New Supernova Constraints on Sterile Neutrino Production

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Abstract

We consider the possibility that a light, sterile-neutrino species \( \nu_S \) can be produced by \( \nu_e \) scattering during the cooling of a proto-neutron star. If we parameterize the sterile-neutrino production cross section by a parameter \( A \) as

\[
\sigma(\nu_e X \rightarrow \nu_S X) = A \sigma(\nu_e X \rightarrow \nu_e X),
\]

where \( X \) is an electron, neutron, or proton, we show that \( A \) is constrained by limits to the conversion of \( \nu_e \) to \( \nu_S \) in the region between the sterile-neutrino trapping region and the electron-neutrino trapping region. This consideration excludes values of \( A \) in the range \( 10^{-4} \lesssim A \lesssim 10^{-1} \).
The possibility that the solar-neutrino problem [1] may be solved via the oscillation of electron neutrinos to “sterile” neutrinos (so named because they have superweak interaction with the $W$ and the $Z$ and thus avoid the LEP bound on the number of neutrinos) has been widely discussed in literature. The most compelling case for sterile neutrinos [2] arises when one tries to solve simultaneously the solar-neutrino problem and the atmospheric-neutrino deficit, as well as accommodating either (or both) the reported $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations at LSND [3] or the idea that neutrinos constitute about 20% of the total matter content of the universe [4]. In the sterile-neutrino hypothesis the solar-neutrino deficit is resolved via MSW oscillations between $\nu_e$ and $\nu_S$.

In constructing realistic gauge models [2, 5, 6] that lead to the mixings between the electron neutrino and the sterile neutrino one generally introduces various new interactions which can lead to the desired mixing without conflicting with known low-energy constraints as well as cosmological and astrophysical ones [7]. A well known astrophysical phenomenon that leads to strong constraints on the static properties of the neutrino is the dynamics of supernovae inferred from the neutrino signal from supernova 1987A (SN1987A) observed by the underground detectors of the IMB and Kamiokande collaborations [8]. Two classes of restrictions on sterile neutrino properties may be obtained. One is on $\nu_e$ oscillating into $\nu_S$, thereby depleting the $\nu_e$ signal and contradicting observations. This possibility has been analysed by Kainulainen et al. [9], who showed that the high density in the supernova suppresses such oscillations for the range of masses and mixing angles needed to solve the solar neutrino problem. The other class of constraints may arise in models where there exist direct interactions of electron neutrinos with visible particles such as $e, p, n, \nu_e, \mu, \tau$ that can convert a $\nu_e$ into a $\nu_S$. This can also potentially deplete the $\nu_e$ luminosity. It is this class of effects that we discuss in this letter. We will also
apply our techniques to restrict the mixing between the photon and a hypothetical mirror (or para) photon.

The observed neutrino signal from SN1987A appears to in agreement with expectations from the standard picture of type II supernovae [10], with neutrino interactions of the standard model of particle physics. Any new interaction of the neutrino will therefore be constrained by these observations. Some examples of constraints already discussed in the literature are limits on the magnetic moment [11], the strength of right-handed interactions [12], and the magnitude of the Dirac mass [13] of the neutrinos [14]. Similar considerations can be applied to new sterile-neutrino interactions. In this paper we study limits to the production of sterile neutrinos in electron-neutrino collisions with normal matter.

Before starting we emphasize that there are two different ways of producing sterile neutrinos in the supernova: (i) $\nu_e-\nu_S$ mixing, which can convert electron neutrinos already in the supernova to sterile neutrinos via oscillations; and (ii) direct production of sterile neutrinos in the electron neutrino collisions with matter in the supernova. We will be concerned only with the second one since the first effect has been shown by Kainulainen et al. [9] to be unimportant for our range of parameters due to MSW suppression.

