grc4f v1.1: a Four-fermion Event Generator for $e^+ e^-$ Collisions

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Abstract

grc4f is a Monte-Carlo package for generating $e^+ e^- \rightarrow 4$-fermion processes in the standard model. All of the 76 LEP-2 allowed fermionic final state processes evaluated at tree level are included in version 1.1. grc4f addresses event simulation requirements at $e^+ e^-$ colliders such as LEP and up-coming linear colliders. Most of the attractive aspects of grc4f come from its link to the GRACE system: a Feynman diagram automatic computation system. The GRACE system has been used to produce the computational code for all final states, giving a higher level of confidence in the calculation correctness. Based on the helicity amplitude calculation technique, all fermion masses can be kept finite and helicity information can be propagated down to the final state particles. The phase space integration of the matrix element gives the total and differential cross sections, then unweighted events are generated. Initial state radiation (ISR) corrections are implemented in two ways, one is based on the electron structure function formalism and the second uses the parton shower algorithm called QEDPS. The latter can also be applied for final state radiation (FSR) though the interference with the ISR is not yet taken into account. Parton shower and hadronization of the final quarks are performed through an interface to JETSET. Coulomb correction between two intermediate $W$'s, anomalous coupling as well as gluon contributions in the hadronic processes are also included.
1. Introduction

The main purpose of the LEP-2 experiments is to measure the properties of the W-boson with a high level of precision by its direct pair production. At the $Z^0$ peak (LEP-1 energy), two-fermion final states produced by the $Z^0$ decay are, by far, the most important processes. At higher energy, charged W-boson being essentially created in pair at $e^+e^-$ colliders, the cross-section is dominated by four-fermion processes. However, other production mechanisms may also lead to four-fermion final states like $Z^0$ pair or two-photon like production. In addition, some four-fermion processes contribute heavily to the background for new particle searches due to their large missing energy. In this paper, a four-fermion event generator, grc4f, is presented. It provides a convenient way of computing cross-sections under the complex cuts, acceptance and resolution of the collider experiments.

It is based on the GRACE system which generates automatically the matrix element in terms of helicity amplitudes (supplied by the CHANEL library) for any process once the initial and the final states have been specified. At present, the system has been completed only at tree level in the framework of the standard model for electroweak and strong interactions. Fermion masses are non-zero and helicity information can be traced down to the final state particles. In addition, a kinematics library has been developed to cover the requirements of each process topology (weeding out multidimensional singularities) to get a faster convergence of the Monte-Carlo integration over the phase space. The grc4f package is actually a collection of 76 $e^+e^- \rightarrow 4$-fermion processes consisting of 16 hadronic, 36 semi-hadronic and 24 leptonic processes, presented in a coherent and uniform environment. Flavor mixing is set to zero. The coupling

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1 In version v1.1, final states with top quarks or those obtained by complex conjugation are not explicitly included. For generating $t$-quark final state processes, the user may try to change the $c$-quark mass to the top mass together with an interchange $s \leftrightarrow b$, but the integration convergence is not warranted because $t \rightarrow bW$ is a real process contrary to $c \rightarrow sW$. 

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between electron and Higgs boson is suppressed. Once a process has been selected, the total and differential cross sections are computed with the Monte-Carlo integration package BASES \( ^{\text{BASES}} \). Then SPRING \( ^{\text{SPRING}} \), a general purpose event generator, provides unweighted events.

The matrix element generated by GRACE corresponds to the genuine tree level process. However, several corrections must be introduced to produce realistic cross sections. First of all the most basic correction, inherent to heavy boson production, is due to the finite width of the \( W \) or \( Z^0 \) boson to ensure a finite cross section. It should be noted that if one introduces the width in a naive way, it violates the gauge invariance and in some case may lead to a divergent cross-section. The \( e^+ e^- \rightarrow e^- \bar{v}, q \bar{q} \) cross section, for example, blows up at \( \theta_e \approx 0 \). To cure this problem, the terms which satisfy the Ward identity are properly subtracted from the electron current which is connected to the \( t \)-channel photon \( ^{\text{GRAC}} \). The computation of the constant and the running decay widths of \( W \) and \( Z^0 \) is presented in the section 2.1.

Two additional effects, although of different origins, can be, in first approximation, introduced into any selected process: radiative corrections and hadronization of the final quarks.

For the radiative corrections two techniques are proposed in the program. The first one uses the well-known analytic form of the \( e^+ e^- \) structure function \( ^{\text{QEDPS}} \), a radiative correction generator producing an indefinite numbers of photons according to the parton shower algorithm in the leading-logarithmic (LL) approximation. Originally this algorithm has been developed to simulate QCD parton shower. One important point here is that QEDPS reproduces the radiative photon transverse momentum distributions. The details of these two methods shall be given in section 2.2. Final state radiation (FSR) for the electron and muon can also be generated with QEDPS. However the contribution of the interference between ISR and FSR is not yet included. Hence one should not use the FSR mode when the final state contains electron for kinematical regions where the \( t \)-channel photon exchange is the dominant diagram. The FSR for the quarks is not allowed in grc4f, because the QCD evolution scale is much shorter than the QED one, therefore the photonic correction on quarks might be meaningless. We recommend to use the FSR mode only for the case where the \( W \)-pair or \( Z \)-pair (\( Z \gamma \)) production diagrams are dominant.

We assume that the hadronization of partons can be separated from the hard interaction studied here. Under this assumption, the calculation of cross sections is exact in grc4f. Final state parton hadronization is performed, in grc4f, through the LUND mechanism as implemented in JETSET \( ^{\text{JETSET}} \). A set of color bases is defined to select final state color flow. Possible ambiguities in this procedure as well as other QCD related issues are discussed in details in section 2.3.

In order not to waste cpu time and disk space in the generation of events that will be eventually rejected by detector acceptance or threshold, a set of general cuts whose values can be set by the user have been implemented directly in the kinematics as presented in section 2.4.

The Coulomb correction \( ^{\text{grc4f}} \) due to the exchange of a virtual photon between two intermediate \( W \) is important for non-relativistic \( W \), namely close to the production threshold. It may reach 4% of the total cross-section. The introduction of this effect is described in section 2.5.

Anomalous coupling in the interaction among vector bosons is introduced in section 2.6.

The basic input parameters used in the program are listed hereafter. Fermion masses are taken from the report of Particle Data Group \( ^{\text{PDG}} \), but can be set to zero if necessary.

\[
\begin{align*}
M_W &= 80.23 \text{ GeV}, \\
M_Z &= 91.1888 \text{ GeV}, \\
\Gamma_W &= 3G_F M_W^3/(\sqrt{8\pi}), \\
G_F &= 1.16639 \cdot 10^{-5} \text{ GeV}^{-2},
\end{align*}
\]
Here $\alpha(Q)$ is the QED fine structure constant at the energy scale $Q$ and $\alpha_S$ is the QCD coupling. These values can be changed through control data. The weak mixing angle is calculated by $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$.

The matrix elements are given in term of helicity amplitudes, it is therefore possible to select any helicity state configuration (initial and final).

This paper is organized as follows. Theoretical details for the problems mentioned in the introduction will be given in section 2. The structure of the program is described in section 3. In section 4, all details about running the program are presented. Some improvement for future releases are summarized in the section 5. Three appendices present the parameters and options which can be changed by the user, the list of all processes and the program installation procedure followed by a run example.

2. Theoretical aspects

This section covers the treatment of the boson width, the introduction of the initial and final state radiation, the QCD related issues such as hadronization and the color problems, the description of kinematics and cuts, the Coulomb correction and the anomalous coupling of heavy vector bosons.

2.1 Gauge boson width

The $O(\alpha)$ self-energy of the gauge boson, $\Sigma^{(1)}(k^2)$, generally satisfies

\[
\begin{align*}
\text{Re}\Sigma^{(1)}(M^2) &= 0, \\
\text{Im}\Sigma^{(1)}(M^2) &= -M\Gamma(M^2),
\end{align*}
\]

where $M$ is the mass of the gauge boson and $\Gamma(M^2)$ is the lowest order decay width. This induces the replacement of the gauge boson propagator by the following simple Breit-Wigner form:

\[
\frac{1}{k^2 - M^2} \to \frac{1}{k^2 - M^2 + iM\Gamma(M^2)}
\]

This form is used for the fixed width scheme in grc4f whenever the propagator appears with positive momentum squared.

Another way is to take into account the energy dependence of the self-energy. Based on the observation that the contribution of fermion pairs to $\text{Im}\Sigma^{(1)}$ is $k^2$ times a constant in any gauge as long as all the light fermion masses are neglected, the following energy dependent width has been proposed $[1,2,3]$.

\[
\frac{1}{k^2 - M^2} \to -\text{Im}\Sigma^{(1)}(k^2)|_{\text{fermion pairs}} \approx \frac{k^2}{M}\Gamma(M^2).
\]

In grc4f this form is used as the default, but the constant width can be selected as well.

