Virtual Compton Scattering from the Proton and the Properties of Nucleon Excited States

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Abstract

We calculate the $N^*$ contributions to the generalized polarizabilities of the proton in virtual Compton scattering. The following nucleon excitations are included: $N^*(1535), N^*(1650), N^*(1520), N^*(1700), \Delta(1232), \Delta^*(1620)$ and $\Delta^*(1700)$. The relationship between nucleon structure parameters, $N^*$ properties and the generalized polarizabilities of the proton is illustrated.

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1. INTRODUCTION

The study of Virtual Compton Scattering (VCS), $e + p \rightarrow e' + p' + \gamma$, at CEBAF and MAMI (Audit et al. 1993) could provide valuable information on the structure of the nucleon, complementing the information obtained from elastic form factors, real Compton scattering, and deep inelastic scattering. In this paper we concentrate on the kinematic region where the final photon has low energy — i.e., below the threshold for $\pi^0$ production. As shown by Guichon et al. (1995) the low energy cross sections are parametrized by 10 generalized polarizabilities (GP), functions of the virtual photon mass. Their evaluation requires the knowledge of the nucleon excited states. This sensibility to the nucleon spectrum can provide substantial insight into the non-perturbative aspects of the QCD Hamiltonian.

Guichon et al. (1995) made an initial evaluation of the GP’s so as to provide an order of magnitude estimate of these new quantities and to illustrate their variation as a function of the virtual photon mass. In that calculation we neglected all recoil effects, that is terms which go like the velocity of the nucleon. As a result of that approximation only 7 GP’s were non-zero.

In this paper we extend the calculations to include the recoil corrections which turn out to contribute only when the final photon is magnetic. We also study the relationship of the nucleon excited states to the GP’s. We use the Non-Relativistic Quark Model (NRQM) to take advantage of its simplicity and the ready availability of its wave functions. Also the separation of the center of mass and internal motion greatly simplifies the calculation, making it analytically tractable. In principle, it is possible to use other wave functions but this can be prohibitively laborious and messy. The NRQM estimate should be a useful guide to the analysis of the soon available experimental data on the $p(e, e'p)\gamma$ reaction.

2. GENERAL FORMS OF GP’S IN TERMS OF CURRENT DENSITIES

We first briefly outline the formalism for the definition and the calculation of the GP’s. We refer to Guichon et al. (1995) for a detailed account of the problem as well as for the
notations and conventions. The hadronic tensor (see Figure 1) is defined by

\[ H_{NB}^{\mu \nu}(q'm', qm_s) = \int d\rho X \sum_{X \neq N} \left[ \langle N(p')|J^\mu(0)|X(p_X)\rangle \frac{\delta(p_X - p' - q')}{E_N(q') + q' - E_X(p_X)} \langle X(p_X)|J^\nu(0)|N(p)\rangle + \langle N(p')|J^\nu(0)|X(p_X)\rangle \frac{\delta(p_X - p + q')}{E_N(q') - q' - E_X(p_X)} \langle X(p_X)|J^\mu(0)|N(p)\rangle \right] + H_{seagull}^{\mu \nu}, \tag{1} \]

where \( J^\mu \) is the hadronic current, \( X \) the intermediate baryon excitations and \( H_{seagull}^{\mu \nu} \) the contact term generally required by gauge invariance. The reduced multipoles are then defined according to

\[ H_{NB}^{(\rho L', \rho L)S}(q', q) = \frac{1}{2S + 1} \sum_{m'_s m_s M'M} (-)^{\frac{1}{2} + m'_s + L + M} \langle \frac{1}{2} - m'_s, \frac{1}{2} m_s | S S \rangle \langle L' M' | L M | S s \rangle H_{NB}^{\rho L'M', \rho LM}(q'm'_s, qm_s). \tag{2} \]

with

\[ H_{NB}^{\rho L'M', \rho LM}(q'm'_s, qm_s) = (4\pi)^{-1} \int d\rho d\rho' V^*_{\rho L'M'}(\rho L'M', \rho') H_{NB}^{\mu \nu}(q'm'_s, qm_s) V_{\rho LM}(\rho LM, \rho'), \tag{3} \]

where \( V_{\rho LM}(\rho LM, \rho') \) are the charge (\( \rho = 0 \)), magnetic (\( \rho = 1 \)) and electric (\( \rho = 2 \)) basis vectors defined by Guichon et al. (1995).

