Anomalous Chromomagnetic Moments of Quarks 
and Large Transverse Energy Jets

Dennis Silverman

Department of Physics and Astronomy 

University of California, Irvine 

Irvine, CA 92717-4575 

(June 7, 1996)

Abstract

We consider the jet cross sections for gluons coupling to quarks with an anomalous chromomagnetic moment. We then apply this to the deviation and bounds from QCD found in the CDF and D0 Fermilab data, respectively, to find a range of possible values for the anomalous moments. The quadratic and quartic terms in the anomalous moments can fit to the rise of a deviation with transverse energy. Since previous analyses have been done on the top quark total cross section, here we assume the same moment on all quarks except the top and find the range $|\kappa'| \equiv |\kappa/(2m_q)| = 1.0 \pm 0.3$ TeV$^{-1}$ for the CDF data. Assuming the anomalous moment is present only on a charm or bottom quark which is pair produced results in a range $|\kappa'_{b,c}| = 3.5\pm1.0$ TeV$^{-1}$. The magnitudes here are compared with anomalous magnetic moments that could account for $R_b$ and found to be in the same general range, as well as not inconsistent with LEP and SLD bounds on $\Delta \Gamma_{\text{had}}$. 
There are several higher dimensional operators that can be used to evaluate structure from a theory beyond the standard model [1]. In this paper we use the time-honored anomalous magnetic moment as the indicator [2–5], applied to the quark-gluon vertex, or the anomalous chromomagnetic moment. Several papers have been written analyzing the contribution of such a moment to the top production cross section [6–10]. In this paper, we assume such a moment could apply to any or all of the mainly annihilated, scattered or produced quarks other than the top quark. The results of this will be different from those of the four-Fermion contact interaction operators, and should also be included in evaluations of possible structure. Along with the exchange of a spin 3/2 excited quark state [11], the $E_T$ dependence and angular two-jet behavior could help further limit or establish a composite mechanism, or new interactions from higher mass scales. The same theory that gives an anomalous chromomagnetic moment may also give a four-Fermion contact interaction, but here we only analyse the anomalous moment mechanism in a general context. For a complete analysis of all color current contributions in a particular model, we refer to the recent analysis of the Minimal Supersymmetric Standard Model [12]. The key point in our analysis is that the anomalous moment term $\kappa'\sigma_{\mu\nu}q^\nu$, when compared to the Dirac current $\gamma_\mu$, grows at high $E_T$ as $O(\kappa'E_T)$.

In the top production papers, gluon fusion or quark fusion to a virtual gluon are calculated to produce the $t-\bar{t}$ pair. Here we include all quark gluon hadronic processes since quarks commonly present in protons and antiprotons could have the small anomalous moments.

Since next-to-leading order QCD corrections have not been calculated for this general set of anomalous moment processes, to compare with the data we follow the CDF [13,14] procedure of calculating the ratio of the theory with structure divided by lowest order QCD and comparing it with the ratio of data divided by NLO QCD. We find that an anomalous chromomagnetic coefficient $|\kappa'| = |\kappa/(2m_q)| = 1.0 \, \text{TeV}^{-1}$ fits the CDF rise, and would be
roughly upper bounded by 1.3 TeV$^{-1}$, and lower bounded by 0.7 TeV$^{-1}$. The central value for the rise in CDF is not directly supported by the D0 [15] data, but is within the one sigma systematic energy calibration error curve for D0, which rises to 120% at $E_T = 450$ GeV.

We use $\kappa'$ since in the general case the internal diagram or dynamics might not involve the light external quark, and a new physics model calculation will give $\kappa'$ directly. The use of the breakup into a vector current ($\gamma_\mu$) and an anomalous chromomagnetic moment term ($i\kappa'\sigma_{\mu\nu}$) includes all anomalous moment vertex corrections in $\kappa'$ including those of QCD. However, at the very large momentum transfers we are considering here, there are form factors on the QCD vertex corrections making up the anomalous chromomagnetic moment either for virtual gluons or for high $E_T$ virtual quarks (“sidewise form factors” [16]) which will damp like ($\kappa'_{\text{QCD}} \approx \mathcal{O}(m_q/p_{T}^2)$) and become irrelevant. At some $q^2$ the anomalous moment from new or composite interactions will also evidence a form factor. That is automatically included in the analysis by considering $\kappa'(q^2)$ a function of $q^2$, but in the comparison to the data we do not need to invoke that dependence yet. We test whether an anomalous magnetic moment equal to the anomalous chromomagnetic moment possibly indicated here would be in conflict with the $\Gamma_{\text{had}}$ accuracy at LEP, and find it would not be.