As mentioned in the previous section, the introduction of the boson width may raise difficulties, particularly when the electron is scattered in the very forward direction. Such final
states usually come from the generation of the so-called single $W$ or $Z$ process. To get rid of the divergent cross section, we apply the following method.

With an arbitrary gauge parameter $\xi$ the electron current can be written as:

$$ I_\mu = \bar{u}(p') \gamma^\mu u(p)[g_{\mu\nu} + (\xi - 1)k_\mu k_\nu/k^2], \quad \xi = \bar{u}(p')\gamma_\mu u(p), $$

where $p_\mu(p'_\mu)$ is the four-momentum of the initial (final) electron and $k_\mu = p_\mu - p'_\mu$, the momentum of the virtual photon. After squaring the amplitude and averaging over spin states, the matrix element can be written as

$$ M = I_{\mu\nu}T^{\mu\nu}, $$

with

$$ I_{\mu\nu} = \sum_{\text{spin}} I_{\mu\nu}^S = 2 \left[ p_\mu p'_\nu + p_\nu p'_\mu + \frac{k^2}{2} g_{\mu\nu} \right]. $$

Let’s assume that $T_{\mu\nu}$ is gauge invariant. Then one can replace $I_{\mu\nu}$ by

$$ I_{\mu\nu} \rightarrow I'_{\mu\nu} = 4p_\mu p_\nu + k^2 g_{\mu\nu}. $$

In this form the first term is responsible for the blow-up of the cross section. Thanks to the gauge invariance, one can further replace the vector $p_\mu$ by:

$$ p_\mu \rightarrow P_\mu = p_\mu - (p_0/k_0)k_\mu. $$

By substituting $P_\mu$ in Eq. (5) and dropping $k_\mu$, one gets the final form

$$ I'_{\mu\nu} \rightarrow I''_{\mu\nu} = 4P_\mu P_\nu + k^2 g_{\mu\nu}. $$

It is known that a product of $P$ with an arbitrary vector $A$, $P \cdot A$, can be expressed by a sum of terms proportional to either $m_0^2$, $1 - \cos \theta_c$, or $\sin \theta_c$. The part $T_{\mu\nu}$ is expressed in terms of some momenta which are to be contracted with $P_\mu$ or $P_\nu$. Hence in the region $\theta_c \approx 0$, any product behaves effectively like $k^2$, because $1 - \cos \theta_c$ vanishes almost like $k^2$. If the current $I''_{\mu\nu}$ is used instead of the original one $I_{\mu\nu}$, the cross-section remains finite down to $\theta_c = 0$.

### 2.2 Radiative corrections

In the first approach, the simple electron structure function is used. The electron structure function at $O(\alpha^2)$ [2] which is to be convoluted to the cross section for a primary process is given by

$$ D(x, s) = \left[ 1 + \frac{3}{8} \beta + \left( \frac{9}{128} - \frac{\zeta(2)}{8} \right) \beta^2 \right] \frac{\beta}{2} (1 - x)^{\beta/2 - 1} $$

$$ - \frac{\beta}{4} (1 + x) - \frac{\beta^2}{32} \left[ 4(1 + x) \ln(1 - x) + \frac{1 + 3x^2}{1 - x} \ln x + (5 + x) \right], $$

$$ \beta = \frac{2\alpha}{\pi}((\ln(s/m^2_e) - 1), $$

where $s$ is the square of the total energy of the system, $x$ is the momentum fraction of the electron, and $\zeta(n)$ is Riemann’s $\zeta$-function. It should be noted that, when compared with the
exact $O(\alpha)$ calculation, the radiative corrected cross section obtained by this function does not contain the overall multiplying $K$-factor:

$$1 + \frac{\alpha}{\pi} \left( \frac{\pi^2}{3} - \frac{1}{2} \right) = 1.006480 \ldots.$$  

(10)

In $\text{gfc4f}$ this factor is missing both for the structure function mode and $\text{QEDPS}$ mode. If necessary, the final result can be multiplied by this factor for better accuracy.

The QED Parton Shower approach, $\text{QEDPS}$, is primarily based on the fact that $D(x,Q^2)$ obeys the Altarelli-Parisi equation:

$$\frac{dD(x,Q^2)}{d\ln Q^2} = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} P_+(x/y) D(y,Q^2),$$  

(11)

in the leading-log(LL) approximation $[11]$. This is equivalent to the following integral equation:

$$D(x,Q^2) = \Pi(Q^2,Q_x^2) D(x,Q_x^2) + \frac{\alpha}{2\pi} \int_{Q_x^2}^{Q^2} \frac{dK^2}{K^2} \Pi(Q^2, K^2) \int_x^1 \frac{dy}{y} P(y) D(x/y, K^2),$$  

(12)

where the small quantity $\epsilon$ will be specified later. In these equations $P(x)$ is the split function noted $P_+(x)$ when regularized at $x = 1$. $Q_x^2$ is the initial value of $Q^2$. For simplicity the fine structure constant $\alpha$ is assumed not running with $Q^2$. The Sudakov factor $\Pi$ is given by:

$$\Pi(Q^2,Q_x^2) = \exp \left( -\frac{\alpha}{2\pi} \int_{Q_x^2}^{Q^2} \frac{dK^2}{K^2} \int_0^{1-\epsilon} \frac{dy}{y} P(x) \right).$$  

(13)

This is the probability that an electron evolves from $Q_x^2$ to $Q^2$ without emitting hard photon. In other words, $\Pi$ already contains the soft photon component, which causes the change in the electron virtuality, and the loop correction contribution at all orders of perturbation.

The integral form Eq. (12) can be solved by iteration. It is clear that the emission of $n$ photons corresponds to $n$ iterations. Hence it is possible to regard the process as a stochastic mechanism suggesting the following shower algorithm $[12]$.

(a) Set $x_b = 1$. The variable $x_b$ is the fraction of the light-cone momentum of the virtual electron that annihilates.

(b) Choose a random number $\eta$. If it is smaller than $\Pi(Q^2,Q_x^2)$, then the evolution stops. If not, one can find the virtuality $K^2$ that satisfies $\eta = \Pi(K^2,Q_x^2)$ at which a branching takes place.

(c) Fix $x$ according to the probability $P(x)$ between 0 and $1 - \epsilon$. Then $x_b$ is replaced by $x_b x$.

Go back to (b) by substituting $K^2$ into $Q_x^2$ and repeat until the evolution stops.

Once a radiative process is fixed by this algorithm, each branching of a photon is a real process, that is, an electron with $x, K^2$ decays like:

$$\epsilon^- (x, -K^2) \rightarrow \epsilon^- (xy, -K'^2) + \gamma (x(1 - y), Q_0^2).$$  

(14)

Here we have introduced a cutoff $Q_0^2$ to avoid the infrared divergence. The momentum conservation at the branching gives: $-K^2 = -K'^2/y + Q_0^2/(1 - y) + k_T^2/(y(1 - y))$ which in turn
determines the photon transverse momentum relative to the parent, \( k_T^2 \), from \( y, K^2, K'^2 \). This technique gives the \( k_T^2 \) distribution as well as the shape of \( x \).

The kinematical boundary \( y(K^2 + Q^2_0/(1 - y)) \leq K'^2 \), equivalent to \( k_T^2 > 0 \), fixes \( \epsilon \) as \( \epsilon = Q^2_0/K'^2 \) since \( K^2 \ll K'^2 \) is expected. In ref.\( [3] \) the important role played by this \( \epsilon \) is discussed in more details.

The above description of the algorithm concerns the case where either \( e^- \) or \( e^+ \) radiates photons when the axial gauge vector is chosen along the momentum of the other electron, namely \( e^+ \) or \( e^- \). In the program, however, we use the double cascade scheme to ensure the symmetry of the radiation between \( e^+ \) and \( e^- \). These two are mathematically equivalent in the LL approximation.

The two parameters \( Q^2_s \) and \( Q^2_0 \) are given as follows:

\[
Q^2_s = m^2 Z = m^2 \times 2.71828 \ldots, \quad Q^2_0 = 10^{-12} \text{ GeV}^2.
\] (15)

\( Q^2_s \) is defined so as to include the constant term \( -1 \) of \( \beta \) in such a way that \( \beta = (2\alpha/\pi)(\ln(s/m^2 e) - 1) = (2\alpha/\pi)\ln(s/(m^2 e)) \). Since \( Q^2_0 \) is unphysical, any observable should not depend on it. It has been checked that increasing \( Q^2_0 \) up to \( O(m^2/10) \) leaves the result unchanged within the statistical error of the event generation [6].

This scheme can be applied for the radiation from the final state charged particles as well. The lower bound of the virtuality integration, \( Q^2_s \), is now the mass square of the final particle instead of the initial electron mass. The upper bound is the four-momentum squared of the lepton pair. Here we assume that the lepton-pairs are created from the gauge boson. The FSR should be used for the processes in which the \( W \)-pair or \( Z \)-pair (\( Z \gamma \)) production diagrams are dominating, but not for the processes such as multi-peripheral two-photon like diagrams.

