Non-Abelian Antisymmetric-Vector Coupling from Self-Interaction

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Abstract

A non-abelian coupling between antisymmetric fields and Yang-Mills fields proposed by Freedman and Townsend several years ago is derived using the self-interaction mechanism.

1 INTRODUCTION

Abelian second-rank antisymmetric fields [1] play an essential role in strings and supergravity theories and have been extensively studied in the last decades [2] [3] [4] [5]. In free theories they describe massless and spinless particles and appear in many contexts, for instance, arising as mediators of the interaction between open strings with charged particles [2] and in ten dimensions, coupling with the Chern-Simons 3-form to achieve an elegant unification of Yang-Mills and supergravity [6]. In particular the Cremmer-Sherk theory [3] has received considerable attention [7] [8] due to the fact that the coupling between the abelian antisymmetric field and a Maxwellian field through a topological $BF$ term leads to massive propagations which are compatible with gauge invariances. Moreover, Allen, et. al. [7] have shown unitarity and renormalizability of the Cremmer-Sherk theory. This fact motivates the non-abelian generalization of the model and several attempts have been proposed [9]. Simultaneously, other alternatives for non-abelian massive vector bosons without the presence of Higgs field have been proposed in the last year [10].

The non-abelian extension of antisymmetric theories was achieved by Freedman and Townsend [4] starting from a first-order formulation where

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the antisymmetric field $B^a_{mn}$ and an auxiliary vector potential are independent variables. It is worth recalling that the non-abelian generalization of the abelian S-duality theory [11] is a Freedman-Townsend theory [12]. In their work, Freedman and Townsend proposed the non-abelian generalization of the Cremmer-Sherk theory. In this letter, starting from an appropriate first-order formulation for the Cremmer-Sherk theory, we will derive the non-abelian generalization using the self-interaction mechanism [13], which has been successfully applied to formulate Yang-Mills, gravity [13], supergravity [14], topologically massive Yang-Mills [15] and Chapline-Manton [16] theories.

2 THE ABELIAN MODEL

Our starting point will be a first-order formulation for the Cremmer-Sherk theory. This is realized introducing an auxiliary vector field $(v_m)$ a la Freedman-Townsend. The action is written down as [17]

$$I = -\frac{1}{4} \mu \epsilon^{mnpq} B_{mn} [\partial_p v_q - \partial_q v_p] - \frac{1}{2} \mu^2 v^m v_m - \frac{1}{2} \mu \epsilon^{mnpq} B_{mn} \partial_p A_q$$

$$+ \frac{1}{4} F_{mn} F^{mn} - \frac{1}{2} F_{mn} [\partial_m A_n - \partial_n A_m] >$$

where $<>$ denotes integration in four dimensions. All the fields involved have mass dimensions and $\mu$ is a mass parameter. There are two sets of abelian gauge invariances:

$$\delta_\lambda A_m = \partial_m \lambda, \quad \delta_\lambda F_{mn} = 0$$

$$\delta_\zeta B_{mn} = \partial_m \zeta_n - \partial_n \zeta_m, \quad \delta_\zeta v_m = 0.$$  

Independent variations in $v_m, B_{mn}, F_{mn}$ and $A_m$ lead to the following equations of motion

$$v^m = -\frac{1}{6\mu} \epsilon^{mnpq} H_{npq},$$

$$\epsilon^{mnpq} \partial_p [v_q + A_q] = 0,$$

$$F_{mn} = \partial_m A_n - \partial_n A_m,$$

$$\partial_p F^{pm} = \frac{1}{6\mu} \epsilon^{mnpq} H_{npq}.$$
where $H_{mnp} \equiv \partial_m B_{np} + \partial_n B_{pm} + \partial_p B_{mn}$ is the field strength associated with the antisymmetric field. The Cremmer-Scherk action is obtained after substituting equations (4) and (6) in (1):

$$I_{CrSc} = -\frac{1}{4} F_{mn[A]} F_{mn[A]} - \frac{1}{12} H_{mnp[B]} H_{mnp[B]} - \frac{1}{4} \mu \epsilon_{mnpq} B_{mn} F_{pq[A]}.$$

(8)

On the other hand, equation (5) can be solved (locally) for the $v$ field,

$$v_m = -[A_m + \frac{1}{\mu} \partial_m \phi],$$

(9)

where $\phi$ is a scalar field. Substituting this solution in the action $I$, the Stuckelberg formulation for massive abelian vector bosons is obtained

$$I_{St} = -\frac{1}{4} F_{mn[A]} F_{mn[A]} - \frac{1}{2} \mu^2 [A_m + \frac{1}{\mu} \partial_m \phi][A^m + \frac{1}{\mu} \partial^m \phi].$$