Due to discrepancies in the total hadronic cross sections for charm and bottom production at LEP and SLD, $R_c$ and $R_b$, we also find a separate range for either the charm or bottom quark only having an anomalous chromomagnetic moment, using the quark-antiquark production cross sections. The range for the CDF data is $|\kappa'_{b,c}| = 3.5 \pm 1$ TeV$^{-1}$. We note that if the $R_b$ discrepancy is accounted for by an anomalous magnetic moment, it is the same order of magnitude as the anomalous chromomagnetic moment found here. $A^b_{FB}$ is not inconsistent with this interpretation and rules out one of two possible anomalous magnetic moment values. For comparison with results from the top quark total cross section from Ref. [8], we note from their Fig. 3, using present CDF and D0 data on $\sigma_{\text{top}}$, that $0 \leq \kappa^g_t \leq 0.35$. This corresponds to $0 \leq \kappa'^b_t \leq 1$ TeV$^{-1}$, and is better than the inclusive jet limits for the $b$ or $c$ quark alone calculated here.
II. ANOMALOUS CHROMOMAGNETIC MOMENT CROSS SECTIONS

The top gluon fusion production $[7, 9, 10]$ with anomalous chromomagnetic moments has already been calculated. We use the simplifications of these for the case of zero quark mass in the various channels in which quark-quark-gluon-gluon processes occur. For completeness we repeat these formulas here for the massless case along with the massless QCD contribution. We also present here the result with anomalous chromomagnetic moments for quark-antiquark annihilation and scattering to the same quark-antiquark with interference, and its crossed quark-quark scattering. We then present the cross sections for different quarks scattering and annihilating. The cross sections are even in powers of $\kappa'$ due to the zero quark mass limit being used. This is because at zero mass only gamma matrices occur in the QCD trace and substitution of an anomalous moment $\sigma_{\mu\nu}$ term for a $\gamma_{\mu}$ would give an odd number of gamma matrices and resulting in a zero trace, unless accompanied by an additional $\sigma_{\mu\nu}$ term. In other terms, at zero quark mass, there is no interference between helicity conserving and helicity flip processes: helicity conserving ones have either no anomalous moment terms or two such terms; helicity flip ones have one anomalous moment for each helicity flipped quark line, and this gets squared in the cross section. Neglect of the mass in a propagator term could give an error of order $m_b/E_T \approx 2.3\%$ in the amplitude at $E_T = 200$ GeV, or 4.5% in the cross section.

A. Gluon Processes

The well known result $[17]$ for gluon-gluon scattering is

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{\pi\alpha_s^2}{s^2} \frac{9}{2} \left( 3 - \frac{\hat{t}\hat{u}}{s^2} - \frac{\hat{s}\hat{t}}{t^2} - \frac{\hat{s}\hat{t}}{u^2} \right)$$

(1)

For quark-antiquark to gluon-gluon $[18, 17, 7, 9]$ the cross section is

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{\pi\alpha_s^2}{s^2} \frac{16}{72} \left( \frac{8}{3} \left( \frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + 12 \frac{\hat{t}\hat{u}}{s^2} - 6 \right) + \frac{28}{3} \kappa'^2 \hat{s} + \frac{16}{3} \kappa'^4 \hat{t}\hat{u} \right)$$

(2)
For gluon-gluon to quark-antiquark we use the above replacing the coefficient 16/72 by 16/256, for either the quark or antiquark being observed.

For gluon-quark to gluon-quark with the gluon observed (or gluon-antiquark to gluon-antiquark) the cross section is

\[
\frac{d\hat{\sigma}}{d\hat{t}} = -\frac{\pi\alpha_s^2}{\hat{s}^2} \frac{16}{96} \left( \frac{8}{3} \hat{t} + \frac{\hat{u} \hat{s}}{\hat{s}^2} + 12 \frac{\hat{u} \hat{s}}{\hat{t}^2} - 6 \right) + \frac{28}{3} \kappa'^2 \hat{t} + \frac{16}{3} \kappa'^4 \hat{u} \hat{s} \right) \]  

(3)

For gluon-quark to gluon-quark with the quark observed (or gluon-antiquark to gluon-antiquark) we interchange \( \hat{t} \) with \( \hat{u} \) in the above equation.