### 2.3 QCD related issues

#### 2.3.1 Color bases

We calculate the matrix element using {\textsc{grace}} which may be written as

\[
M = | \sum T_j |^2.
\] (16)

Each amplitude \( T_j \) includes a color factor. The color indices of the external particle must be summed in the final state and averaged in the initial state.

If a diagram has four gluon vertices, the diagram is separated into three pieces, the so-called \( s \)-, \( t \)- and \( u \)-channels, respectively, for each vertex. Let’s now consider the \( T_j \)’s not as amplitude but as a component of an amplitude after this decomposition.

For each \( T_j \), the color factor can be factorized out and expanded on a set of color bases, \( C_k \):

\[
T_j = \left( \sum_k w_j^{(k)} C_k \right) \hat{T}_j
\] (17)

where \( w_j^{(k)} \)’s are numbers. Then the matrix element is given by

\[
M = \sum_{k,k'} \left( C_k C^\dagger_{k'} \right) F_k F_{k'}^*, \quad F_k = \sum_j w_j^{(k)} \hat{T}_j
\] (18)

Therefore, for the four-gluon vertices, the colored part of an amplitude consists of three-gluon vertices (\( -i \delta_{abc} \)), quark-gluon vertices (\( t^a \)) and non-colored vertices. The ghost-gluon
vertex bears the same color structure as the three-gluon vertex and do not require a specific treatment.

The choice of the color bases and the technique used to introduce color flow at the event generation level are not all equivalent. Let’s describe one of the simplest approach. The following algorithm has been introduced in GRACE.

1. We consider a process in which \( n_q \) quark pairs and \( n_g \) gluons exist among external particles. For any pair of amplitudes, \( T_j \) and \( T_k \), the color index of an external gluon is contracted between them. Making use of this, each gluon is converted into a pair of quark and antiquark creating a quark-gluon vertex. An overall factor \( 2^{n_g} \) is assigned to \( M \) from:

\[
\delta^{ab} = \text{Tr} t^a t^b / T_R, \quad T_R = \frac{1}{2}.
\] (19)

2. External particles are \( n \) pairs of quarks and antiquarks where \( n = n_q + n_g \). Quarks and antiquarks are denoted as \( 1, 2, 3, \ldots, n \) and \( \overline{1}, \overline{2}, \overline{3}, \ldots, \overline{n} \), respectively. The color base here has the form:

\[
C_k = \delta_{j_1j_2} \delta_{j_2j_3} \cdots \delta_{j_n}\overline{j}_n
\]

where \( j_1, j_2, \ldots, j_n \) is a permutation of \( 1, 2, 3, \ldots, n \). The number of color bases is hence \( n! \).

3. Each three-gluon vertex is converted into a pair of quark loops by

\[
- i f_{abc} = (\text{Tr} t^a t^b t^c + \text{Tr} t^b t^c t^a) / T_R
\] (21)

4. Each gluon propagator is replaced by quark lines by use of the Fiertz transformation:

\[
(t^a)_{ij} \delta^{ab} (t^b)_{kl} = - \frac{1}{2N_C} \delta_{ij} \delta_{kl} + \frac{1}{2} \delta_{il} \delta_{jk}, \quad (N_C = 3)
\] (22)

5. After the above procedures, only quark lines remain. For each closed quark loop, a factor \( \text{Tr} 1 = 3 \) is assigned.

When the calculation is done in the covariant gauge, the diagrams including external ghost particles, if any, are to be considered separately.

A different approach can be followed without gluon-quark conversion. For example, let us denote quarks, antiquarks, and gluons as \( 1, 2, 3, \ldots, n_q, \overline{1}, \overline{2}, \overline{3}, \ldots, \overline{n_q} \), and \( 1, 2, 3, \ldots, n_g, \overline{1}, \overline{2}, \overline{3}, \ldots, \overline{n_g} \), respectively. Instead of Eq. (22), the color bases can be represented by a product of the following objects,

\[
\delta_{j_1} \delta_{k_1}, \quad (t^k_1)_{1j_1}, \quad ((t^k_1 t^k_2)_{1j_1}, \ldots
\]

where \( j_1, j_2, \ldots \) and \( k_1, k_2, \ldots \) are permutation of \( 1, 2, 3, \ldots, n_q \) and \( 1, 2, 3, \ldots, n_g \), respectively.

### 2.3.2 Interface to hadronization

As mentioned in the previous section, there are many possible choice for selecting a color base. As far as the total cross section is concerned they give the same result. However, the choice of the color base and the technique used to introduce color flow at the generation level are not equivalent and may induce noticeable discrepancies after hadronization.

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2 This does not happen in grc4f.
We take the string picture used in JETSET as a concrete example of hadronization. The color flow pattern can be assigned to each color base intuitively. However, an event is generated by matrix element which is the square of the amplitude, i.e., a linear combination of color bases. For interference terms, assignment of a color flow pattern is ambiguous. One possible solution is to introduce a set of orthogonal bases.

To illustrate this point, we consider the four-quark final state, $q_1\bar{q}_2q_3\bar{q}_4$ which is produced by grc4f. We label them 1, 2, 3, and 4. Among possible candidates, let us consider the following three types of color bases.

1. Primitive base
   \[
   \begin{align*}
   C_1 &= \delta_{12}\delta_{34} \\
   C_2 &= \delta_{14}\delta_{32}
   \end{align*}
   \] (23)

2. Orthogonal base
   \[
   \begin{align*}
   C_1^{(o)} &= C_1 + C_2 \\
   C_2^{(o)} &= C_1 - C_2
   \end{align*}
   \] (24)

3. Extended orthogonal base
   \[
   \begin{align*}
   C_1^{(e)} &= C_1 - \delta_{1234} \\
   C_2^{(e)} &= C_2 - \delta_{1234} \\
   C_3^{(e)} &= \delta_{1234}
   \end{align*}
   \] (25)

Here $\delta_{1234} = 1$ only when all four indices are equal and $= 0$ otherwise.

For each primitive base, one can assign a configuration of two strings naturally. They are, however, not diagonal; the interpretation of interference term is obscure. In the second case, the orthogonal bases, there is no interference term. However, each base $C_k^{(e)}$ do not relate directly to the string picture and each $C_k^{(o)}$ contains $|C_1|^2$ and $|C_2|^2$ with equal weight. The same thing happens in elementary quantum mechanics: The information on left/right circular polarization is lost after the light goes through a linear polarization filter. In the third example, the first two $C_k^{(e)}$ can be assigned to two strings of different colors. The third one is a kind of 4-quark loop worm. They are orthogonal to each other but they do not fit well with the JETSET approach.

The situation does not change for the case with more external particles, and for both cases with or without the gluon conversion. We have to say that there is some distance between the color bases and the definition of color singlet sub-system (e.g., strings) in the hadronization model.

In the present scheme of grc4f, we use the primitive base case and the color flow is chosen for each event stochastically with weight $C_k^\dagger C_k |F_k|^2$.

### 2.3.3 Options

There is an option to include the overall QCD correction factor. Here the cross section is multiplied by a simple factor $(1 + \alpha_s/\pi)$ for each quark vertex.

When the final state is a four-quark state, gluon exchange diagrams are included. It is possible to remove these by setting control data.

### 2.4 Kinematics and cuts

For the phase-space integration by the adaptive Monte Carlo method, a proper treatment of the singular behavior of the amplitude is necessary. It may come from:
1. s-channel W, Z resonances and \( \gamma \) propagators.

2. t-channel electron and electron neutrino propagators in \( Z \)-pair \((Z\gamma, \gamma\gamma)\) production and in \( W \)-pair production.

3. The electron (positron) forward scattering in two-photon processes.

4. Mixed resonance of \( W \)-pair and \( Z \)-pair \((Z\gamma, \gamma\gamma)\) processes, such as \( u\bar{d}\bar{d} \) process.

5. Identical particles in the final state, such as \( \mu^+\mu^-\mu^+\mu^- \).

\( \text{grc4f} \) includes the mapping routine from the eight integration variables to the four-momenta of final particles with proper treatment of these singular behavior. (A convergence of the numerical integration for the forward scattering of the two-photon processes is not so good.) Higgs boson diagrams are included in the amplitude, however the kinematics does not treat the Higgs boson resonance yet. (For cross section with very forward electrons or with Higgs boson resonance, contact the authors [\(^5\)]) For the identical particles in the final state, some momentum ordering is assumed in the kinematics routine, then, for instance, the first particle has always larger momentum than the second particle.

One can apply some experimental cuts for the phase space integration; energy and angle cuts on all final particles and an invariant-mass cut on any pair of final particles. These parameters can be specified by the user. For comparison with other programs, \( \text{grc4f} \) has the canonical cut option used in ref.\([15]\), which is:

1. the energy of charged leptons must be greater than 1 GeV.
2. the polar angle of charged leptons must be between 10 and 170 degree.
3. the energy of quarks must be greater than 3 GeV.
4. the opening angle between charged leptons and quarks must be greater than 5 degree.
5. the invariant mass of quark pairs must be greater than 5 GeV.

Here, the charged leptons include \( \tau \)'s too.

