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Simulation Results of Bent Crystal Extraction in the Tevatron^{*}

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Abstract

We present simulation results describing parasitic extraction of a high energy proton beam from the Tevatron via a bent Silicon crystal – demonstrated recently by E853 experiment at Fermilab. In a spirit of parasitic extraction the transverse phase-space of the beam is initially prepared, so that only high amplitude particles from the beam halo interact with the Silicon crystal (inserted into the beam), without affecting the 'core' of the beam. As a result of the beam halo – crystal interaction (multiple scattering and wide angle nuclear interaction), some fraction of the beam halo channels into the crystal – a multi-turn extraction occurs. The ultra strong focusing environment of a crystal channel combined with the externally imposed curvature (vertically bent crystal) provide excellent guiding features of an efficient extraction line. Here, we use a realistic model of the Tevatron lattice to describe the beam optics of the machine. Presented phase-space simulation describes a 'kick' mode of the E853 experiment. To enhance the beam halo interaction with the crystal, the proton beam is initially excited by firing a transverse kicker, which induces horizontal oscillations of the beam centroid on the subsequent turns. A realistic phase-space tracking yields turn-by-turn channeling efficiency, which sheds light on multi-turn extraction mechanisms. It also allows us to optimize experimental configuration parameters for maximum extraction efficiency.

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1 Introduction

This study is motivated by E853 experiment (bent crystal extraction experiment) conducted at the Fermilab's Tevatron. A realistic simulation reflecting details of the experimental set up was carried out to better understand extraction mechanisms and the transverse phase-space configuration as well as to optimize the performance of the experiment. A clear physical picture of the transverse phase-space evolution under extraction conditions is needed to explore and evaluate various schemes/ideas of halo generation – an essential ingredient of a truly parasitic extraction.

The simulation is meant to cover all of the relevant accelerator physics issues involved (matching of the initial beam distribution to the machine beam optics, trajectories of individual particles, dynamics of the beam excited by the kicker, etc.), as well as fundamental physics processes – characteristics of the proton beam interaction with the Silicon crystal. The 'accelerator' issues can be well represented in the form of the TWISS functions and corresponding matrix transformations describing particle transport between any given locations of the Tevatron lattice. The beam – crystal interaction, however remains largely an open issue as to how one may treat relevant interactions within approximations that are consistent with results that one can measure. Here, the major issue is an appropriate implementation and modelling of multiple scattering, channeling [1] and beam loss due to inelastic nuclear collisions.

Beyond these issues, the remaining questions are related to possible diffusion mechanisms; like that of using a thin pre-scatterer so as to enhance the initial step size of particles approaching a 'channeling window' in the

transverse phase-space (a proton entering a crystal channel with the 'pitch' angle smaller than the critical angle for the planar channeling).

Similar simulation will be conducted to examine more exotic schemes of crystal extraction, e.g. to employ noise induced perturbations or multiple resonance crossing to drive a large amplitude phase-space diffusion, which would enhance the halo formation and would boost the overall parasitic extraction efficiency without 'disturbing' the core of the beam – no luminosity degradation for the collider operation.

2 Crystal Channeling Experiment at the Tevatron

E853 experiment aims to test the feasibility and efficiency of low intensity beam extraction from the halo of the Tevatron high intensity beam. It employs planar channeling of relativistic protons in a bent Silicon crystal [5], [7].

The original motivation for this effort is to use this a scheme as a tool to extract protons parasitically at trans-TeV energies for the purpose of fixed target beauty physics. Similar efforts in this direction are in progress to develop a fixed target beamline at the LHC – a bent crystal extraction of 120 GeV proton beam at SPS [8], [9]. Such a scheme could be employed to create a low intensity test-beamline at Fermilab.

The goal of the experiment is to extract one million 900 GeV/c protons per second (out of 10^{12} protons circulating in the Tevatron). Furthermore, one would like to study the extraction efficiency, as well as to show that the luminosity life-time of the circulating beam is not adversely affected by the extraction process and that there are no intolerable backgrounds created for the two Tevatron collider experiments (CDF and D0).