When \( \rho, \rho' \) are equal to 0 or 1 the GP’s are defined by

\[ P^{(\rho L', \rho L)S}(q) = \left[ \frac{1}{q L' q L} H_{NB}^{(\rho L', \rho L)S}(q', q) \right]_{q' = 0}. \tag{4} \]

In the case of a virtual electric charge the analogous definition does not yield a GP with the usual photon limit as \( q \to 0 \). As explained by Guichon et al. (1995), one must therefore introduce mixed GP’s according to

\[ \hat{H}_{NB}^{\rho L'M', LM}(q'm'_s, qm_s) = (4\pi)^{-1} \int d\rho d\rho' V^*_{\rho L'M'}(\rho L'M', \rho') \sum_i H_{NB}^{\mu i}(q'm'_s, qm_s) \left( \gamma_{L+1}^{LM}(\rho') \right)^i, \tag{5} \]

\[ \hat{P}^{(\rho L', L)S}(q) = \left[ \frac{1}{q L' q L + 1} \hat{H}_{NB}^{(\rho L', L)S}(q', q) \right]_{q' = 0}, \tag{6} \]

where \( \gamma_{LM}(\rho) \) is the vector spherical harmonic. The 10 independent GP’s needed to describe the low energy regime are then the following,

\[ P^{(10,00)}_1, P^{(11,02)}_1, P^{(11,11)}_0, P^{(11,11)}_1, \hat{P}^{(11,2)}_1, \]

\[ P^{(10,00)}_0, P^{(10,02)}_0, P^{(11,11)}_0, P^{(11,11)}_1, \hat{P}^{(11,2)}_0. \]
In the low energy regime the following excited states contribute: $N^*(\frac{1}{2}^-, 1535)$, $N^*(\frac{1}{2}^-, 1650)$, $N^*(\frac{3}{2}^-, 1520)$, $N^*(\frac{3}{2}^-, 1700)$, $\Delta(\frac{3}{2}^+, 1232)$, $\Delta^*(\frac{1}{2}^-, 1620)$ and $\Delta^*(\frac{3}{2}^-, 1700)$.

In the NRQM the current density in Eqn.(1) takes the form

$$\langle X(p_X)|J^0(0)|N(p)\rangle = N_0 \rho_X(p_X - p)$$

and

$$\langle X(p_X)|J(0)|N(p)\rangle = N_0 \left[ \frac{P_X + P}{6m_q} \rho_X(p_X - p) + P_X(p_X - p) + \frac{i}{2m_q} \Sigma_X(p_X - p) \times (p_X - p) \right].$$

Here $\rho, P$ and $\Sigma$ are overlap integrals of current operators. If one takes into account the factorisation of the c.m. and internal baryon wave functions, they can be written in the following forms.

$$\rho_X(p_X - p) = \int dp d\lambda e^{-i \sqrt{\frac{2}{3}}(p_X - p) \cdot \lambda} \phi^\dagger_X(\rho, \lambda, J_X, M_X, T_X, \tau_X) \hat{Q} \phi_N(\rho, \lambda, m_N, \tau_N),$$

$$P_X(p_X - p) = \sqrt{\frac{2}{3}} \frac{1}{2m_q} \int dp d\lambda e^{-i \sqrt{\frac{2}{3}}(p_X - p) \cdot \lambda} \phi^\dagger_X(\rho, \lambda, J_X, M_X, T_X, \tau_X) (i \nabla_\lambda - i \nabla_\lambda) \hat{Q} \phi_N(\rho, \lambda, m_N, \tau_N),$$

$$\Sigma_X(p_X - p) = \int dp d\lambda e^{-i \sqrt{\frac{2}{3}}(p_X - p) \cdot \lambda} \phi^\dagger_X(\rho, \lambda, J_X, M_X, T_X, \tau_X) \sigma_3 \hat{Q} \phi_N(\rho, \lambda, m_N, \tau_N),$$

where $\hat{Q} = (\frac{1}{6} + \frac{\tau_3}{2})$ is the charge operator of the third quark and $\sigma_3$ is the twice the spin operator of the third quark.