### 2.5 Coulomb correction

This effect has been originally discussed for the off-shell \( W \)-pair production\([8]\) corresponding to the three resonant diagrams. The set of these diagrams cannot satisfy the gauge invariance and has no physical meaning in principle. In \( \text{grc4f} \), however, all relevant diagrams are taken into account and the gauge invariance is restored from the contribution of the non-resonant diagrams (besides an effect from the finite width of \( W \) boson). To maintain the invariance we adopt the following prescription: the Coulomb factor is applied to the minimum set of gauge invariant diagrams containing the \( W \)-pair production\([11]\). Hence the Coulomb multiplicative factor appears in some diagrams even without \( W \)-pair. The following formula is used:

\[
\sigma_{\text{Coul}} = \sigma_{\text{gauge inv.}} \frac{\alpha}{2\beta} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{|\beta_M + \Delta|^2 - \beta}{2\beta |\beta_M|} \right) \right],
\]

\(^3\) E-mail address: grc4f@minami.kek.jp
where

\[ \beta = \frac{1}{s} \sqrt{s^2 - 2s(k_+^2 + k_-^2) + (k_+^2 - k_-^2)^2}, \]

\[ \beta_M = \sqrt{1 - 4M^2/s}, \quad M^2 = M_W^2 - iM_W\Gamma_W, \quad \Delta = \frac{|k_+^2 - k_-^2|}{s}, \]

and \(-\pi/2 < \text{arctan}< \pi/2\). Here \(\beta\) is the average velocity of the \(W\) boson in its center-of-mass system. Two squared momentum \(k_+^2\) and \(k_-^2\) are the virtualities of the intermediate \(W\) bosons.

2.6 Anomalous coupling

In the program, the anomalous coupling of heavy boson is available for the convenience of the user who may be interested to study such an effect, although there is no definite and reliable model beyond the standard model at present. The program includes only those terms which conserve \(C\) and \(P\) invariance which correspond to the following effective Lagrangian\(\text{[16]}\):

\[ L_{\text{eff}} = -ig_V(W_{\mu\rho}^+ W_{\rho\nu}^+ V^{\mu\nu} - W_{\mu\nu}^+ W_{\rho\nu}^+ V^{\mu\rho}) - ig_V\kappa_V W_{\mu\nu}^+ W_{\rho\nu} V^{\mu\rho} - ig_V \frac{\lambda_V}{m_W^2} W_{\lambda\mu}^+ W_{\rho\nu}^+ V^{\mu\rho}, \]

\[ W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu, \quad V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu, \quad V = Z^0 \text{ or } \gamma, \quad g_V = \begin{cases} -e & V = \gamma \\ -\text{csc}\theta_W & V = Z^0 \end{cases} \]

Here \(\kappa_V\) and \(\lambda_V\) stand for the anomalous couplings parameters being \(1\) and \(0\), respectively, in the standard model. Deviation from these values corresponds to the introduction of anomalous coupling.

3. Structure of the program

The generator \texttt{grc4f} enables us to generate any \(e^+e^- \rightarrow 4\)-fermions events with or without radiative corrections. This requires different calculation in some part of the program. In addition there are many options covering theoretical and experimental requirements. Since all program components are distributed as source code, users can set all options by editing the relevant subprograms directly. However, an interface program \texttt{grc4f} has been created to lighten the user’s burden. It selects and/or corrects the components which are affected by the various options and create a ”Makefile” according to the user requirements. This procedure is called the source generation phase.

In the integration step, the matrix element of a selected process is integrated over the phase space by the subprogram \texttt{BASES}, which gives the effective total and differential cross sections and the probability distribution used in the event generation phase. There, the subprogram \texttt{SPRING} samples a point in the phase space and test if it can be accepted as a new event according to its probability. When an event is accepted the program control returns from \texttt{SPRING} to the main program, where further analysis is performed by using the resultant four-momenta of the final state particles.

There are therefore, three steps in the generator \texttt{grc4f}, the first is the source generation, the second is the integration, and the third is the event generation. In addition to the user interface program \texttt{grc4f}, the following programs are available in the \texttt{grc4f} package:

i) The main programs \texttt{MAINBS} and \texttt{MAINSP}, and all program components for the integration and event generation steps.
ii) The interface programs to CERNLIB (GRC2CL) and to JETSET (GRC2SH).

iii) 76 function programs FUNCs for processes $e^+ e^- \rightarrow 4$-fermions, each of which calculates the numerical value of the differential cross section for each process.

iv) The kinematics subprograms KINMOQ and KINEMO for the processes with radiative correction using the QED parton shower model or the electron structure function, respectively.

v) The library CHANEL used to calculate the numerical values of basic components of Feynman diagrams in terms of helicity amplitudes.

vi) The numerical integration and event generation program package BASES/SPRING v5.1[3].

The relationship among these program components and their function are presented in the next two sections.

3.1 Source generation step

The user interface program grc4f reads the parameters from control data prepared by users, which contains process selection, type of radiative corrections, physical options, experimental cuts, etc., and generates all necessary components:

i) A main program MAINBS for the integration.

ii) Interface programs GRC2CL and GRC2SH.

iii) Four initialization subprograms USRPRM, MODMAS, KINIT, USERSP, and

iv) a “Makefile”.

3.2 Integration step

Before starting the numerical integration the main program MAINBS invokes an initialization subprogram USERIN, where the following subprograms are called in this order:

USRPRM : To define the set of so-called "canonical cut" authorized by LEP200 working group and some additional optional parameters.

SETMAS : To set masses and decay widths of particles.

MODMAS : To alter the default values of all parameters defined in SETMAS.

AMPARM : To set the coupling constants and others parameters.

KINIT : To set the parameters for the integration, kinematics, cuts, histograms etc.

The subprograms SETMAS and AMPARM are generated by the GRACE system. There are no consistency checks among the constants, e.g. $M_Z$, $M_W$ and $\sin \theta_W$, so all modifications on these parameters in the subprograms USRPRM, MODMAS and KINIT are under the user responsibility alone.

The integration program BASES calculates the scattering cross section by sampling the function FUNC on the allowed phase space segmented by an self adapted grid where finer cells are clustered on the high gradient zones. This is an iterative process running until either the maximum number of allowed iteration is reached or the required accuracy is obtained. In the function FUNC, the kinematics subroutine KINEMO or KINMOQ is used to map the integral variables with the four-momenta of the final state particles. KINEMO is used for reaction with no radiative
corrections or those involving initial radiation treated with the structure function techniques. KINMOQ is called for processes where radiative corrections are computed with the QED parton shower method.

It is recommended to check the integration result carefully, especially the convergence behaviors both for the grid optimization and integration steps. When the accuracy of each iteration fluctuates, iteration by iteration, and, in some case, it may jump up suddenly to a large value compared to the other iterations, the resultant estimate of integral may not be reliable. There are two possible origins of this behavior: too few sampling points or an unsuitable choice of the kinematics.

After the numerical integration by BASES, the subprograms BSINFO and BHPLOT are called to print the result of integration and the histograms, respectively. Before terminating the integration procedure the probability distribution obtained by the integration can be saved in a file by invoking BSWRIT, which is then used for the event generation by SPRING.

3.3 Event generation step

After integrating the differential cross section and saving the probability distribution, the main program, MAINSP, handles the event generation program. The subprogram BSREAD is invoked to restore the probability distribution and then the subprogram USERIN is called. Each call to SPRING generates one event by sampling a point in the phase volume. It calculates first the differential cross section at that point using the same function FUNC seen in the integration phase and returns the weight of this sampling point. A weight-one event is finally generated using the usual unweighting technique. When an event is generated, SPRING returns the particle types and four momenta. The event information is stored in the labelled common LUJETS by calling the subprogram SP2LND. Here, the information for the color connection to be referred by JETSET is also supplied. Among the final states in Appendix B, those in Table 3 have non-trivial color flow which is determined as is described in the section 2.3.

Then parton shower and hadronization of quarks and gluons can be performed by calling LUSHOW and LUEXEC. At the end of the event generation, the routine SPINFO and SHPLOT are invoked successively for printing event generation information and histograms.