The experiment is located in the C0 straight section of the Tevatron. The abort line consists of a three-bend magnetic dogleg which provides a 4 milliradian horizontal kick, so that the abort line can clear the magnets at the downstream end of the straight section. The crystal is placed at the outer side of the circulating beam with an upward curvature of 640 microradians, sufficient to deflect the beam halo that strikes the crystal into the field-free region of the Lambertson magnets.

Then the extracted beam traverses two instrumented air gaps [5] (AG1

and AG2) 40 meters apart and approximately 100 meters downstream of the position of the crystal and it is finally deposited into the beam dump. The air-gaps are instrumented with two scintillators and two silicon-strip planes; each one counts the extracted beam flux, it also measures the extracted beam trajectory. Furthermore, the scintillators provide the trigger for the Silicon planes. In addition, a fluorescent screen coupled to a CCD camera serves as a digital readout of the beam-profile for the run-time diagnostics.

One of the most critical parameters is the alignment of the vertical angle of the crystal with the beam angle. This alignment is necessary for the channeling to occur. An intrinsic (crude) crystal alignment is maintained by mounting the upstream end of the crystal on a 1 meter long beam pipe, terminated with articulating bellows. Further tuning is done by two precision motors (horizontal and vertical motion) at each end of the pipe allowing for the alignment of the crystal with four degrees of freedom. There is also a coarse horizontal stepping motor used to move the crystal holder in and out of the beam line.

3 Tevatron Lattice

An up-to-date description of the Tevatron lattice at 'low beta squeeze' was used to model the beam optics of the machine [14]. A recent version of a lattice design code, MAD 8.14, [11] was used to find the periodic solution for the betatron motion in both planes. Quantitative characteristics of the periodic betatron motion were calculated in terms of the TWISS functions, α , β and γ , as well as the betatron phase advance μ at various locations around the ring. Some global lattice characteristics (tunes, natural chromaticities, maximum dispersion, maximum beta, etc.) are summarized in Table 1.

3.1 Transport Properties

Beam transport between two locations in the lattice is given by the appropriate transfer matrix, $M_{1,2}$, determined uniquely by the TWISS functions and betatron phase advance between these two locations [2]. The transfer map can be written in the following compact form

$$M_{1,2} = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}(\cos\Delta\mu + \alpha_1\sin\Delta\mu) & \sqrt{\beta_1\beta_2}\sin\Delta\mu \\ -\frac{(1+\alpha_1\alpha_2)\sin\Delta\mu + (\alpha_2 - \alpha_1)\cos\Delta\mu}{\sqrt{\beta_1\beta_2}} & \sqrt{\frac{\beta_1}{\beta_2}}(\cos\Delta\mu - \alpha_2\sin\Delta\mu) \end{pmatrix} \quad (1)$$

where,

$$\Delta\mu = \mu_2 - \mu_1. \quad (2)$$

The phase-space points transform via the transfer matrix according to the following formula

$$\underline{x}_2 = M_{1,2}\underline{x}_1 \quad (3)$$

Table 1:

TEVATRON injection at 150 GeV Collider Mode	Quantity	Value
<u>Tunes</u>		
horizontal	ν_x	20.864
vertical	ν_y	20.855
<u>Maximum Betas</u>		
horizontal	β_x^{max}	1107.30 m
vertical	β_y^{max}	1077.10 m
<u>Maximum Dispersion</u>		
horizontal	D_x^{max}	9.392 m
vertical	D_y^{max}	0.0 m
<u>Natural Chromaticity</u>		
horizontal	χ_x	-56.99
vertical	χ_y	-56.49

where,

$$\underline{x} = \begin{pmatrix} x \\ x' \end{pmatrix}, \quad (4)$$

represents the position-angle coordinate of a generic point in the transverse phase-space.

Similarly, turn-by-turn phase space evolution at a given location in the lattice is described by the one turn map, R , defined in the following compact form [2]

$$R = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix} \quad (5)$$

Here, μ is the machine tune for a given plane.

The turn-by-turn transformation of a generic phase-space is governed by the following formula

$$\underline{x}^{(n+1)} = R\underline{x}^{(n)} \quad (6)$$

where n is the turn number.