**B. Quark Processes**

For quark-antiquark to the same quark-antiquark including t channel exchange as well as s channel annihilation with their interference, and observing the quark we have

\[
\frac{d\hat{\sigma}}{d\hat{t}} = \frac{\pi\alpha_s^2}{\hat{s}^2} \frac{16}{36} \left( \frac{\hat{u}^2 + \hat{t}^2}{\hat{s}^2} + \frac{(\hat{s}^2 + \hat{u}^2)}{\hat{t}^2} - \frac{2}{3} \frac{\hat{u}^2}{\hat{s} \hat{t}} \right) + \kappa'^2 \left( 4 \hat{s} \hat{t} \frac{1}{\hat{s}^2} + \frac{1}{\hat{t}^2} \right) + \frac{1}{3} \hat{u} (9 + \frac{\hat{s}}{\hat{t}} + \frac{\hat{t}}{\hat{s}}) \right) 
\]

\[+ \frac{1}{2} \kappa'^4 \left( (\hat{t} - \hat{u})^2 + (\hat{s} - \hat{u})^2 - \frac{1}{3} (\hat{t} - \hat{u}) (\hat{s} - \hat{u}) \right) \]  

(4)

For the above process observing the antiquark, we interchange \( \hat{t} \) and \( \hat{u} \) in the above formula.

For identical quarks scattering to the same identical quarks, we interchange \( \hat{s} \) and \( \hat{u} \) in the above equation, and add a factor of 1/2 for the identical final state quarks.

For a quark-antiquark annihilation to a virtual gluon creating a different quark-antiquark pair, as in charm and bottom production, summing over identical cross-sections for either quark or antiquark observed (which will later be multiplied by 4 for four other light quarks being created) we have for the s-channel gluon exchange

\[
\frac{d\hat{\sigma}}{d\hat{t}} = \frac{\pi\alpha_s^2}{\hat{s}^2} \frac{16}{36} \left( \frac{(\hat{u}^2 + \hat{t}^2)}{\hat{s}^2} + 4 \kappa'^2 \hat{s} \hat{u} \frac{1}{\hat{s}^2} + \frac{1}{2} \kappa'^4 (\hat{t} - \hat{u})^2 \right) \]  

(5)

For \( q + \bar{q}' \to q + \bar{q}' \) observing the final quark we have for the t-channel gluon exchange
\begin{align}
\frac{d\sigma}{dt} &= \frac{\pi\alpha_s^2}{s^2} \frac{16}{36} \left( \frac{(\hat{s}^2 + \hat{u}^2)}{t^2} + 4\kappa'^2 \hat{s} \hat{u} 1 + \frac{1}{2} \kappa'^4 (\hat{s} - \hat{u})^2 \right) \tag{6}
\end{align}

For observing the final antiquark, we use the above equation with \( \hat{t} \) and \( \hat{u} \) exchanged. For \( q + q' \) to \( q + q' \) observing the quark we use the above formula and interchange \( \hat{s} \) and \( \hat{u} \) in the large parenthesis. For \( q + q' \) to \( q + q' \) observing \( q' \), we interchange \( \hat{t} \) and \( \hat{u} \) in the large parenthesis of the above formula.

**III. COMPARISON WITH POSSIBLE ANOMALOUS MAGNETIC MOMENTS IN Z COUPLINGS**

The possibility of anomalous magnetic moments of quarks appearing at LEP and SLD at the Z peak and their forward-backward asymmetry has been considered [4,8]. Here we just get values to indicate the order of magnitude, referring the reader to the more careful and complete analyses by T.G. Rizzo [8], which gives \(-0.012 \leq \kappa_b^e \leq -0.002 \) at 95% CL, updated to Moriond '96 data [19]. With an anomalous electric dipole moment as well, it allows positive \( \kappa_b^e \leq 0.025 \). Using the integrated cross-section [4,8] for \( b \) production at the Z peak we find with \( m_b = 4.5 \text{ GeV} \) (and with \( F_2' = \kappa_b^e \))

\begin{align}
\sigma &\propto (v_b^2 + a_b^2 + 3v_b\kappa_b^e + m_b^2\kappa_b^{e2}/(8m_b^2)) \tag{7} \\
\sigma &\propto (0.365 - 1.03\kappa_b^e + 51.1\kappa_b^{e2}) \tag{8}
\end{align}

Letting the \( \kappa_b^e \) terms account for the discrepancy [20] \( \Delta R_b/R_b = 0.026 \) gives two solutions, \( \kappa_b^e = 0.027 \), and \( \kappa_b^e = -0.0069 \), corresponding to \( \kappa_b^e = 3.0 \text{ TeV}^{-1} \) and \( \kappa_b^e = -0.77 \text{ TeV}^{-1} \).