A simplified model of neutrino interactions in the proto-neutron star will be adequate for our purposes. We assume that the core consists of a sphere $10^6$ cm in radius at a constant temperature of about 10 MeV, and density approximately equal to that of nuclear matter, $\rho = 3 \times 10^{14}$ g cm$^{-3}$. The neutrino scattering cross section is roughly $\sigma_{ee} \simeq G_F^2 E_\nu^2$. Using $E_\nu = 3T$, the mean-free-path of the neutrino is about $\lambda_e \sim 10^2$ cm. This means that the neutrino random walks out of the core, taking on

\[ \text{Note:} \] A matter of notation: by $\sigma_{ij}$ we mean the cross section for $\nu_i + X \rightarrow \nu_j + X$, where $X$ is a normal matter particle. For example, $\sigma_{ee}$ is the cross section for $\nu_e + X \rightarrow \nu_e + X$, while $\sigma_{eS}$ is the cross section for $\nu_e X \rightarrow \nu S X$. 

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average \((R_C/\lambda_e)^2 \sim 10^8\) steps. So a typical neutrino travels \(10^8\lambda_e \sim 10^{10}\) cm through the core, requiring about a second.

Since electron neutrinos are trapped, effectively they are emitted from a neutrinosphere, analogous to the familiar photosphere, where the optical depth for a neutrino travelling out of the core is unity. Observations by IMB and KII of neutrinos from SN1987A verify the prediction of Colgate and White [15] that almost all the binding energy of the neutron star is emitted in the form of neutrinos. Furthermore, a fair fraction must have been in the form of \(\nu_e\bar{\nu}_e\) pairs. Thus, if there are additional weakly interacting particles produced under the conditions of the proto-neutron star, they cannot modify the fact that a significant fraction of the binding energy must be radiated in the form of \(\nu_e\bar{\nu}_e\) pairs. For instance, if the energy loss due to a new hypothetical particle were too rapid, then the core would cool too rapidly without emitting the observed neutrinos, leading to a conflict with observation. Or if a process somehow prevented \(\nu_e\bar{\nu}_e\) emission, then that process would be disallowed.

If sterile neutrinos are produced in the core, then in order that they not carry away a disproportionate share of the binding energy, we must either require that they are hard to produce, or else require that it be difficult for them to escape. Since the sterile-neutrino mean-free-path is \(A^{-1}\) larger than that for the electron neutrino, trapping will obtain for \(A > 10^{-4}\).

Let’s first examine the situation where the sterile neutrinos are trapped. If they are trapped and form their own neutrinosphere, the ratio of the electron-neutrino luminosity to the sterile-neutrino luminosity would be \(r_{\nu} \equiv \mathcal{L}(\nu_e)/\mathcal{L}(\nu_S) = R_e^2 T_e^4/R_S^2 T_S^4\) where \(R_e, T_e\) and \(R_S, T_S\) are the radius and temperature of the electron-neutrino (sterile-neutrino) neutrinosphere. If the sterile neutrinosphere is deep in the core the temperature will be higher, but let’s assume for a moment that
the temperature is the same as the electron-neutrinosphere. From the universality of the weak interactions we know that $A$ must be much less than unity in realistic models [2, 5, 6], e.g., the effective Fermi constant for sterile neutrinos must be smaller than $G_F$ from the fact that the sterile neutrino interactions generally contribute additional modes to muon and tau lepton decays. Since we expect $A < 1$ from low-energy weak-interaction data, we would have $R_S \lesssim R_e$. Therefore, so long as sterile neutrinos are trapped, the emission from the sterile neutrinosphere will be less than that from the electron neutrinosphere. That is, if $10^{-4} \lesssim A$, then neutrinos will be trapped, and radiation from the sterile neutrinosphere will result in a sterile-neutrino luminosity less than the electron-neutrino luminosity. The point of this paper, however, is to observe that the sterile-neutrinosphere would not be the dominant source of sterile neutrinos.