(10)

As it is well known, both formulations (Stuckelberg and Cremmer-Scherk) are equivalent descriptions of massive abelian gauge invariant vectorial theories and propagate three degrees of freedom. This equivalence is reflected by the fact that they are connected by duality [18]. Indeed, since the scalar field appears in equation (10) only through its derivative, we can apply the dualization method due to Nicolai and Townsend [19], which consist in replacing $\partial_m \phi$ by $\frac{1}{2} l_m$ and adding a new term to equation (10): $\epsilon B \partial l$, i.e.

$$I_{Stmod} = -\frac{1}{4} F_{mn[A]} F_{mn[A]} - \frac{1}{2} \mu^2 [A_m + \frac{1}{2 \mu} l_m][A^m + \frac{1}{2 \mu} l_m] + \frac{1}{4} \epsilon_{mnpq} B_{mn} \partial_p l_q.$$  

(11)

At this stage, $B_{mn}$ is a Lagrange multiplier forcing the constraint $\partial_m l_n - \partial_n l_m = 0$ whose local solution is $l_m = 2 \partial_m \phi$. Now, if we eliminate $l_m$ via its equation of motion

$$l^m = \frac{1}{3} \epsilon_{mnpq} H_{npq} - 2 \mu A^m$$

(12)

and go back to equation (11), the Cremmer-Scherk action is recovered.

Finally, let us recall that the second-order field equations can be written as

$$\partial_p F^{pm} = J^m, \quad \partial_p H^{pmn} = J^{mn},$$

(13)

where

$$J^m = \frac{1}{6} \mu \epsilon_{mnpq} H_{npq} \quad \text{and} \quad J^{mn} = \frac{1}{2} \mu \epsilon_{mnpq} F_{pq}$$

(14)

are "topological" currents in the sense that they are conserved without using the equations of motion.
3 THE SELF-INTERACTION PROCESS

Now, we extend the first-order action, equation (1), by introducing a triplet of free abelian antisymmetric fields $B_{mn}^a$ coupled with a triplet of free abelian vector fields $A_m^a$, ($a = 1, 2, 3$)

$$
I_o = -\frac{1}{4} \mu \epsilon^{mpq} B_{mn}^a [\partial_m v_n^a - \partial_n v_m^a] - \frac{1}{2} \mu^2 v^a_m v^a_m - \frac{1}{2} \mu \epsilon^{mpq} B_{mn}^a \partial_p A_q^a \tag{15}
$$

$$
+ \frac{1}{4} F_{mn}^{a} F^{amn} - \frac{1}{2} F^{amn} [\partial_m A_n^a - \partial_n A_m^a] >
$$

Besides the local gauge transformations

$$
\delta \lambda A_m^a = \partial_m \lambda^a, \quad \delta \lambda F_{mn}^a = 0 \tag{16}
$$

$$
\delta \zeta B_{mn}^a = \partial_m \zeta_n^a - \partial_n \zeta_m^a, \quad \delta \zeta v_m^a = 0, \tag{17}
$$

our action has two global invariances: one is a global $SU(2)$ rotation and the other is a a global symmetry associated with the Freedman-Townsend theory:

$$(I) \quad \delta \omega X^a = g_1 \epsilon^{abc} X^b \omega^c \tag{18}$$

$$(II) \quad \delta \rho B_{mn}^a = g_2 \epsilon^{abc} [v_m^b + A_m^b \rho_n^c - m \leftrightarrow n], \tag{19}$$

and

$$
\delta \rho v_m^a = \delta \rho A_m^a = \delta \rho F_{mn}^a = 0,
$$

$\omega$ and $\rho$ being global parameters. In principle the coupling constants $g_1$ and $g_2$ are different. We note that under type II transformations the action changes by a total derivative. The Noether currents associated to these invariances are given by

$$
g_1^{-1} j_{am} = \epsilon^{abc} F^{bmn} A_n^c + \frac{1}{2} \mu \epsilon^{mpq} \epsilon^{abc} B_{pq}^b [A_n^c + v_n^c] \tag{20}
$$

and

$$
g_2^{-1} K^{amn} = \frac{1}{2} \mu \epsilon^{mpq} \epsilon^{abc} [A_p^b + v_p^b] [A_q^c + v_q^c]. \tag{21}
$$

