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NEW LARGE NUMBER HYPOTHESES
AND GEOMETRICAL RELATIONS
ON SUBSTRUCTURES OF THE UNIVERSE

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Summary

In analogy of Dirac's large number hypothesis (LNH), proposed is the “less large number hypothesis (LLNH)” of $N_G \sim N_S$ where N_G and N_S are the total number of galaxies in the Universe and the average total number of stars in a galaxy since both of them are of the order of 10^{11} . Also suggested are the two “geometrical LLNH relations” of $N_G \sim (R_U/R_G)^2$ and $N_S \sim R_G/R_S$ where R_U , R_G and R_S are the “radius of the Universe”, the average radius of a galaxy and that of a star. Furthermore, it is pointed out that the “even less large number hypothesis (ELLNH)” of $n_1 \sim n_2 \sim n_3$ may also work for $n_1 \equiv R_U/D_G$, $n_2 \equiv D_G/R_G$ and $n_3 \equiv R_G/D_S$ where D_G and D_S are the average distance between two neighboring galaxies and that between two neighboring stars since all of them are of the order of 10^3 .

Dirac's large number hypothesis (LNH) [1] states that the Eddington large numbers [2], $N_1 (\equiv \alpha/Gm_em_p \cong 10^{39})$, $N_2 (\equiv m_e/\alpha H \cong 10^{40})$ and $N_3 (\equiv 4\pi\rho/3m_p H^3 \cong 10^{80})$, are not independent but related with each other. It has recently been shown that not only the “dynamical LNH relation” of $N_3 \sim N_1 N_2$ [3] but also the “geometrical LNH relation” of $N_3 \sim (N_2)^2$ [4] holds so that the LNH may not be taken as a hypothesis but become the large number rule (LNR). In this paper, we shall propose the “less large number hypothesis (LLNH)” of $N_G \sim N_S$ where N_G and N_S are the total number of galaxies in the Universe and the average total number of stars in a galaxy since both of them are of the order of 10^{11} . We shall also suggest the two “geometrical LLNH relations” of $N_G \sim (R_U/R_G)^2$ and $N_S \sim R_G/R_S$ where R_U , R_G and R_S are the “radius of the Universe”, the average radius of a galaxy and that of a star. Furthermore, we shall point out that the “even less large number hypothesis (ELLNH)” of $n_1 \sim n_2 \sim n_3$ may also work for $n_1 \equiv R_U/D_G$, $n_2 \equiv D_G/R_G$ and $n_3 \equiv R_G/D_S$ where D_G and D_S are the average distance between two neighboring galaxies and that between two neighboring stars since all of them are of the order of 10^3 .

Let us first briefly review how to derive the dynamical and geometrical LNH relations so that we may derive the geometrical LLNH relations later in a similar way. The dynamical LNH relation,

$$N_3/N_1 N_2 (\equiv 4\pi G\rho/3H^2 \equiv \Omega/2) \cong 1/2, \quad (1)$$

means that the gravitational self-mass of the Universe has the same order of magnitude as the total mass of the Universe. It is now well known that this is always the case in the standard Friedmann cosmology in the usual Einstein theory of gravity. More

precisely, let us recall the “initial-value equation” of Einstein field equations given by

$$(2) \quad H \left(\equiv \frac{\dot{a}}{a} \right) = \left(\frac{8\pi G\rho}{3} - \kappa \frac{a^2}{a^2} \right)^{1/2}$$

where $\kappa = 0, +1$ or -1 for the spatially flat, closed or open universe and a is the scale

factor defined in the Friedmann-Robertson-Walker space-time metric of

$$(3) \quad ds^2 = -dt^2 + a^2(t) [dr^2/(1 - \kappa r^2) + r^2(d\theta^2 + \sin^2\theta d\phi^2)].$$

Eq. (2) can be re-written in terms of the large numbers as

$$(4) \quad \frac{2N_3}{N_1N_2} - \frac{a^2H^2}{\kappa} = 1.$$

Therefore, the exact LNH relation of $N_3/N_1N_2 = 1/2$ holds for $\kappa = 0$ (or the spatially

flat Universe). For $\kappa = 1$ (or the closed Universe), the approximate relation can be

transformed into another approximate LNH relation of

$$(5) \quad N_3'/N_1N_2' (\equiv 4\pi G\rho a^2/3) \approx 1/2$$

where N_2' and N_3' are the modified large numbers defined by

$$(6) \quad N_2' \equiv m_e a/\alpha, \quad N_3' \equiv 4\pi\rho a^3/3m_p.$$

Even for $\kappa = -1$ (or the open Universe), Eq. (4) indicates that the approximate