With specific internal wave functions for the nucleon ($\phi_N$) and the intermediate excitations ($\phi_X$), one obtains explicit forms for the current density and hence the hadronic tensor.
3. CURRENT DENSITY AND HADRONIC TENSOR IN NRQM

To calculate the integrals $\rho(p_X - p)$, $P(p_X - p)$, and $\Sigma(p_X - p)$ for intermediate states $X$ being $N^*(\frac{1}{2}^-, 1535)$, $N^*(\frac{1}{2}^-, 1650)$, $N^*(\frac{3}{2}^-, 1520)$, $N^*(\frac{3}{2}^-, 1700)$, $\Delta(\frac{3}{2}^+, 1232)$, $\Delta^*(\frac{1}{2}^-, 1620)$ and $\Delta^*(\frac{3}{2}^-, 1700)$, we use the wave functions from Isgur and Karl (1978). The calculation is straightforward though fairly lengthy. We arrive at the following expressions.

$$\rho_{N^{(2S)}}(q) = -\frac{i\sqrt{8\pi}}{9} \frac{q}{\alpha} e^{-q^2/6\alpha^2} \tau_N (-1)^{1/2} M_X \sqrt{2J_X + 1} \left( \begin{array}{cc} 1 & J_X \\ M_X - m_N & m_N \end{array} \right) Y^*_{1M_X - m_N}(\hat{q}).$$

(12)

$$P_{N^{(2S)}}(q) = -\frac{i\sqrt{6}}{9} \frac{\alpha}{m_q} e^{-q^2/6\alpha^2} \tau_N (-1)^{1/2} M_X \sqrt{2J_X + 1} \left( \begin{array}{cc} 1 & J_X \\ M_X - m_N & m_N \end{array} \right) e^*_{M_X - m_N}.$$  

(13)

$$\Sigma_{N^{(2S)}}(q) = -\frac{i\sqrt{\pi}}{18} \frac{q}{\alpha} e^{-q^2/6\alpha^2} (1 + 4\tau_N) \sum_{\mu} (-1)^{1/2} M_X \sqrt{2J_X + 1} \left( \begin{array}{cc} 1 & J_X \\ M_X - \mu & \mu \end{array} \right) Y^*_{1M_X - \mu}(\hat{q}) \langle \chi^\lambda_{\mu} | \sigma_3 | \chi^\lambda_{m_N} \rangle + Y^*_{1M_X - \mu}(\hat{q}) \langle \chi^\lambda_{\mu} | \sigma_3 | \chi^\lambda_{m_N} \rangle$$

(14)

$$\Sigma_{N^{(4S)}}(q) = -\frac{i\sqrt{\pi}}{18} \frac{q}{\alpha} e^{-q^2/6\alpha^2} (1 - 2\tau_N) \sum_{\mu} (-1)^{-1/2} M_X \sqrt{2J_X + 1} \left( \begin{array}{cc} 1 & J_X \\ M_X - \mu & \mu \end{array} \right) Y^*_{1M_X - \mu}(\hat{q}) \langle \chi^\lambda_{\mu} | \sigma_3 | \chi^\lambda_{m_N} \rangle$$

(15)

$$\Sigma_\Delta(q) = \frac{1}{3} e^{-q^2/6\alpha^2} \langle \chi^\lambda_{m_\Delta} | \sigma_3 | \chi^\lambda_{m_N} \rangle$$

(16)

$$\rho_{\Delta^*}(q) = \frac{1}{2\tau_N} \rho_{N^{(2S)}}(q)$$

(17)

$$P_{\Delta^*}(q) = \frac{1}{2\tau_N} P_{N^{(2S)}}(q)$$

(18)
where $\chi$ are spin wave functions, $\tau_n$ the isospin quantum number of the nucleon (i.e., $\pm \frac{1}{2}$ for $(p_n)$), and $e_m$ is the $m$th component of the spherical basis vectors. Eqns. (12-19) are for intermediate states with the same isospin quantum number as the proton.

The main characteristics of these integrals are summarized below:

a.) The $\Delta(1232)$ and the $4^8$ component of the $N^*$'s contribute only to $\Sigma(p_X - p)$.

b.) The $2^8$ component of the $N^*$'s, and the $\Delta^*$'s contribute to all $\rho(p_X - p)$, $P(p_X - p)$, and $\Sigma(p_X - p)$.

c.) For small $x = |p_X - p|$ the behavior of the integrals are $\Sigma_{\Delta}(x), P(x) \propto O(1)$, and $\Sigma_{N*,\Delta^*}(x), \rho(x) \propto O(x)$.