Now consider the possibility that sterile-neutrino interactions are so feeble that they are not trapped. Then one must limit the production of the sterile neutrinos. This results in $A \lesssim 10^{-10}$. An easy way to see the origin of this bound is to note that the sterile neutrino luminosity $\mathcal{L}(\nu_S)$ in the non-trapped case is directly given by the total number of $\nu_S$’s produced in the supernova core times the average neutrino energy. This is given by

$$\mathcal{L}(\nu_S) \simeq n_e n_{\nu_e} A \sigma_{ee} V \langle E \rangle,$$  

where $n_i$ represent the number density of relevant particles and $V$ is the volume of the supernova core. Using $n_i \simeq 10^{38} \text{cm}^{-3}$ and $E = 3T$ with $T \simeq 50$ MeV, and demanding that $\mathcal{L}(\nu_S) \leq 10^{53} \text{erg s}^{-1}$, we obtain $A \lesssim 10^{-10}$.

So to review the standard analysis, values of $A$ in the range $10^{-4} \lesssim A$ result in a sufficiently small luminosity from the sterile neutrinosphere because of the small trapping radius. Values of $A$ in the range $10^{-4}$ to $10^{-10}$ are not allowed because in this regime the sterile neutrinos free stream and have a sufficiently large production.
cross section to be dangerous. Thus, the usual supernova analysis allowed regions for $A$ are $10^{-4} \lesssim A$, and $A \lesssim 10^{-10}$.

We, however, find that if the new interactions that convert $\nu_e$'s to $\nu_S$'s are strong enough to satisfy the trapping criteria, then a new consideration appears to lead to more stringent limits on the strength of these interactions than one would obtain using the familiar arguments discussed above [14]. This new consideration will exclude $A \gtrsim 10^{-4}$, so the final result will be that the only allowed range of $A$ is $A \lesssim 10^{-10}$.

Consider the trapped sterile-neutrino scenario in the "flat-star" approximation [16], i.e., as a one-dimensional problem. We know that the sterile neutrinosphere is well within the electron neutrinosphere. Consider the fate of an electron neutrino between the two neutrinospheres. Let $n$ be the density of scatterers $(e, n, p, \nu_{e,\mu,\tau})$ and $\sigma_{ij}$ the cross section for $\nu_i$ scattering into $\nu_j$ as before. We assume that, for the sterile $\nu_S$, $\sigma_{SS}$ is negligible. The $\nu_e$ and $\nu_S$ mean-free-paths are then given by

$$
\lambda_{ee} = 1/(n\sigma_{ee}); \quad \lambda_{eS} = 1/(n\sigma_{eS}) = \lambda_{ee}/A.
$$

(2)

Near $R$, the radius of the SN core, let $R_e = R - \lambda_{ee}$ and $R_S = R - \lambda_{Se}$. So long as $\lambda_{ee}$ and $\lambda_{Se}$ are much less than $R$, the relation

$$
\frac{dn_i}{dt} = -n_i c\sigma_{ij} + n_j c\sigma_{ji} \sim 0
$$

(3)

gives that $n_S \sim n_e$ at $R_S$. Thus $1/(2e)$ of neutrinos passing $R_S$ will exit promptly. For $R > R_S$, we will have that $\nu_e$ scattering into $\nu_S$ with $\nu_S$ exiting without further scattering depletes the number of $\nu_e$'s. A $\nu_e$ traveling a distance $\lambda_S$ without changing to a $\nu_S$ will have suffered $1/A^2$ scatterings. The square is because of the random-walk nature of the path. The chances of a $\nu_e$ surviving so many scatterings without changing to a $\nu_S$ are

$$
P(R_S) = [1 - A/(1 + A)]^{1/A^2} \simeq \exp[-1/A(1 + A)].
$$

(4)
As $A$ goes to zero, this result fails when $\lambda_{eS}$ approaches $R$ since the number of scatterings stops decreasing continuously below one; as $A$ approaches one it remains qualitatively correct. Thus, except for $A$ very close to one, essentially no $\nu_e$ survive the trip from $R_S$ to $R$. All exiting $\nu_e$ must be the result of either “local production” ($\nu_e$ absorption followed by $\nu_S$ emission can be considered incorporated in $\sigma_{eS}$), or “regeneration.” We now compute the fraction from regeneration.