4. How to run the program

The user should first prepare the control data to define the process, the options, flags and the experimental cuts. The user interface program grc4f takes this control data (let’s call it control.data) as an input.

```
  process = eNeuD
  energy = 190.0d0
  canon = yes
  type = tree
  massive = yes
  coulomb = no
  anomal = no
  qcdcr = no
  end
```

The first line specifies the process to be calculated and the second is the center of mass energy in GeV unit. The others are options, whose meanings are given in Appendix A. Then the user
may type:

```
% grc4f < control.data
```

If the message "syntax error" is returned, no files will be generated and the contents of the control data file must carefully be checked. After a successful completion, the following messages should be returned:

```
Process is "eNEuD"
Energy is "190.0d0"
CANON <yes>
MASSIV <yes>
COULMB <no>
ANOMAL <no>
QCDCR <no>
bye-bye
```

```
absolute directory name is /home/grc/prc/e1NEuqDQ
```

According to the parameters given in control data, the files, i.e. `usrprmf, modmas.f, kinit.f, mainbs.f, grc2cl.f, grc2sh.f, usersp.f` and `Makefile`, are generated in a specified subdirectory (`e1NEuqDQ` in this case). According to the last four lines in the message, users can proceed the calculations as follows:

i) Change directory by

```
% cd /home/grc/prc/e1NEuqDQ
```

ii) Create an executable `integ` for the integration by typing:

```
% make integ
```

iii) Numerical integration is actually performed by typing:

```
% integ
```

According to the parameters given in control data, the files, i.e. `usrprmf, modmas.f, kinit.f, mainbs.f, grc2cl.f, grc2sh.f, usersp.f` and `Makefile`, are generated in a specified subdirectory (`e1NEuqDQ` in this case). According to the last four lines in the message, users can proceed the calculations as follows:

The results of integration step are displayed as well as written in an output file `bases.result`. The total cross section in \( pb \) with the error are displayed at the last row, under `Cumulative Result`, in the table of the `Convergence Behavior for the Integration step`. The differential cross sections as a function of the energy and angle of each particle and invariant masses of any two final particles will also be printed. The probability distribution is written in a file `bases.data` which will be used in the event generation step by `spring`.  

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Before running the event generation, users should edit `user.sp.f` to set additional parameters like the number of events and call user’s own analysis routines.

The following is the structure of `main.sp.f`, where four-momenta of all particles are stored in the `common/lujets/` in the JETSET format by calling subprogram `sp2lnd` in the event-loop:

```fortran
implicit real*8(a-h,o-z)
........................
real*4  p,v
common/lujets/n,k(4000,5),p(4000,5),v(4000,5)
........................
call usersp(mxtry,mxevnt)
........................
d0 100 nevnt = 1, mxevnt
      call spring( func, mxtry )
........................
  *               ------------
  *      call sp2lnd
  *               ------------
  *               ============
  *      ( user_analysis )
  *               ============
  *          100 continue
........................
stop
end
```

Create an executable `spring` for event generation by typing:

```
% make spring
```

Start the event generation by typing:

```
% spring
```

Information on the event generation will be written in the `spring.result` file. Users should pay special attention to the histograms. The distributions of generated events are superimposed by “0” on the original histograms by BASES. These two distributions should be consistent within the statistical error of the generation. For the detail of the output files of BASES and SPRING, user can consult the Ref.[24].
5. Summary

The generator \texttt{grc4f} enables us to calculate the effective cross section and to generate events for one of 76 $e^+e^- \to 4$-fermions processes listed in appendix B. It is dedicated to the LEP and future linear collider physics studies. The produced quarks can be hadronized according to JETSET. Also processes with initial and final radiations can be generated in terms of the electron structure function or the QED parton shower methods, though the inclusion of the interference between the initial and final radiations requires further study.

There still remain important problems to be solved and further necessary improvements. Among them it will be desirable to extend the program so as to produce several final state processes in a single run like $e^+e^- \to 4$-quarks. In order to get more precise QED corrections an implementation of a complete next-to-leading logarithmic calculation is unavoidable.

In this version, we have assumed that the hadronization of partons takes place independent of the hard interaction which produces partons. However a very important contribution arises from diagram where a gluon is exchanged between quarks produced in the two $W$ decays. Taking into account this effect implies higher order calculations or the implementation of a specific phenomenologic correction.

Finally the present version provides only 76 processes, the missing processes will be prepared soon by using GRACE.

Acknowledgements

The authors would like to thank G. Coignet, F. Boudjema, B. Mele for their interest and encouragement and colleagues in Minami-Tateya group of KEK for their help. This work was done in the KEK-LAPP collaboration supported in part by Mombusho in Japan under the Grant-in-Aid for International Scientific Research Program No.07044097, and CNRS/IN2P3 in France.

References


GRACE manual v2.0, in preparation.


Appendix A. Option table in Control data

In the table below, the default values are underlined and the relation between commands and variable/array in Fortran sources is also described. Variable names are written in bold letters and filenames in italic.

i) Process selection.

<table>
<thead>
<tr>
<th>Process</th>
<th>eNEuD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>abbreviation of process name</td>
</tr>
</tbody>
</table>

This specifies the subdirectory name, where process related subroutines are stored. Tables 1, 2, and 3 in Appendix B shows the abbreviation of process names and the subdirectory names where they are stored are listed.

ii) Center of mass energy.

<table>
<thead>
<tr>
<th>energy</th>
<th>j90.d0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CMS energy in GeV</td>
</tr>
<tr>
<td></td>
<td>w in kinit.f</td>
</tr>
</tbody>
</table>

iii) Global options

<table>
<thead>
<tr>
<th>helicity1</th>
<th>average, left, right</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Helicity state for the initial electron.</td>
</tr>
<tr>
<td>helicity2</td>
<td>average, left, right</td>
</tr>
<tr>
<td></td>
<td>Helicity state for the initial positron.</td>
</tr>
<tr>
<td>helicity3</td>
<td>sum, left, right</td>
</tr>
<tr>
<td></td>
<td>Helicity state for 3rd particle.</td>
</tr>
<tr>
<td>helicity4</td>
<td>sum, left, right</td>
</tr>
<tr>
<td></td>
<td>Helicity state for 4th particle.</td>
</tr>
<tr>
<td>helicity5</td>
<td>sum, left, right</td>
</tr>
<tr>
<td></td>
<td>Helicity state for 5th particle.</td>
</tr>
<tr>
<td>helicity6</td>
<td>sum, left, right</td>
</tr>
<tr>
<td></td>
<td>Helicity state for 6th particle.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>type</th>
<th>tree, sf, qedpsi, qedpsif</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type of calculation:</td>
</tr>
<tr>
<td></td>
<td>Without radiation(tree), ISR with structure function(sf), ISR with QEDPS(qedpsi) and ISR and FSR with QEDPS(qedpsif).</td>
</tr>
<tr>
<td></td>
<td>jqedps = 0, isr = 0 without radiation(tree).</td>
</tr>
<tr>
<td></td>
<td>jqedps = 0, isr = 1 for sf.</td>
</tr>
<tr>
<td></td>
<td>jqedps = 1, ips = 1 for qedpsi.</td>
</tr>
<tr>
<td></td>
<td>jqedps = 1, ips = 2 for qedpsif.</td>
</tr>
<tr>
<td></td>
<td>jqedps in usrprm.f, isr and ips in kinit.f.</td>
</tr>
</tbody>
</table>
iv) Physical options

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>canon</td>
<td>yes, no</td>
<td>Apply canonical cuts or not: jcanon = 1 or 0 in usrprm.f.</td>
</tr>
<tr>
<td>massiv</td>
<td>yes, no</td>
<td>Quarks are massive or massless: jqmass = 1 or 0 in usrprm.f.</td>
</tr>
<tr>
<td>width</td>
<td>run, fixed</td>
<td>Running width or fixed width: jwidth = 0 or 1 in usrprm.f.</td>
</tr>
<tr>
<td>coulomb</td>
<td>yes, no</td>
<td>Coulomb correction: jcolmb = 1 or 0 in modmas.f.</td>
</tr>
<tr>
<td>qcdcr</td>
<td>yes, no</td>
<td>Include overall QCD correction factor: jqcdcr = 1 or 0 in modmas.f.</td>
</tr>
<tr>
<td>gluon</td>
<td>yes, no</td>
<td>Include diagrams with gluon exchange: jgluon = 1 or 0 in modmas.f.</td>
</tr>
<tr>
<td>anomal</td>
<td>yes, no</td>
<td>janco3v = 1 or 0 in modmas.f.</td>
</tr>
<tr>
<td>ankaa</td>
<td>1.0d0</td>
<td>Anomalous coupling:</td>
</tr>
<tr>
<td>anlma</td>
<td>0.0d0</td>
<td></td>
</tr>
<tr>
<td>ankaz</td>
<td>1.0d0</td>
<td></td>
</tr>
<tr>
<td>anlmz</td>
<td>0.0d0</td>
<td></td>
</tr>
</tbody>
</table>

v) Masses, widths and $\alpha$ (only if “canon = no”).