LNH relation (1) still holds unless $a^2H^2 \approx 1$ to a very good accuracy. The latter

relation can be transformed into $a \approx 1$ which would lead to a very special case where

$a \approx t$ and, therefore, the deceleration parameter $q \left(\equiv -\ddot{a}/a^2 \right)$ almost vanishes. In

any case, to sum up, the approximate LNH relation (1) must hold in the standard

Friedmann cosmology in the usual Einstein theory of gravity unless there occurs a

certain accidental coincidence.

Let us now move on to derivation of the geometrical LNH relation

$$(7) \quad N_3/(N_2)^2 \sim 1.$$

Since the initial-value equation (2) has already been used in deriving the dynamical

LNH relation (1), the “acceleration (or dynamic) equation” of

$$(8) \quad q \left(\equiv -\frac{\ddot{a}}{a} \right) = \frac{1}{2} + \frac{2a^2H^2}{\kappa} + \frac{4\pi G\rho}{H^2}$$

is the only one independent equation left among the Einstein field equations. However,

the latter equation relates the deceleration parameter q with the pressure p but does

not directly relate the Hubble parameter H with the mass-energy density ρ as the

initial-value equation (2). Although ρ and p are related with each other by the “law of

conservation of energy” as

$$(9) \quad (\rho a^3)_{,t} = -p(a^3)_{,t},$$

these three Eqs. (2), (8) and (9) are not independent with each other. Therefore, we

cannot expect to derive any other LNH relations from the usual Einstein field equations

in the standard Friedmann cosmological model but can hope to do from some unknown

“initial conditions”. What kind of “initial condition” may lead to such LNH relation

as (7)?

As the product of $m_p/N_1N_2 \left(\equiv Gm_p^2H \right)$ has been taken as the magnitude of the

Newtonian gravitational (potential) energy of two protons at a distance of H^{-1} in

deriving the dynamical LNH relation (1), the product of $\pi/(N_2)^2 \left(\equiv \pi\alpha^2H^2/m_e^2 \right)$ can

be taken as the solid angle of a particle whose radius is $r_e \left(\equiv \alpha/m_e \right) \approx 2.8 \times 10^{-13}$ cm,

the classical radius of the electron, seen from a distance of $H^{-1} \left(\approx 10^{10}$ yr). More

precisely, let r_p be the charge radius of the proton. Then, the product of $\pi N_3 r_p^2 H^2 [\equiv \pi(r_p/r_e)^2 N_3/(N_2)^2]$ can be approximately taken as the sum of solid angles of all the protons (or baryons) seen from any point in the Universe since the average distance between two baryons in the Universe is of the order of H^{-1} . Furthermore, since the experimental value of r_p ($\cong 0.81 \times 10^{-13}$ cm) has the same order as that of r_e ($r_p/r_e \cong 0.29$), the geometrical LNH relation (7), if it holds, means that the sum of solid angles of all the baryons seen from any point in the universe is of the order close to 4π .

This remarkable observation would lead us to either one of the following assumptions for the initial condition of the Universe: 1) In the present Universe (and probably at any time of the history of our Universe), the amount of matter in the Universe and the size of the Universe are related with each other in such a way as any light (or observer) may be geometrically blocked (or blinded) once (and probably only once) by a baryon (or a particle in general) before it travels for a period of the order of H^{-1} (or a distance of the order of H^{-1}) to any direction from any point in the Universe. This possibility may be closely related to the fact that all the possible measurements of the cosmological parameters of the Universe such as H^{-1} are naturally affected by the attenuation of light by the inter-galactic matter in the Universe. 2) In the history of our Universe, there must be the era in which the space-time was of three dimensions and all the baryons (or particles in general) were most packed in the two-dimensional space in the Universe. After the three-dimensional era, there occurred the phase transition of the space-time to the present four-dimensional Universe, which has naturally left the miraculous situation described in 1). This possibility seems much more exotic than the first possibility but does not yet seem impossible. In fact, the present author has

suggested that such dimensional phase transitions of the space-time (or the Universe) may occur at the minimum action near the Planck scale [5]. If we make either one of these assumptions, we can immediately derive the geometrical LNH relation (7) by tracing back along the deductive way leading to the assumptions. Therefore, we have completed reviewing how to show that not only the dynamical LNH relation (1) but also the geometrical LNH relation (7) holds so that the LNH may not be taken as a hypothesis but become a rule, the LNR.