The leading term and the recoil term of the hadronic tensor in Eqn. (1) are separated in the following way. We work in the initial $N\gamma$ c.m. system so that $p = -q$. In the direct term of Eqn. (1), $p_X = 0$ and $q' = -p'$. So, aside from the energy denominator, the direct amplitude factorizes into a product of a $q$-dependent current density $J_{d,XN}(q)$ and a $q'$ dependent current density $J_{d,NX}(q')$, where

$$J_{d,XN}(q) = \langle X(p_X)|J(0)|N(p)\rangle_{direct} = N_0 \left[ \frac{-q}{6m_q} \rho_X(q) + P_X(q) + \frac{i}{2m_q} \Sigma_{X}(q) \times q \right],$$

$$J_{d,NX}(q') = \langle N(p')|J(0)|X(p_X)\rangle_{direct} = N_0 \left[ \frac{-q'}{6m_q} \rho_X^*(q') + P_X^*(q') - \frac{i}{2m_q} \Sigma_{X}(q') \times q' \right].$$

The current density in the cross term has a more complicated $q$ and $q'$ dependence. Because $p_X = -q - q'$, $p + p_X = -q' - 2q$ and $p + p_X' = -q - 2q'$, it involves terms depending upon both $q$ and $q'$. Let

$$\langle X(p_X)|J(0)|N(p)\rangle_{cross} = J_{c,XN}(q') + \delta J_{XN}(q, q'),$$

and

$$\langle N(p')|J(0)|X(p_X)\rangle_{cross} = J_{c,NX}(q) + \delta J_{NX}(q, q'),$$

then $J_{c,XN}(q')$, $J_{c,NX}(q)$, $\delta J_{XN}(q, q')$ and $\delta J_{NX}(q, q')$ are given by the following expressions:

$$J_{c,XN}(q') = N_0 \left[ \frac{-q'}{6m_q} \rho_X(-q') + P_X(-q') + \frac{i}{2m_q} \Sigma_{X}(-q') \times (-q') \right],$$
\[
J_{c,NX}(q) = N_0 \left[ \frac{-q}{6m_q} \rho_X^*(-q) + P_X^*(-q) - \frac{i}{2m_q} \Sigma_X^*(-q) \times (-q) \right],
\]

\[
\delta J_{XN}(q, q') = N_0 \frac{-q}{3m_q} \rho_X(-q'),
\]

\[
\delta J_{NX}(q, q') = N_0 \frac{-q'}{3m_q} \rho_X^*(-q).
\]

From Eqn.(8) we see that \(J^0\) depends only on \(q\) or \(q'\), so that \(\delta J^0 = 0\). We define the leading term of the hadronic tensor \(H_{NB}^{\mu\nu}(q'm_s', qm_s)\) by neglecting the terms depending on both \(q\) and \(q'\) in the cross term, and the \(q'\) dependence of \(E_X(q + q')\) in the energy denominator of the cross amplitude. That is,

\[
H_{NB-LEADING}^{\mu\nu}(q'm_s', qm_s) = \sum_{X \neq N} \left[ \frac{J_{d,NX}^{\mu}(q') J_{d,NX}^{\nu}(q)}{M - M_X} + J_{c,NX}^{\mu}(q') J_{c,NX}^{\nu}(q) \right] + H_{seagull}^{\mu\nu}.
\tag{20}
\]

The recoil term of \(H_{NB}^{\mu\nu}(q'm_s', qm_s)\) includes all the effects coming from terms depending on both \(q\), and \(q'\), and those arising from the expansion of the energy denominator of the cross term to order \(q'\). The recoil contribution contains terms to order \(q'\) and higher and can be written as follows,

\[
H_{NB-RECOIL}^{\mu\nu}(q'm_s', qm_s) = \sum_{X \neq N} \left[ \frac{J_{c,NX}^{\mu}(q') \delta J_{NX}^{\nu}(q, q') + \delta J_{NX}^{\mu}(q, q') J_{c,NX}^{\nu}(q)}{E(q) - E_X(q)} \right]
+ \frac{1}{[E(q) - E_X(q)]^2} \left( q' + \frac{q' \cdot q}{E_X(q)} \right) J_{c,NX}^{\mu}(q') J_{c,NX}^{\nu}(q).
\tag{21}
\]

To get the GP we need only keep terms to order \(q'\). So the recoil effects contribute only to GP’s with a magnetic final photon, \(\mu \neq 0\).