See section 2.6 for the definition of above variables, if “anomalous = no” is specified, these options affect nothing.
\[
\begin{align*}
\text{amw} &= 80.23 \text{D}0 \\
\text{agw} &= 2.033 \text{D}0 \\
\text{amz} &= 91.1888 \text{D}0 \\
\text{agz} &= 2.4974 \text{D}0 \\
\text{alphai} &= 128.07 \text{D}0 \\
\alpha_s &= 0.12 \text{D}0 \\
\end{align*}
\]

\[
\begin{align*}
\text{The numbering convention of particles follows the GRACE scheme, where the initial electron and positron are 1st and 2nd, respectively, and the four final particles are numbered 3, 4, 5, 6. In the process name of Table 1, 2, and 3, the order of particles corresponds to this numbering convention. For instance, in the process, } e^- \bar{\nu}_e u \bar{d}, \text{ the 3rd is } e^- \text{, the 4th is } \bar{\nu}_e, \text{ the 5th is } u \text{ and the 6th is } \bar{d}. \\
\text{Instead of giving a numerical value, the user can use the keywords as below: amass(i) has the mass for } \text{i-th particle and w is the CM energy.}
\end{align*}
\]
\begin{verbatim}
engcut3 = amass1(3), w
Energy cut for 3rd particle (min., max.)
engyct(1:2,1) in kinit.f

engcut4 = amass1(4), w
Energy cut for 4th particle (min., max.)
engyct(1:2,2) in kinit.f

engcut5 = amass1(5), w
Energy cut for 5th particle (min., max.)
engyct(1:2,3) in kinit.f

engcut6 = amass1(6), w
Energy cut for 6th particle (min., max.)
engyct(1:2,4) in kinit.f

ivmcut34 = amass1(3) + amass1(4), w - (amass1(5) + amass1(6))
Invariant mass cut for 3-4 particles (min., max.)
amasct(1:2,1) in kinit.f

ivmcut56 = amass1(5) + amass1(6), w - (amass1(3) + amass1(4))
Invariant mass cut for 5-6 particles (min., max.)
amasct(1:2,2) in kinit.f

ivmcut35 = amass1(3) + amass1(5), w - (amass1(4) + amass1(6))
Invariant mass cut for 3-5 particles (min., max.)
amasct(1:2,3) in kinit.f

ivmcut46 = amass1(4) + amass1(6), w - (amass1(3) + amass1(5))
Invariant mass cut for 4-6 particles (min., max.)
amasct(1:2,4) in kinit.f

ivmcut36 = amass1(3) + amass1(6), w - (amass1(4) + amass1(5))
Invariant mass cut for 3-6 particles (min., max.)
amasct(1:2,5) in kinit.f

ivmcut45 = amass1(4) + amass1(5), w - (amass1(3) + amass1(6))
Invariant mass cut for 4-5 particles (min., max.)
amasct(1:2,6) in kinit.f
\end{verbatim}

vii) Parameters for integration.

\begin{verbatim}
itmx = 7, 15
Iteration numbers: itmx1, itmx2 in kinit.f

acc = 0.1, 0.05
Accuracies in %:acc1, acc2 in kinit.f

ncall = 40000
Sampling points: ncall in kinit.f
\end{verbatim}

viii) Parameters for event generation.
\( \text{mxtry} = 50 \)

Maximum trial numbers: \text{mxtry} in \textit{usersp.f}

\( \text{maxn} = 10000 \)

Maximum event numbers: \text{mxevnt} in \textit{usersp.f}

\( \text{hadron} = \text{yes, no} \)

Hadronization with JETSET\(^7\) for event generation:
If \text{yes}, then a statement, \texttt{call luexec} is added in \textit{grc2sh.f}. In this case, program JETSET is necessary, and one should specify JETSET file location in this control data.

\( \text{jetset} = /home/jetset/jetset74.o \)

Object file name for JETSET, if \text{hadron} = \text{yes}

\( \text{qcd} = \text{yes, no} \)

QCD parton shower with JETSET\(^7\) for event generation:
If \text{yes}, then a statement, \texttt{call lushow} is added in \textit{grc2sh.f}. In this case, program JETSET is necessary, and one should specify JETSET file location in this control data.

Those parameters affect \textit{Makefile}.

ix) HBOOK interface.

\( \text{cernlib} = \text{yes, no} \)

If \text{yes}, then the \texttt{bases.hbook} in integration and \texttt{spring.hbook} in event generation will be generated.

Those parameters affect \textit{mainbs.f}, \textit{gre2cl.f} and \textit{Makefile}.

x) End of description.

\texttt{end}

After the command \texttt{end} any command is neglected.

\textbf{Appendix B. Process table}

Here, the 76 processes included in \texttt{grc4f} are listed. In the heading, 'abbrev.' and 'dir.' stand for the abbreviated name used in the control card and directory name where the generated code is stored, respectively.
Table 1  Leptonic processes in grc4f.

<table>
<thead>
<tr>
<th>process</th>
<th>abbrev.</th>
<th>dir.</th>
<th>process</th>
<th>abbrev.</th>
<th>dir.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_e, e^+, e^-, \bar{\nu}_e$</td>
<td>eNEEe</td>
<td>eLEnNE</td>
<td>$e^-, \bar{\nu}<em>e, \mu^+, \nu</em>\mu$</td>
<td>eNEEmMU</td>
<td>eLEnMUmm</td>
</tr>
<tr>
<td>$\nu_e, \bar{e}, \tau^+, \nu_\tau$</td>
<td>eNEntAU</td>
<td>eLNTAuT</td>
<td>$\nu_\tau, \mu^+, \mu^-, \bar{\nu}_\mu$</td>
<td>muMUmNM</td>
<td>muMUmmNM</td>
</tr>
<tr>
<td>$\nu_\tau, \tau^+, \bar{\nu}_\tau$</td>
<td>tauTANuNT</td>
<td>taTANtNT</td>
<td>$\mu^-, \bar{\nu}<em>\tau, \tau^+, \nu</em>\tau$</td>
<td>muNTAuNT</td>
<td>muNMTAnt</td>
</tr>
<tr>
<td>$\tau, \tau^+, \tau^+$</td>
<td>tauTATauTAU</td>
<td>taTATaTA</td>
<td>$\mu^-, \mu^+, \mu^-, \mu^+$</td>
<td>muMUmuMU</td>
<td>muMUmuMU</td>
</tr>
<tr>
<td>$\nu_\tau, \bar{\nu}_\tau, \mu^-, \mu^+$</td>
<td>nNTmMU</td>
<td>nNTmMU</td>
<td>$\nu_\tau, \nu_\bar{\tau}, \tau^+$</td>
<td>neNEmtauT</td>
<td>neNEtauT</td>
</tr>
<tr>
<td>$\nu_\tau, \bar{\nu}<em>\tau, \nu</em>\mu, \bar{\nu}_\mu$</td>
<td>ntNTmuNM</td>
<td>ntNTmMuNM</td>
<td>$\nu_\nu_\tau, \nu_\bar{\nu}_\tau, \tau^+$</td>
<td>mmNTmAU</td>
<td>mmNTmTA</td>
</tr>
<tr>
<td>$\nu_\tau, \bar{\nu}<em>\tau, \nu</em>\mu, \nu_\bar{\mu}$</td>
<td>nEntNT</td>
<td>nEntNT</td>
<td>$\nu_\nu_\tau, \nu_\bar{\nu}<em>\tau, \nu</em>\mu, \nu_\bar{\mu}$</td>
<td>mmNnmNM</td>
<td>mmNnmNM</td>
</tr>
<tr>
<td>$\nu_\tau, \nu_{\bar{\tau}}, \nu_\mu, \nu_{\bar{\mu}}$</td>
<td>ntNTnTNT</td>
<td>ntNTnTNT</td>
<td>$\nu_\nu_\tau, \nu_\nu_\bar{\tau}, \nu_\nu_\mu, \nu_\nu_{\bar{\mu}}$</td>
<td>nmNnmNM</td>
<td>nmNnmNM</td>
</tr>
</tbody>
</table>

Table 2  Semi-hadronic processes in grc4f.