\( A_{FB}^b \) isolates the cos \( \theta \) term in the Z cross-section which can be partly due to the anomalous magnetic moment [4,8] \( (A_b^{SM} \equiv 2v_ba_b/(v_b^2 + a_b^2)) \)

\begin{align}
A_{FB}^b &= \frac{3}{4} A_b^{SM} A_e (1 + \kappa_b^e/g_V)/(1 + \Delta R_b/R_b) \tag{9}
\end{align}

where the inverse power term has been taken here to match \( 1 + \Delta R_b/R_b \). Using the SM value for \( A_b^{SM} \), the positive \( \kappa_b^e = 0.027 \) gives a 3.1\( \sigma \) discrepancy with [20] \( A_{FB}^b = 0.1002 \pm 0.0028 \), and is eliminated. The negative \( \kappa_b^e = -0.0069 \) \( (\kappa_b^e = -0.77 \text{ TeV}^{-1}) \) value only gives a 0.4\( \sigma \)
deviation and is consistent. The error on $A_b = A_b^{\text{SM}}(1 + \kappa_b^3/v_b)/\left(1 + \Delta R_b/R_b\right)$ is about 6%, and it is 11% below theory. The correction of the anomalous moment only lowers theory by 0.5%.

Our calculated CDF jet cross-sections do not depend on the sign of $\kappa'^g$, and for the case of only the $b$ quark having the anomalous chromomagnetic moment $|\kappa_b'^g| = 3.5 \pm 1 \text{ TeV}^{-1}$ is not inconsistent with the $\Delta R_b$ anomalous magnetic moment since there will be a numerical factor depending on how the electric and color charges are distributed among the internal constituents with different masses.

For the case where all quarks except top are given the anomalous chromomagnetic moment, allowing an equal anomalous magnetic moment for each, we find changes in $\Delta \Gamma_{\text{had}}/\Gamma_{\text{had}}$ of 0.0017, 0.0005, and -0.0005, corresponding to the $\kappa' = 1.3$, 1.0, and 0.7 TeV$^{-1}$ cases, respectively. This is at most a one $\sigma$ discrepancy since the fractional error [20] on $\Delta \Gamma_{\text{had}}/\Gamma_{\text{had}}$ is $\pm 0.0019$.

**IV. RESULTS AND CONCLUSIONS**

In calculating the jet transverse energy distributions, we have used the parton distribution functions MRSA' [21]. Since we are taking ratios of anomalous moment contributions to strict QCD, both in leading order, most variance with PDFs drops out, only being about 2% at $E_T = 450$ GeV among the MRS set of PDFs, or between MRSA' and CTEQ3M. Assuming all quarks except the top have the same anomalous chromomagnetic moment, we find in Fig. 1 that the range $|\kappa'| = 1.0 \pm 0.3 \text{ TeV}^{-1}$ will fit the CDF data [13,14], and be in the allowed systematic error range of the D0 data [15]. The quadratic and quartic terms in $\kappa'$ give a natural fit to the possible curvature in the data, and indicate why the corrections do not show up significantly until $E_T \geq 200$ GeV. It is not inconsistent with an equal anomalous magnetic moment being present for the quarks at the $Z$ peak.

If the charm or bottom quark alone among the lighter quarks possesses a sizeable anomalous chromomagnetic moment, we find in Fig. 2 the range $|\kappa'_{b,c}| = 3.5 \pm 1 \text{ TeV}^{-1}$ will fit the
CDF data, and be in the allowed range of the D0 data. It is not inconsistent with a comparable anomalous magnetic moment explanation [8] of $R_b$.

ACKNOWLEDGMENTS

The author thanks Prof. Myron Bander for help with the conceptual framework and numerous helpful discussions. We also thank W. Giele for supplying the structure function program. The author thanks the SLAC theory group for their hospitality and acknowledges discussions with S. Drell, S. Brodsky, L. Dixon, and T.G. Rizzo. This research was supported in part by the U.S. Department of Energy under Contract No. DE-FG03-91ER40679.
REFERENCES


FIGURES

FIG. 1. The ratio of the anomalous chromomagnetic moment contribution to the lowest order QCD jet distribution in transverse energy. The dashed, solid, and dotted curves are for $|\kappa'| = 1.3$, 1.0, and 0.7 TeV$^{-1}$, respectively. The crossed data are from CDF Run 1a, and the circle data from Run 1b (preliminary).

FIG. 2. The ratio of the anomalous chromomagnetic moment contribution of either a bottom or charm quark-antiquark pair created to the lowest order QCD jet distribution in transverse energy. The dashed, solid, and dotted curves are for $|\kappa'_{b,c}| = 4.5$, 3.5, and 2.5 TeV$^{-1}$, respectively. The data are as in Fig. 1.