These are conserved on-shell. In order to couple these currents to the action $I_o$ we must add the corresponding self-interaction terms: $I_1$ and $I_2$ defined by:

$$
j_{am} \equiv \frac{\delta I_1}{\delta A_m^a}; \quad K^{amn} \equiv -2 \frac{\delta I_2}{\delta B_{mn}^a}. \tag{22}
$$
These functional differential equations can easily be integrated. In fact, we find that

\[
I_1 = -g_1 < \frac{1}{2} \epsilon^{abc} F_{amn} A^b_m A^c_n + \frac{1}{4} \mu \epsilon^{mnps} \epsilon^{abc} B_{mn}^a A^b_p A^c_q >
\]

and

\[
I_2 = -g_2 < \frac{1}{4} \epsilon^{mnps} \epsilon^{abc} B_{mn}^a v^b_p v^c_q + \frac{1}{4} \mu \epsilon^{mnps} \epsilon^{abc} B_{mn}^a A^b_p A^c_q >
\]

However, these two terms have overlapping parts. This situation is akin to what happens in the derivation of supergravity from self-interaction [14]. In order to overcome this obstacle we must require equality of the coupling constants: \( g \equiv g_1 = g_2 \) and write down the self-interaction action as

\[
I_{SI} \equiv -g < \frac{1}{2} \epsilon^{abc} F_{amn} A^b_m A^c_n + \frac{1}{4} \epsilon^{mnps} \epsilon^{abc} B_{mn}^a v^b_p v^c_q >
\]

Actually, we have that

\[
j_{am} \equiv \frac{\delta I_{SI}}{\delta A^a_m} \quad \text{and} \quad K^{amn} \equiv -2 \frac{\delta I_{SI}}{\delta B^a_{mn}}.
\]

The self-interaction mechanism stops here since no other derivative terms appear in \( I_{SI} \). Finally, the full non-abelian theory is

\[
I = I_o + I_{SI}
\]

\[
= < -\frac{1}{4} \mu \epsilon^{mnps} B_{mn}^a \left[ F_{pq}^a + f_{pq}^a + 2 \epsilon^{abc} A^b_p A^c_q \right] - \frac{1}{2} \mu^2 v^a_m v^a_m - \frac{1}{4} F_{mn}^a F^{amn} >,
\]

where

\[
F_{mn}^a \equiv \partial_m A^a_n - \partial_n A^a_m + g \epsilon^{abc} A^b_m A^c_n
\]
and
\[ f^a_{mn} \equiv \partial_m v^a_n - \partial_n v^a_m + g \epsilon^{abc} v^b_m v^c_n \] (29)
which is just that proposed by Freedman and Townsend (equation (2.15) in their paper). As usual, the self-interaction process combines the abelian gauge transformations with the global ones giving rise to non-abelian local gauge transformations. In our case, we have
\[ \delta_\alpha A^a_m = \partial_m \alpha^a + g \epsilon^{abc} A^b_m \alpha^c \] (30)
\[ \delta_\alpha B^a_{mn} = g \epsilon^{abc} B^b_{mn} \alpha^c \] (31)
\[ \delta_\alpha v^a_m = g \epsilon^{abc} v^b_m \alpha^c \]
and
\[ \delta_\xi B^a_{mn} = \partial_m \xi^a + g \epsilon^{abc} [A^b_m + v^b_m] \xi^c - m \leftrightarrow n \] (32)
\[ \delta_\xi A^a_m = 0 = \delta_\xi v^a_m. \]

The action of Freedman-Townsend, equation (27), is equivalent to massive Yang-Mills (locally) as can be shown after elimination of \( B^a_{mn} \) through its equation of motion, which said us that \( A_m + v_m \) is a pure gauge.

4 CONCLUSION

In this letter, by starting with a nice abelian first-order formulation, and through the application of the self-interaction mechanism we have obtained the Freedman-Townsend theory and its corresponding gauge transformation rules through self-interaction. The first order abelian formulation allowed us to find Cremmer-Scherk and Stuckelberg formulations for massive spin-1 theories, these later formulations are connected by duality. The BRST quantization of the massive Freedman-Townsend has been performed by Thierry-Meig [20]. Since massive Freedman-Townsend theory is equivalent (in topologically trivial manifolds) to massive Yang Mills it should be interesting to attempt to connect Friedman-Townsend with others approaches dealing with massive gauge bosons without the presence of Higgs field [10].
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6 REFERENCES

References


[17] We use the metric $\eta_{mn} = diag(-1, +1, +1, +1)$.

