In view of these considerations, the experimental fact that the lifetime ratio turns out to be very close to unity is somewhat of a surprise. It implies a constraint on a certain combination of the colour-octet matrix elements, which reads $\varepsilon_1 - 0.3\varepsilon_2 = \text{few \%}$. 

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3.2 Lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$

Understanding the low experimental value of the lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$ is one of the major problems in heavy-quark theory. We will now discuss the structure of spectator contributions to this ratio. It is important that the heavy-quark symmetry allows us to reduce the number of hadronic parameters contributing to the decay rate of the $\Lambda_b$ baryon from four to two, and that these parameters are almost certainly positive (unless the quark model is completely misleading) and enter the decay rate with the same sign. Thus, unlike the meson case, the structure of the spectator contributions to the width of the $\Lambda_b$ baryon is rather simple, and at least the sign of the effects can be predicted reliably.

It is useful to distinguish between the two cases where one does or does not allow spectator contributions to enhance the theoretical prediction for the semileptonic branching ratio of $B$ mesons. As we have shown in Section 2, the theoretical prediction for $B_{SL}$, which neglects spectator contributions, is slightly larger than the central experimental value. If spectator effects increased the prediction for $B_{SL}$ further, this discrepancy could become uncomfortably large.

If we do not allow for an increase in the value of the semileptonic branching ratio, the explanation of the low value of $\tau(\Lambda_b)/\tau(B^0)$ must reside entirely in a low value of the $\Lambda_b$ lifetime (rather than a large value of the $B$-meson lifetime). This can be seen by writing

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = \tau(\Lambda_b) \left( \frac{\tau(B^-)}{\tau(B^0)} \right)^{1/2} \frac{1}{\Gamma_{SL}(B)} = \frac{\tau(\Lambda_b)}{B_{SL}} \left( \frac{\tau(B^-)}{\tau(B^0)} \right)^{1/2} \Gamma_{SL}(B), \quad (15)$$

where $B_{SL}$ is the average semileptonic branching ratio of $B$ mesons, and $\Gamma_{SL}(B)$ is the semileptonic width. In the last step we have replaced the geometric mean $[\tau(B^-) \tau(B^0)]^{1/2}$ by the average $B$-meson lifetime, which because of isospin symmetry is correct to order $1/m_b^2$ in the heavy-quark expansion. Since there are no spectator contributions to the semileptonic rate $\Gamma_{SL}(B)$, and since we do not allow an enhancement of the semileptonic branching ratio, in order to obtain a small value for $\tau(\Lambda_b)/\tau(B^0)$ we can increase the width of the $\Lambda_b$ baryon and/or decrease (within the experimental errors) the lifetime ratio $\tau(B^-)/\tau(B^0)$. Allowing for a downward fluctuation of this ratio by two standard deviations, i.e. $\tau(B^-)/\tau(B^0) > 0.94$, and using the estimate of $1/m_b^2$ corrections in (6), we conclude that

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} > 0.97 \times \left( 0.98 - \frac{\Gamma_{spec}(\Lambda_b)}{\Gamma(\Lambda_b)} \right) = 0.95 - (d_1 + d_2 \tilde{B}) r, \quad (16)$$

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where $\Gamma_{\text{spec}}(\Lambda_b)$ is the spectator contribution to the width of the $\Lambda_b$ baryon. The values of the coefficients $d_i$ depend on the scale $\mu$, at which the Wilson coefficients $c_{\pm}(\mu)$ are renormalized.\footnote{For $\mu \neq m_b$ we take into account the evolution of the operators between the scales $\mu$ and $m_b$, so that the parameters defining the matrix elements of the four-quark operators are always renormalized at $m_b$.} For $\mu = m_b$ we find $d_1 = 0.013$ and $d_2 = 0.022$, whereas for $\mu = m_b/2$ we obtain the larger values $d_1 = 0.018$ and $d_2 = 0.024$. If we assume that $r$ and $\tilde{B}$ are of order unity, we find that the spectator contributions yield a reduction of the lifetime of the $\Lambda_b$ baryon by a few per cent, and that $\tau(\Lambda_b)/\tau(B^0) > 0.9$, in contrast with the experimental result. If we try to push the theoretical prediction by taking the large value $\tilde{B} = 1.5$ (corresponding to a violation of the valence-quark approximation by 50%) and choosing a low scale $\mu = m_b/2$, we have to require that $r > r_{\text{min}}$ with $r_{\text{min}} = 3.1, 2.2$ and $1.3$ for $\tau(\Lambda_b)/\tau(B^0) = 0.78, 0.83$ and $0.88$ (corresponding to the central experimental value and the $1\sigma$ and $2\sigma$ fluctuations). Hence, unless we allow for an upward fluctuation of the experimental result by two standard deviations, we need a value of $r$ that is significantly larger than the quark-model prediction in (12). A reliable field-theoretic calculation of the parameters $r$ and $\tilde{B}$ is of great importance to support or rule out such a possibility.