<table>
<thead>
<tr>
<th>process</th>
<th>abbrev.</th>
<th>dir.</th>
<th>process</th>
<th>abbrev.</th>
<th>dir.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^-, \bar{\nu}_e, u, \bar{d}$</td>
<td>eNEUD</td>
<td>eNEUdQ</td>
<td>$e^-, \bar{\nu}_e, c, \bar{s}$</td>
<td>eNEcS</td>
<td>eLEcqSQ</td>
</tr>
<tr>
<td>$\mu^-, \bar{\nu}_\mu, u, \bar{d}$</td>
<td>muMUmU</td>
<td>muMUmQ</td>
<td>$\mu^-, \bar{\nu}_\mu, c, \bar{s}$</td>
<td>muNCMS</td>
<td>muMcmSQ</td>
</tr>
<tr>
<td>$\tau, \tau^+, u, \bar{u}$</td>
<td>tauNTuT</td>
<td>tauNTuQ</td>
<td>$\tau, \nu_\tau, \bar{\nu}_\tau, s - \bar{s}$</td>
<td>tauNTcS</td>
<td>tauTcSQ</td>
</tr>
<tr>
<td>$e^-, e^+, u, \bar{u}$</td>
<td>eEUeU</td>
<td>eEUuQ</td>
<td>$e^-, e^+, c, \bar{\bar{c}}$</td>
<td>eEcC</td>
<td>eELcCQ</td>
</tr>
<tr>
<td>$e^-, e^+, d, \bar{d}$</td>
<td>eEdD</td>
<td>eEdQ</td>
<td>$e^-, e^+, s, \bar{s}$</td>
<td>eScS</td>
<td>eELsQ</td>
</tr>
<tr>
<td>$e^-, e^+, b, \bar{b}$</td>
<td>eEBB</td>
<td>eEBBQ</td>
<td>$\mu^-, \mu^+, u, \bar{u}$</td>
<td>muMUuU</td>
<td>muMUuUQ</td>
</tr>
<tr>
<td>$\mu^-, \mu^+, c, \bar{c}$</td>
<td>muMucC</td>
<td>muMuCQ</td>
<td>$\mu^-, \mu^+, s, \bar{s}$</td>
<td>muMuSQ</td>
<td>muMuBQ</td>
</tr>
<tr>
<td>$\tau, \tau^+, c, \bar{\bar{c}}$</td>
<td>tauTAcC</td>
<td>tauTcQ</td>
<td>$\tau, \tau^+, u, \bar{u}$</td>
<td>tauTAUuU</td>
<td>tauTcUQ</td>
</tr>
<tr>
<td>$\tau, \tau^+, s, \bar{s}$</td>
<td>tauTAD</td>
<td>tauTaqQ</td>
<td>$\tau, \tau^+, s, \bar{s}$</td>
<td>tauTAdS</td>
<td>tauTaqSQ</td>
</tr>
<tr>
<td>$\tau, \tau^+, d, \bar{d}$</td>
<td>tauTAbB</td>
<td>tauTaqBQ</td>
<td>$\nu_\nu_\tau, \nu_\nu_{\bar{\tau}}, u, \bar{u}$</td>
<td>nNEuUE</td>
<td>nNEuUEQ</td>
</tr>
<tr>
<td>$\nu_\nu_\tau, \nu_\nu_{\bar{\tau}}, c, \bar{\bar{c}}$</td>
<td>nNEcC</td>
<td>nNEcQ</td>
<td>$\nu_\nu_\tau, \nu_\nu_{\bar{\tau}}, d, \bar{\bar{d}}$</td>
<td>nNEdC</td>
<td>nNEdQ</td>
</tr>
<tr>
<td>$\nu_\nu_\tau, \nu_\nu_{\bar{\tau}}, s, \bar{s}$</td>
<td>nNEsS</td>
<td>nNEsQ</td>
<td>$\nu_\nu_\tau, \nu_\nu_{\bar{\tau}}, b, \bar{b}$</td>
<td>nNEbB</td>
<td>nNEbBQ</td>
</tr>
<tr>
<td>$\nu_\nu_\mu, \nu_\nu_{\bar{\mu}}, u, \bar{u}$</td>
<td>nmMuU</td>
<td>nmMuUQ</td>
<td>$\nu_\nu_\mu, \nu_\nu_{\bar{\mu}}, c, \bar{\bar{c}}$</td>
<td>nmNcC</td>
<td>nmNcCQ</td>
</tr>
<tr>
<td>$\nu_\nu_\mu, \nu_\nu_{\bar{\mu}}, d, \bar{\bar{d}}$</td>
<td>nmMuD</td>
<td>nmMuDQ</td>
<td>$\nu_\nu_\mu, \nu_\nu_{\bar{\mu}}, s, \bar{s}$</td>
<td>nmMuS</td>
<td>nmMuSQ</td>
</tr>
<tr>
<td>$\nu_\mu, \nu_{\bar{\mu}}, b, \bar{b}$</td>
<td>nmMBB</td>
<td>nmMBBQ</td>
<td>$\nu_\mu, \nu_{\bar{\mu}}, d, \bar{\bar{d}}$</td>
<td>nmMdB</td>
<td>nmMbDQ</td>
</tr>
<tr>
<td>$\nu_\mu, \nu_{\bar{\mu}}, s, \bar{s}$</td>
<td>ntNTsS</td>
<td>ntNTsSQ</td>
<td>$\nu_\mu, \nu_{\bar{\mu}}, b, \bar{b}$</td>
<td>ntNtbB</td>
<td>ntNtbBQ</td>
</tr>
<tr>
<td>process</td>
<td>abbrev.</td>
<td>dir.</td>
<td>process</td>
<td>abbrev.</td>
<td>dir.</td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
<td>------</td>
<td>---------</td>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>u, d, d, \bar{\nu}</td>
<td>uDUd</td>
<td>uqDdqUQ</td>
<td>e, \bar{s}, s, \bar{c}</td>
<td>cCSs</td>
<td>cqSQsqCQ</td>
</tr>
<tr>
<td>u, \bar{d}, s, \bar{c}</td>
<td>uDsC</td>
<td>uqDqaqCQ</td>
<td>u, \bar{\nu}, u, \bar{\nu}</td>
<td>u\bar{u}U</td>
<td>u\bar{q}UqUqUQ</td>
</tr>
<tr>
<td>c, \bar{\nu}, c, \bar{c}</td>
<td>cCcC</td>
<td>cCcqCcqCQ</td>
<td>d, \bar{d}, d, \bar{d}</td>
<td>d\bar{D}</td>
<td>d\bar{q}DqDqDQ</td>
</tr>
<tr>
<td>s, \bar{s}, s, \bar{s}</td>
<td>sSSs</td>
<td>sqSQsqSQ</td>
<td>b, \bar{\bar{b}}, b, \bar{\bar{b}}</td>
<td>b\bar{B}bBbB</td>
<td>b\bar{q}bQbQbQ</td>
</tr>
<tr>
<td>u, \bar{\nu}, c, \bar{\nu}</td>
<td>uCcC</td>
<td>uqUcqCcqCQ</td>
<td>u, \bar{s}, s, \bar{s}</td>
<td>uUsS</td>
<td>uqUqsqSQ</td>
</tr>
<tr>
<td>u, \bar{\nu}, b, \bar{b}</td>
<td>uUbB</td>
<td>uqUqqbBQ</td>
<td>c, \bar{\nu}, d, \bar{d}</td>
<td>cCdD</td>
<td>cqCqDqDQ</td>
</tr>
<tr>
<td>c, \bar{\nu}, b, \bar{b}</td>
<td>cCbB</td>
<td>cCcqBcqBQ</td>
<td>d, \bar{d}, s, \bar{s}</td>
<td>dDsS</td>
<td>d\bar{q}DqDsSQ</td>
</tr>
<tr>
<td>d, \bar{d}, b, \bar{b}</td>
<td>dDbB</td>
<td>d\bar{q}DqBqBQ</td>
<td>s, \bar{s}, b, \bar{b}</td>
<td>sSbB</td>
<td>s\bar{q}bQbBQ</td>
</tr>
</tbody>
</table>

Table 3 Hadronic processes in grc4f.

Appendix C. Installation

The source code is available by anonymous ftp from ftp.kek.jp in the directory kek/minami/grc4f. The grc4f package contains the complete set of Fortran sources for 76 processes, the three libraries, i.e., BASES/SPRING, CHANEL and utilities for kinematics. Those source codes are written in FORTRAN77. In addition, grc4f provides the interface program to generate a few Fortran source files according to the control data specified by the user. This program is written in C, YACC and LEX. grc4f has been developed on HP-UX, but should run on any UNIX platform with a fortran compiler.

The procedure of installation is as follows:

1. Editing Makefile.

The following macros in Makefile should be taken care of by users themselves. For example, in the right hand side of GRC4FDIR the directory name where grc4f is installed should be given, and for FC and FOPT the relevant compiler name and option for your system should be given. The other macros can be left as they are.

   - GRC4FDIR = directory where grc4f are installed.
   - PRCDIR = directory where process files are installed.
   - LIBDIR = directory where libraries are installed.
   - BINDIR = directory where an executable is installed.
   - MACHINE = [hpx|hiux|sgi|dec|sun]
   - FC = FORTRAN compiler command name.
   - FOPT = FORTRAN compiler options.

2. Compilation.

By executing command make install one executable, i.e. the interface program(grc4f), is generated at BINDIR. Furthermore three libraries, i.e. BASES/SPRING, CHANEL and kinematics utility library, are generated in LIBDIR.

24
3. Install default *Makefiles*.

By executing `src01gen.sh` command, all *Makefiles* for 76 processes will be generated according to the environment where `grc4f` has been installed.

The sample control data files will be found at the directory `sample`.