Now we are ready to present the following three main points: 1) If the Universe is closed, the total number of galaxies N_G is well defined and experimentally known to be of the order of 10^{11} . Also, the average total number of stars in an ordinary galaxy N_S is experimentally known to be of the order of 10^{11} . Therefore, in analogy of LNH, this coincidence on the “less large numbers”, N_G and N_S , tempts us to propose the “less large number hypothesis (LLNH)” of

$$N_G/N_S \sim 1. \quad (10)$$

2) If the sum of solid angles of all the galaxies seen from any point in the Universe is of the order close to 4π , a “geometrical LLNH relation” of

$$N_G(R_G/R_U)^2 \sim 4 \quad (11)$$

must hold for the total number of galaxies in the Universe N_G , the “radius of the Universe” R_U and the average radius of an ordinary galaxy R_G . In fact, this relation holds remarkably well since experimentally $N_G \sim 10^{11}$, $R_U \sim 10^{10}$ yr [6,7] and $R_G \sim 10^4$ yr. 3) If the sum of arcs in the two-dimensional plane of galaxy of all the stars in an ordinary galaxy seen from any point in the galaxy is of the order close to 2π , another

“geometrical LLNH relation” of

$$N_S(R_S/R_G) \sim \pi \quad (12)$$

must hold for the average total number of stars in an ordinary galaxy N_S , the average radius of an ordinary galaxy R_G and the average radius of an ordinary star R_S . In fact, this relation also holds remarkably well since experimentally $N_S \sim 10^{11}$, $R_G \sim 10^4$ yr and $R_S \sim 1$ sec. Note that all of these three points are in a perfect analogy of the LNH and geometrical LNH relations.

Furthermore, let us define the “even less large numbers”, n_1 , n_2 and n_3 , by

$$n_1 \equiv R_U/D_G, n_2 \equiv D_G/R_G \text{ and } n_3 \equiv R_G/D_S \quad (13)$$

where D_G and D_S are the average distance between two neighboring galaxies and that between two neighboring stars. Then, we are tempted to suggest the “even less large number hypothesis (ELLNH)” of

$$n_1 \sim n_2 \sim n_3 \quad (14)$$

since experimentally $R_U \sim 10^{10}$ yr, $D_G \sim 10^7$ yr, $R_G \sim 10^4$ yr, $D_S \sim 10$ yr and, therefore, all of n_1 , n_2 and n_3 are of the order of 10^3 . Note that this remarkable regularity existing in substructures of the Universe may be exactly the same as what the Indian philosopher knew already in B.C.5C as “San-Zen-Dai-Sen-Sekai”, which means “Triply-Thousand-Times-Folded-Great-World” in Chinese [8].

What is left as an important subject for future investigations is to find (or formulate) a theory in which these hypotheses and relations can be accommodated. To this end, we must at least extend the Einstein field equations in the standard Friedmann cosmology

by taking into account the finite non-vanishing sizes of particles, stars and galaxies as van der Waals extended the equation of state for ideal gas (or liquid) by taking into account those of atoms and molecules. Also, Eq. (9) must be extended to the “extended law of conservation of energy”,

$$[\rho(a^3 - b)]_{,t} = -[(p + \frac{A}{a^6})(a^3 - b)]_{,t}, \quad (15)$$

where A and b are the constant parameters representing the finite non-vanishing sizes. In conclusion, we must admit that we are still far from understanding the profound substructures of our Universe. To make cosmology a real science, we should wait for more experimental discoveries on the substructures of our Universe such as recently made by Ikebe et al. [9] and for more phenomenological ones such as lately made by B. Moore et al. [10].

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