On the other hand, the low experimental value of the semileptonic branching ratio may find its explanation in a low renormalization scale (see Figs. 2 and 3), or it may be caused by the effects of New Physics, such as an enhanced rate for flavour-changing neutral currents of the type $b \rightarrow sg$. Hence, one may be misled in using the semileptonic branching ratio as a constraint on the size of spectator contributions. Then there is the possibility to decrease the value of $\tau(\Lambda_b)/\tau(B^0)$ by increasing the lifetime of the $B^0$ meson, i.e. in (16) we can allow for spectator contributions to the width of the $B^0$ meson. At first sight, this seems to make it possible (with a suitable choice of $\varepsilon_1$ and $\varepsilon_2$) to gain a contribution of about $-0.1$, which would take away much of the discrepancy between theory and experiment. However, the experimental result for the lifetime ratio $\tau(B^-)/\tau(B^0)$ imposes the powerful constraint $\varepsilon_1 \simeq 0.3\varepsilon_2$. Using this to eliminate $\varepsilon_1$ from the result, and allowing the parameters $B_i$ to take values between 0 and 2, we find

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} \simeq 0.98 \pm 0.02 + 0.15\varepsilon_2 - (d_1 + d_2\tilde{B}) r > 0.88 - (d_1 + d_2\tilde{B}) r,$$

where in the last step we have assumed that $|\varepsilon_2| < 0.5$, which we consider to be a conservative bound. Even in this case, a significant contribution must still come from the parameters $r$ and $\tilde{B}$.

In view of the above discussion, the short $\Lambda_b$ lifetime remains a potential problem for the heavy-quark theory. If the current experimental value persists, there are two possi-
bilities: either some hadronic matrix elements of four-quark operators are significantly larger than naive expectations based on large-$N_c$ counting rules and the quark model, or (local) quark–hadron duality, which is assumed in the calculation of lifetimes, fails in non-leptonic inclusive decays. In the second case, the explanation of the puzzle lies beyond the heavy-quark expansion. Let us, therefore, consider the first possibility and give a numerical example for some possible scenarios. Assume that $\mu = m_b/2$ is an appropriate scale to use in the evaluation of the Wilson coefficients, and that $\tilde{B} = 1.5$. Then, to obtain $\tau(\Lambda_b)/\tau(B^0) = 0.8$ without enhancing the prediction for the semileptonic branching ratio requires $r \simeq 3$, i.e. three times larger than the quark-model estimate in (12). If, on the other hand, we consider $r = 1.5$ as the largest conceivable value, we need $\varepsilon_2 \simeq -0.5$, corresponding to a rather large matrix element of the colour-octet operator $T_{S-P}$. Such a value of $\varepsilon_2$ leads to an enhancement of the $B$-meson lifetime, and hence to an enhancement of the semileptonic branching ratio of $B$ mesons, by $\Delta B_{SL} \simeq 1\%$ [17]. As shown in Figs. 2 and 3, this is still tolerable provided yet unknown higher-order corrections confirm the use of a low renormalization scale. Although in both cases some large parameters are needed, we find it important to note that until reliable field-theoretic calculations of the matrix elements of four-quark operators become available, a conventional explanation of the $\Lambda_b$-lifetime puzzle cannot be excluded.

4 Conclusions

The heavy-quark expansion, supplemented by the assumption of quark–hadron duality, provides the theoretical framework for a systematic calculation of inclusive decay rates of hadrons containing a heavy quark. Whereas this formalism works well for the description of the total decay rate and the lepton and neutrino spectra in semileptonic decays, two potential problems related to non-leptonic decays have become apparent in recent years: the low experimental value of the semileptonic branching ratio of $B$ mesons, and the low value of the lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$.

We have shown that the semileptonic branching ratio can be explained if QCD radiative corrections are properly taken into account. The exact formulae at order $\alpha_s$ are known since last year, and only very recently have correct numerical analyses of these formulae been presented. As the situation is now, the experimental results for the semileptonic branching ratio and for the charm-counting rate obtained in high-energy measurements are in perfect agreement with theory (see Fig. 3), whereas the results of low-energy measurements can be explained at the $1\sigma$ level by using a low renormalization scale.
In order to obtain a detailed understanding of beauty lifetimes, it is necessary to go to order $1/m_b^3$ in the heavy-quark expansion, at which the matrix elements of four-quark operators appear. They describe the physics of spectator effects, i.e. contributions in which a light quark in a beauty hadron is actively involved in the weak interaction. We have presented a model-independent study of such contributions, introducing a minimal set of hadronic parameters, which eventually may be determined using some field-theoretic approach such as lattice gauge theory. We find that in $B$-meson decays the coefficients of the colour-octet non-factorizable operators are much larger than those of the colour-singlet factorizable operators, and therefore the contributions from the non-factorizable operators cannot be neglected. The theoretical prediction for the ratio $\tau(B^-)/\tau(B^0)$ is in agreement with experiment; however, our present ignorance about the matrix elements of four-quark operators does not allow us to calculate this ratio with an accuracy of better than about 20%. The short $\Lambda_b$ lifetime, on the other hand, remains a potential problem for the heavy-quark theory. If the current experimental value persists, either some hadronic matrix elements of four-quark operators must be significantly larger than naive expectations, or (local) quark–hadron duality fails in non-leptonic inclusive decays. We stress that at present the first possibility is not yet ruled out, although it requires large values of at least some hadronic matrix elements in the baryon and/or meson sector. In the second case, the explanation of the puzzle of the $\Lambda_b$ lifetime lies beyond our present capabilities, as there is no known way to estimate duality violations in a quantitative way.

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