**TEST RUN OUTPUT**

control data

```
process = eNEuD
end
```

Followings are the output files from BASES and SPRING. Only one histogram, the energy distribution of the particle 1, is shown since the whole output is too lengthy to be included here.
Parameters for BASES

(1) Dimensions of integration etc.
   # of dimensions : Ndim = 8 (50 at max.)
   # of Wilds      : Nwild = 8 (15 at max.)
   # of regions    : Nregion = 3 / variable
   # of Hypercubes : Ncube = 6561

(2) About the integration variables
   i       XL(i)       XU(i)       IG(i)   Wild
           1       .000000E+00    1.000000E+00     1      yes
           2       .000000E+00    1.000000E+00     1      yes
           3       .000000E+00    1.000000E+00     1      yes
           4       .000000E+00    1.000000E+00     1      yes
           5       .000000E+00    1.000000E+00     1      yes
           6       .000000E+00    1.000000E+00     1      yes
           7       .000000E+00    1.000000E+00     1      yes

(3) Parameters for the grid optimization step
   Max.# of iterations : ITMX1 = 7
   Expected accuracy  : Acc1 = 0.1000

(4) Parameters for the integration step
   Max.# of iterations : ITMX2 = 15
   Expected accuracy  : Acc2 = 0.0500

Computing Time Information

(1) For BASES
   Overhead : 0:00:0.04
   Grid Optm. Step : 0:16:0.53
   Integration Step : 2:63:0.43
   Go time for all : 3:03:1.00

(2) Expected event generation time
   Expected time for 1000 events : 18.36 Sec
Convergency Behavior for the Grid Optimization Step

<table>
<thead>
<tr>
<th>Result of each iteration</th>
<th>Cumulative Result</th>
<th>CPU time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.014559E-01</td>
<td>0.211</td>
<td>0:54:00</td>
</tr>
</tbody>
</table>

Convergency Behavior for the Integration Step

<table>
<thead>
<tr>
<th>Result of each iteration</th>
<th>Cumulative Result</th>
<th>CPU time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.012611E-01</td>
<td>0.183</td>
<td>3:00:30</td>
</tr>
</tbody>
</table>

**Contents of the histogram Header************

1. Actual Buffer size = 9269 Words
2. Contents of Histograms
   - Max. No. of Histograms = 50
   - Number of Histograms = 12
   ID X_min X_max X_bin Hash Net#
   1 .000E+00 1.000E+02 50 2 2 1
   14 .000E+00 1.000E+02 50 2 2 11
   2 .000E+00 1.000E+02 50 3 2 2
   15 .000E+00 1.000E+02 50 3 2 12
   3 .000E+00 1.000E+02 50 4 1 3
   4 .000E+00 1.000E+02 50 1 5 4
   5 .000E+00 1.000E+00 50 6 1 5
   6 .000E+00 1.000E+00 50 7 1 6
   7 .000E+00 1.000E+00 50 8 1 7
   8 .000E+00 1.000E+00 50 9 1 8
   9 .000E+00 1.000E+00 50 10 1 9
   10 .000E+00 1.000E+02 50 1 1 10

3. Contents of Scatter Plots
   - Max. No. of Scatter Plots = 50
   - Number of Scatter Plots = 0

**Histogram (ID = 1) for Energy of Particle 3**

<table>
<thead>
<tr>
<th>x</th>
<th>d(log(sigma))/dx</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0E-06</td>
</tr>
<tr>
<td>1.0E-05</td>
<td>1.0E-05</td>
</tr>
<tr>
<td>1.0E-04</td>
<td>1.0E-04</td>
</tr>
<tr>
<td>1.0E-03</td>
<td>1.0E-03</td>
</tr>
</tbody>
</table>

Linear Scale indicated by "*"
Logarithmic Scale indicated by "O"
**Date: 96/2/6 10:41**

- **Process**: $e^+e^- \rightarrow (3)\text{e}^-c\overline{d}(5)u(6)d\overline{b}$
- **Canonical CUT**: YES
- **Mass (Width)**:
  - $W$-boson: 80.230 +/- 0.034
  - $Z$-boson: 91.189 +/- 2.497
- **Cuts**:
  - $u$-quark: $0.05$, $d$-quark: $0.10$
  - $c$-quark: $1.300$, $a$-quark: $0.200$
  - $t$-quark: $170.000$, $b$-quark: $4.300$
- **Experimental Cuts**:
  - **Angle Cuts**:
    - Particle 3: $0.985$ to $0.985$
    - Particle 4: $1.000$ to $1.000$
    - Particle 5: $1.000$ to $1.000$
    - Particle 6: $1.000$ to $1.000$
  - **Energy Cuts**:
    - Particle 3: $1.000$ to $190.000$
    - Particle 4: $0.000$ to $190.000$
    - Particle 5: $3.000$ to $190.000$
    - Particle 6: $3.000$ to $190.000$
  - **Invariant mass Cuts**:
    - Inv. Mass 3-4: $0.001$ to $189.985$
    - Inv. Mass 5-6: $5.000$ to $189.999$
    - Inv. Mass 3-5: $0.006$ to $189.990$
    - Inv. Mass 4-6: $0.010$ to $189.994$
    - Inv. Mass 3-6: $0.011$ to $189.995$
    - Inv. Mass 4-5: $0.005$ to $189.989$
- **Options**:
  - Calculation: TREE
  - Anomalous Coupling: NO
  - Coulomb Correction: NO
  - QCD Correction: NO
  - Gluon graph: NO (only for hadronic process)

---

**Number of generated events**: 10000
**Generation efficiency**: 16.538 Percent
**Computing time for generation**: 836.960 Seconds
**Max. number of trials MXTRY**: 50 per event
**Number of miss-generation**: 56 times

---

**Copyright Minami•Tateya Collaboration**

*J. Fujimoto, et al. LEPII Physics, WM-Generator, CERN 1996.*

*Minami•Tateya Collaboration, KEK, JAPAN*

*E-mail: scr@minami.kek.jp*

*Last date of change: 2 Feb 1996*

*Version 1.0 (2)*

*Spring Version 5.1*

*coded by S. Kawabata KEK, March 1994*
*** Number of trials to get an event ***

<table>
<thead>
<tr>
<th>x</th>
<th>Lg(dN/dx)</th>
<th>dN/dx</th>
<th>e+</th>
<th>e+</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**Total = 10056 events**

*** Contents of the histogram Header ******

(1) Actual Buffer size = 9269 Words

(2) Contents of Histograms

<table>
<thead>
<tr>
<th>Max. No. of Histograms</th>
<th>Number of Histograms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ID</th>
<th>X_min</th>
<th>X_max</th>
<th>X_bin</th>
<th>Hash Hst#</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0000</td>
<td>1.9000</td>
<td>E+02</td>
<td>50 2 2 1</td>
</tr>
<tr>
<td>3</td>
<td>0.0000</td>
<td>1.9000</td>
<td>E+02</td>
<td>50 3 2 2</td>
</tr>
<tr>
<td>5</td>
<td>0.0000</td>
<td>1.9000</td>
<td>E+02</td>
<td>50 6 1 3</td>
</tr>
<tr>
<td>6</td>
<td>0.0000</td>
<td>1.9000</td>
<td>E+02</td>
<td>50 6 1 4</td>
</tr>
<tr>
<td>8</td>
<td>0.0000</td>
<td>1.9000</td>
<td>E+02</td>
<td>50 9 1 8</td>
</tr>
<tr>
<td>9</td>
<td>0.0000</td>
<td>1.9000</td>
<td>E+02</td>
<td>50 10 1 9</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
<td>1.9000</td>
<td>E+02</td>
<td>50 11 1 10</td>
</tr>
</tbody>
</table>

(3) Contents of Scatter Plots

<table>
<thead>
<tr>
<th>Max. No. of Scat_Plots</th>
<th>Number of Scat_Plots</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0</td>
</tr>
</tbody>
</table>

---

**O** : No. of Events in Log. scale.
<table>
<thead>
<tr>
<th>x</th>
<th>d(Sig/dx)</th>
<th>dN/dx</th>
<th>1.E-06</th>
<th>1.E-05</th>
<th>1.E-04</th>
<th>1.E-03</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1.0E-06</td>
<td>1.0E-05</td>
<td>1.0E-04</td>
<td>1.0E-03</td>
</tr>
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<td>1.0E-06</td>
<td>1.0E-05</td>
<td>1.0E-04</td>
<td>1.0E-03</td>
</tr>
<tr>
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<td>1.0E-06</td>
<td>1.0E-05</td>
<td>1.0E-04</td>
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<td>1.0E-04</td>
<td>1.0E-03</td>
</tr>
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<td>1.998E-01</td>
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<td>1.0E-06</td>
<td>1.0E-05</td>
<td>1.0E-04</td>
<td>1.0E-03</td>
</tr>
<tr>
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<td>1.0</td>
<td>1.0E-06</td>
<td>1.0E-05</td>
<td>1.0E-04</td>
<td>1.0E-03</td>
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<td>1.0E-05</td>
<td>1.0E-04</td>
<td>1.0E-03</td>
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<td>1.0</td>
<td>1.0E-06</td>
<td>1.0E-05</td>
<td>1.0E-04</td>
<td>1.0E-03</td>
</tr>
<tr>
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<td>1.0</td>
<td>1.0E-06</td>
<td>1.0E-05</td>
<td>1.0E-04</td>
<td>1.0E-03</td>
</tr>
<tr>
<td>0.96</td>
<td>2.295E-01</td>
<td>1.0</td>
<td>1.0E-06</td>
<td>1.0E-05</td>
<td>1.0E-04</td>
<td>1.0E-03</td>
</tr>
<tr>
<td>0.95</td>
<td>2.356E-01</td>
<td>1.0</td>
<td>1.0E-06</td>
<td>1.0E-05</td>
<td>1.0E-04</td>
<td>1.0E-03</td>
</tr>
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<td>1.0E-06</td>
<td>1.0E-05</td>
<td>1.0E-04</td>
<td>1.0E-03</td>
</tr>
<tr>
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<td>1.0E-04</td>
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