For a $\nu_S$ approaching $R$, the chance of it scattering into a $\nu_e$ in a length $dx$ at a distance $x$ before $R$ is $dx/\lambda_{ee}$. The chance of the $\nu_e$ produced surviving the distance from $x$ to $R$ is

$$P(x) = \left[1 - A/(1 + A)\right]^{x^2/\lambda_{e_e}} \sim \exp[-x^2/(1 + A)\lambda_{ee}\lambda_{eS}]. \quad (5)$$

The fraction, $f$, of exiting $\nu_e$’s is then the product of these two probabilities summed over distances $x$:

$$f = \int P(x)dx/\lambda_{es} = \sqrt{\pi A(1 + A)} / 2 \sim \sqrt{A}. \quad (6)$$

Thus, if the coupling constant to the sterile neutrino is $1/3$ that to the electron neutrino ($A = 0.1$), only one-third of the exiting neutrinos will be electron neutrinos. As a result, the range in the parameter $A$ for which a “sterile” neutrino can be confined in a supernova and permit a reasonable number of $\nu_e$ to exit is limited to $A$ close to one, roughly $A > 0.1$.

There are several possible corrections to this result. They include: (i) production of $\nu_e$ by $\nu_\mu$ or $\nu_\tau$ pairs; and (ii) MSW [17] oscillations which may regenerate the electron neutrinos from the sterile neutrinos as they pass through the dense remainder of the neutrino sphere. As for possibility (i), one may show by arguments similar to those above that, because the $\nu_\mu$ and $\nu_\tau$ mean-free-paths are larger than that of $\nu_e$, they tend to decrease the $\nu_e$ flux as the $\nu_S$ does, but because neutrino densities are small compared to matter density near $R_S$, the effect is small. Let
us briefly comment on the second aspect. As the converted sterile neutrinos pass through the neutrino sphere (or what is left of it after they are produced), they experience a matter potential due to MSW effect which differs from those of the $\nu_e$ and $\nu_\mu$ as follows:

\[
\begin{align*}
V(\nu_S) &= 0 \\
V(\nu_e) &= V_0(3Y_e - 1 + 4Y_{\nu_e}) \\
V(\nu_\mu) &= V_0(Y_e - 1 + 2Y_{\nu_e}),
\end{align*}
\]

where $V_0 = 18$ eV ($\rho/5 \times 10^{14}$ g cm$^{-3}$), the factors of $Y$ represent the fraction of the corresponding species relative to the total number of nucleons [9], and $\rho$ is the density. The MSW resonance therefore may occur if the expression within the parentheses above vanishes. If it does vanish, one may have conversion of $\nu_S$ to $\nu_e$ if the latter are lighter. It is hard to estimate the precise number of the $\nu_S$’s that would be converted. And in any case, this will require accidental fine tuning of the particle densities at the right distance from the surface of the supernova.

Let us now discuss the implications of our result:

(i) In order that $\nu_e \bar{\nu}_e$ pairs be emitted from the supernova, if sterile neutrinos interact strongly enough to be trapped, the sterile neutrinosphere must be close to the electron neutrinosphere. This requires $A > \sim 10^{-1}$, which would be in conflict with what we know about the weak interactions. In particular, we know that $\nu_e^e$ neutral current scattering agrees with the standard-model result to a few percent. Setting a precise limit to $A$, however, would require a specific sterile-neutrino model. Thus, the true bound on $A$ is the upper bound derived from luminosity discussion in SN1987A, i.e., $A \lesssim 10^{-10}$.

(ii) Our result also has several implications for models incorporating sterile neutrinos that must be ultra light. Any model that has effective Four-Fermi interactions of $\nu_S$
with $e, \nu_e, \nu_\mu$, etc., with strength above $G_F \times 10^{-5}$, will be ruled out. It is interesting that the mirror model [5] for sterile neutrino is among the models that are consistent with the above constraints, since all interactions between the visible sector particles and the $\nu_S$'s in this model are Planck-scale suppressed. The two models [2] that use two-loop graphs to suppress the $m_{\nu_S}$'s generically involve larger couplings but use $A \simeq 10^{-10}$ so that they are also barely consistent with these constraints. On the other hand several models constructed to explain the 17 KeV neutrino had larger $\nu_s$-$e$ cross-sections with $A$ in the range of $10^{-4}$ or so and are inconsistent with our improved supernova limit.