With Eqns.(20,21) for the leading and recoil terms, one can carry out the partial wave projection using the definitions of section 2. The partial wave projection poses no particular difficulty except the need for careful book-keeping. We therefore skip the details of the partial wave decomposition and give the final expressions for the GP’s in section 4.

4. GENERALIZED POLARIZABILITIES IN THE NRQM

Here we give the final results for the 10 GP’s in the NRQM. We also study the properties of the GP’s in relation to the parameters determining the nucleon structure in
the NRQM. Our aim is to find those properties of the nucleon structure to which the GP’s are most sensitive. This should help identify the most useful aspects of VCS in studying the nucleon structure.

The analytic expressions for the 10 GP’s can be written in the following form, where the leading contributions are the same as in Guichon et al. (1995). (Note, however, that the curve for \( P^{(01,1)S} \) in Guichon et al. has an error where the cross term was a factor of 2 too large. We correct it here. The overall sign for \( P^{(01,1)S} \) is also changed to conform with the definition for the electric virtual photon case.)

The GP’s are plotted in Figure 2.

4.1 Leading Contributions To The Generalized Polarizabilities

\[
P^{(0101)_S} = \frac{1}{2S + 1} \frac{1}{118} \frac{1}{\alpha^2} e^{-q^2/6\alpha^2} \sum_{x=N^*,\Delta^*} a_X^2 \left( \frac{Z_d^{S,Jx}}{M - M_X} + \frac{Z_c^{S,Jx}}{E(q) - E_X(q)} \right)
\]

(22)

\[
P^{(0112)_1} = \frac{1}{108} \sqrt{\frac{3}{5}} \frac{1}{m_q \alpha^2} e^{-q^2/6\alpha^2} \sum_{x=N^*,\Delta^*} a_X^2 \left( \frac{Z_{ad}^{2,S,Jx}}{2I_x} \right) \left( \frac{Z_{ad}^{2,S,Jx}}{M - M_X} - \frac{Z_{ac}^{2,S,Jx}}{E(q) - E_X(q)} \right)
\]

(23)

\[
P_{para}^{(1111)_S} = \frac{1}{2S + 1} \frac{4}{27} \frac{1}{m_q} e^{-q^2/6\alpha^2} \left( \frac{Z_\Delta^S}{M - M_\Delta} + \frac{Z_\Delta^S}{E(q) - E_\Delta(q)} \right)
\]

(24)

\[
P_{dia}^{(1111)_S} = \delta_S \frac{7\sqrt{6}}{54} \frac{1}{m_q \alpha^2} e^{-q^2/6\alpha^2}
\]

(25)

The mixed GP is the sum of two terms:

\[
\hat{P}^{(01,1)_S} = \hat{P}_F^{(01,1)_S} + \hat{P}_S^{(01,1)_S}
\]

(26)

\[
\hat{P}_F^{(01,1)_S} = \frac{1}{2S + 1} \frac{1}{118} \frac{1}{\alpha^2} e^{-q^2/6\alpha^2} \sum_{x=N^*,\Delta^*} a_X^2 \left( \frac{Z_d^{S,Jx}}{M - M_X} - \frac{Z_c^{S,Jx}}{E(q) - E_X(q)} \right)
\]

(27)

\[
\hat{P}_S^{(01,1)_S} = \frac{1}{2S + 1} \frac{1}{36\sqrt{3}} \frac{1}{m_q \alpha^2} e^{-q^2/6\alpha^2} \sum_{x=N^*,\Delta^*} a_X^2 \left( \frac{Z_{ad}^{1,S,Jx}}{2I_x} \right) \left( \frac{Z_{ad}^{1,S,Jx}}{M - M_X} - \frac{Z_{ac}^{1,S,Jx}}{E(q) - E_X(q)} \right)
\]

(28)
Angular functions $Z^{S,J_X}$, $Z^{L,S,J_X}$ and $Z^{S}_{\Delta_{132}}$ in the above summation are given by:

<table>
<thead>
<tr>
<th>L</th>
<th>S</th>
<th>J_X</th>
<th>$Z^{S,J_X}_d$</th>
<th>$Z^{S,J_X}_c$</th>
<th>$Z^{L,S,J_X}_{ad}$</th>
<th>$Z^{L,S,J_X}_{ac}$</th>
<th>$Z^{S}<em>{\Delta</em>{132}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1/2</td>
<td>$\sqrt{2}/3$</td>
<td>$\sqrt{2}/3$</td>
<td>$-2/\sqrt{3}$</td>
<td>$2/\sqrt{3}$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>3/2</td>
<td>$2\sqrt{2}/3$</td>
<td>$2\sqrt{2}/3$</td>
<td>$2/\sqrt{3}$</td>
<td>$-2/\sqrt{3}$</td>
<td>$\sqrt{6}$</td>
</tr>
<tr>
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<td>1</td>
<td>1/2</td>
<td>2</td>
<td>$-2/\sqrt{2}$</td>
<td>$-2\sqrt{2}/3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3/2</td>
<td>$-2/\sqrt{2}$</td>
<td>$-2\sqrt{2}/3$</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3/2</td>
<td>$\sqrt{30}$</td>
<td>$\sqrt{30}/3$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.2 Recoil Contributions To Generalized Polarizabilities

$$P^{(1100)}_{recoil} = -\frac{1}{3\sqrt{3}} \frac{q^2}{m_q} e^{-q^2/6\alpha^2} \sum_{X=N^*,\Delta^*} a_X^2 Z^{J_X}_{1100} \frac{E_X(q)^2}{E_X(q)[E(q) - E_X(q)]^2} \left[ 1 - \frac{E_X(q)[E(q) - E_X(q)]}{3\alpha^2} \right]$$

(29)

$$P^{(1102)}_{recoil} = -\frac{1}{3\sqrt{3}} \frac{1}{m_q} e^{-q^2/6\alpha^2} \sum_{X=N^*,\Delta^*} a_X^2 Z^{J_X}_{1102} \frac{E_X(q)^2}{E_X(q)[E(q) - E_X(q)]^2} \left[ 1 - \frac{E_X(q)[E(q) - E_X(q)]}{3\alpha^2} \right]$$

(30)

$$P^{(1111)}_{recoil} = \frac{1}{3} \frac{\alpha^2}{m_q} e^{-q^2/6\alpha^2} \sum_{X=N^*,\Delta^*} a_X^2 Z^{J_X}_{1111} \frac{E_X(q)^2}{E_X(q)[E(q) - E_X(q)]^2}$$

(31)

$$P^{(1121)}_{recoil} = -\frac{1}{6\sqrt{15}} \frac{1}{m_q} e^{-q^2/6\alpha^2} \sum_{X=N^*,\Delta^*} a_X^2 Z^{J_X}_{1102} \frac{E_X(q)^2}{E_X(q)[E(q) - E_X(q)]^2} \left[ 1 - \frac{E_X(q)[E(q) - E_X(q)]}{3\alpha^2} \right]$$

(32)

where

<table>
<thead>
<tr>
<th>J_X</th>
<th>$Z^{J_X}_{1100}$</th>
<th>$Z^{J_X}_{1102}$</th>
<th>$Z^{J_X}_{1111}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>2/27</td>
<td>$\sqrt{2}/27$</td>
<td>3$\sqrt{6}/27$</td>
</tr>
<tr>
<td>3/2</td>
<td>$-2/27$</td>
<td>$-\sqrt{2}/27$</td>
<td>6$\sqrt{6}/27$</td>
</tr>
</tbody>
</table>

The $^4S_8$ component of the excited state wavefunctions contribute nothing to the proton GP’s, because of the isospin factor $(1 - 2\tau)$. However, they do contribute in the neutron case, which we do not study here. The parameters $m_q = 350$ MeV and $\alpha = 320$ MeV are taken from Isgur and Karl (1978). The P-wave intermediate states are ordered as in
the following table, together with their representation mixing parameters $a_X$ for the $^{28}O$ representation.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$N^*({\frac{1}{2}^-}, 1535)$</th>
<th>$N^*({\frac{1}{2}^-}, 1650)$</th>
<th>$N^*({\frac{3}{2}^-}, 1520)$</th>
<th>$N^*({\frac{3}{2}^-}, 1700)$</th>
<th>$\Delta^*({\frac{1}{2}^-}, 1620)$</th>
<th>$\Delta^*({\frac{3}{2}^-}, 1700)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_X$</td>
<td>0.85</td>
<td>-0.53</td>
<td>0.99</td>
<td>0.11</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
5. *N*\(^\ast\) PROPERTIES AND THE GP’S