(iii) The above bound is independent of the mass of the sterile neutrino as long it is light enough to be produced in supernova temperatures (say, $m \lesssim 10$ MeV or so).

(iv) Another application of our bound is to the Dirac magnetic moment of the tau neutrino with mass in the MeV range, which has sometimes been considered in literature to be large so that it could be the dark matter of the universe [18] or have an effect on big-bang nucleosynthesis [19]. In this case, if the magnetic moment is larger than $10^{-8} \mu_B$, Giudice showed that the mean free path for $\nu_\tau$ will be less than that of $\nu_e$, and the $\nu_e$ neutrinosphere will be within the $\nu_\tau$ neutrinosphere. Hence, by Eq. (6), $\nu_\tau$ and $\bar{\nu}_\tau$ will tend to be converted to $\nu_e$ and $\bar{\nu}_e$ (as well as $\nu_\mu$ and $\bar{\nu}_\mu$) while traversing the region between the neutrinospheres. This would enhance the low energy $\nu_e$ and $\bar{\nu}_e$ signal in the underground detectors.

(v) A final implication of our discussion is that one could apply the techniques of our paper to constrain the mixing of of a new photon [20] with the known photon using the supernova luminosity information. The discussion is very similar to the case of neutrinos. Let us consider only the effect of photon scattering off electrons. If we denote the shadow photon as $\gamma'$ and the $\gamma - \gamma'$ mixing to be $\epsilon$, then the rate
of supernova cooling via $\gamma'$ emission per unit volume is given by:

$$Q_{\gamma'} = n_e \int \frac{dn}{d\omega} \sigma_{\gamma\gamma'} \omega \, d\omega.$$  \hspace{1cm} (8)

The cross section for $\gamma + e \longrightarrow \gamma' + e$ is just $\epsilon^2$ times the Compton cross section: $\sigma_{\gamma\gamma'} = \epsilon^2 \pi \alpha^2 / 2\omega^2$. Integrating over the photon spectrum, one finds for the total luminosity via the $\gamma'$ channel to be

$$Q_{\gamma'} = \frac{\alpha^2 \epsilon^2}{6\pi} n_e \int \frac{\omega d\omega}{e^{\omega/T} - 1} = \frac{\alpha^2 \epsilon^2 \pi}{36} n_e T^2.$$ \hspace{1cm} (9)

The supernova luminosity is obtained from this by multiplying the volume of the supernova. Assuming $T \simeq 50$ MeV as the temperature of the supernova core, and the number density of electrons in the supernova to be $1.5 \times 10^{38} \text{cm}^{-3}$, we get

$$Q_{\gamma'} V \simeq 10^{72} \epsilon^2 \text{ erg s}^{-1}.$$ \hspace{1cm} (10)

Demanding that this be less than $10^{53} \text{erg s}^{-1}$, we obtain the bound $\epsilon \leq 10^{-9.5}$. In principle in the supernova as well as the solar case, there would have been a region close to $\epsilon \geq 10^{-8}$ where trapping arguments would have said that shadow photon is allowed. However our discussion here can be applied to the supernova after breakaway when the first electromagnetic signals were observed\textsuperscript{2). It is simpler, however, just to consider the sun. If $\epsilon = 0.5$ or less, three quarters or more of the solar luminosity would be in para-photons ($\gamma'$). This essentially rules out the region $\epsilon \geq 10^{-9.5}$. We note that $\epsilon$ would exhibit itself as a minicharge of the same magnitude on mirror electrons and a recent experiment at SLAC \textsuperscript{21} has ruled out minicharged particles above charge $7 \times 10^{-5} e$.

In conclusion, from considerations of the electron neutrino luminosity from SN 1987A we have pointed out a new effect that considerably limits the allowed couplings of the sterile neutrinos to ordinary matter.

\textsuperscript{2}Similarly, Primakoff process conversion of photons to axions should lead to constraints, although necessarily new ones, on the axions.
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