The NRQM parameters are well determined by fitting the static properties of baryons (Isgur and Karl, 1978). Nonetheless, there are other phenomenological models than the NRQM, such as the bag models. Different models may not necessarily give exactly the same properties for the nucleon and its excitations. We do not intend to survey all the different nucleon models in this paper, but just to investigate those nucleon and *N*\(^\ast\) properties which exert the most important influence on the behavior of the GP’s in the NRQM. Two main factors are studied here: the mass spectrum of the nucleon and the size parameter \(\alpha\). The results are displayed in Figures 2, 3, and 4.

The GP’s have a strong dependence on the mass and energy spectrum of the excited states of the nucleon and the \(\Delta\). In Figure 2 we used the average masses of the *N*\(^\ast\) and \(\Delta^\ast\) from the particle data tables (1994). However, these masses are all determined within a range and may be different from the predictions of the NRQM. We study the effects of the *N*\(^\ast\) mass spectrum by comparing the GP’s calculated with the lower and upper limits of the masses from the particle data tables (1994) and also with those predicted in NRQM of Isgur and Karl (1978). Figure 3 shows that some GP’s are very sensitive to the *N*\(^\ast\) masses, particularly at small \(q\). The effect on \(\hat{P}^{(01,1)0}\) is especially large as compared with the theoretical masses of Isgur and Karl (1978). The change in \(P^{(11,00)1}\), \(P^{(11,02)1}\), and \(\hat{P}^{(11,2)1}\) are quite drastic due to a more complicated factor of the mass and energy differences.

The effects of the hadron size parameter \(\alpha\), are illustrated in Figure 4, where the plot is for a variation of \(\pm 5\%\) in \(\alpha\) from its normal value of 320 MeV. The main influences are again seen to be in the low \(q\) region.

In conclusion, we have presented a calculation of the *N*\(^\ast\) contribution to the generalized polarizabilities for virtual Compton scattering on the proton. The dependence of these GP’s on the *N*\(^\ast\) properties has been studied. We hope that there will soon be experimental data with which these estimates can be compared.

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References


FIGURE CAPTIONS

Figure 1. Direct, Cross and Seagull Terms of the hadronic tensor in the lowest order QED perturbation theory.

Figure 2. The 10 GP’s in the NRQM, The parameters \( m_q = 350 \text{ MeV} \) and \( \alpha = 320 \text{ MeV} \). Note that \( P^{(11,00)} \), \( P^{(11,02)} \), and \( \tilde{P}^{(11,2)} \) are all zero in the absence of the recoil correction. (Note that in Fig. 2c the superscript “para” refers to the paramagnetic contribution from the \( \Delta(1232) \) and “dia” to the seagull contribution which has the opposite sign.)

Figures 3. Effects of the masses of nucleon(\( \Delta \)) excitations on GP’s. Four groups of different masses for P-wave excitations are used.
- dotted line – using the lower limit of the masses from the particle data tables (1994): \( N^*(1520) \ N^*(1640) \ N^*(1515) \ N^*(1650) \ \Delta^*(1615) \ \Delta^*(1670) \)
- dashed line – using the upper limit of the masses from the particle data tables (1994): \( N^*(1555) \ N^*(1680) \ N^*(1530) \ N^*(1750) \ \Delta^*(1675) \ \Delta^*(1770) \)
- dot-dashed line – using the theoretical masses given in Isgur and Karl (1978): \( N^*(1490) \ N^*(1555) \ N^*(1535) \ N^*(1745) \ \Delta^*(1685) \ \Delta^*(1685) \)
- solid line – same as Figs.2, using the average masses from the particle data tables (1994): \( N^*(1535) \ N^*(1650) \ N^*(1520) \ N^*(1700) \ \Delta^*(1620) \ \Delta^*(1700) \)

Figures 4. Effects of a \( \pm 5\% \) variation (304 and 336 MeV) in \( \alpha \) from its normal value of 320 MeV. Other parameters remain the same as in Figs.2. The dotted line is for \( \alpha = 304 \text{ MeV} \), the dashed line is for \( \alpha = 336 \text{ MeV} \), the solid line \( \alpha = 320 \text{ MeV} \), as in Figure 2.