3.2 Kick Mode

Prior to the crystal extraction process, the beam is perturbed by firing a horizontal kicker in E17 location [12]. The kicker is located approximately on the opposite side of the ring with respect to the crystal (upstream of the C0 marker). The amount of angular kick of $\Delta x' = 30$ micro radians adds the following offset, $\underline{\Delta x}$ to every phase-space point at E17 location

$$\underline{\Delta x} = \begin{pmatrix} 0 \\ \Delta x' \end{pmatrix} \quad (7)$$

For the purpose of particle tracking in our simulation one can mark the following two location of interest: the location of the crystal – labelled by (1) and the location of the E17 kicker labelled by (2). Using this convention one can define turn-by-turn phase-space tracking algorithm for the kick mode in the following compact matrix notation

$$\underline{x}_1^{(n)} = \underline{x}_1^{(0)} + R_1^{n-1} M_{2,1} \underline{\Delta x} \quad (8)$$

Here, n is the turn number, R_1 is the one-turn transfer matrix (at the crystal location) and $\underline{x}_1^{(0)}$ is the initial phase-space configuration at the crystal location on the turn 'just before firing the kicker'. Above transformation is used for particle tracking in our simulation, where the transfer matrices R_1 and $M_{2,1}$ are calculated via Eqs. (5-9) using values of the TWISS functions collected in Table 2.

Table 2: Lattice TWISS functions

location	α	β	μ
kick	-2.181	109.851	7.767
crystal	0.145	104.678	20.44

As one can see from Eq.[8], the phase-space will be displaced on all subsequent turns as a result of the applied kick. The beam centroid position on a given turn is illustrated in Figure 1.

Figure 1: Beam centroid motion on subsequent turns after firing the horizontal kicker. Angular kick of 30 micro radian induces in-and-out horizontal oscillations with a maximum amplitude of 3 mm.

4 Simulation

The simulation presented here resembles the sequence of events that takes place during a standard data run session for E853 experiment in the kick mode. The beam energy is set at 900 GeV/c. The crystal is positioned in the tail of the beam at 5 mm from the ideal orbit position at the crystal location. The Data Acquisition System is triggered on a signal that is sent out whenever the beam is kicked by the chosen horizontal kicker, located at E17.

4.1 Beam

To model the beam efficiently we start with a bi-Gaussian distribution in x - x' and y - y' – the horizontal and vertical components of the 4-dim transverse phase-space. Since the Tevatron lattice is globally decoupled, the betatron motion in both planes can be treated independently. For simplicity, we will focus on the horizontal phase-space – all details of the initial phase-space distribution can be extended to the vertical plane as well. The initial collection of 80,000 particles populates elliptical equi-density contours carefully matched to the lattice TWISS functions α , β and γ at the crystal location. The elliptical contours satisfy the following matching condition [2]

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon. \quad (9)$$

Here, we assume a Gaussian distribution in x with a sigma of 2 mm.

Before we start the actual simulation, the crystal is placed at its nominal position along the horizontal x -direction and a normally distributed beam transported around the Tevatron lattice will heavily interact with the crys-

tal (multiple scattering and wide angle nuclear interaction). To describe the 'beam scraping' on the crystal, one can introduce the following useful quantity, x^{max} – the maximum horizontal extent of the invariant ellipse for a given value of the transverse emittance, ϵ . From Eq.[9], x^{max} , is expressed as follows

$$x^{max} = \sqrt{\beta\epsilon}. \quad (10)$$

It is natural to assume that all the particles that lie outside of the ellipse defined by $x^{max} \geq x_{crystal}$ are scraped off rapidly by the crystal edge after a few seconds. This time scale is typical of the time involved between the positioning of the crystal close to the tail of the beam and the beginning of a data taking session marked by the application of a kick to the beam to introduce a reasonable step-size into the crystal. Taking equality sign in the above inequality and applying Eq.[10] one can uniquely define the maximum allowed transverse emittance ϵ^{max} .

Assuming a maximum beam centroid shift of $\Delta x = 3$ mm, one can define the 'core' of our distribution, as an ensemble of phase-space points that will never get in 'contact' with the crystal. The core is enclosed by the invariant ellipse defined by the minimum transverse emittance, ϵ^{min} , which is defined by the following condition

$$x_{crystal} - \Delta x = \sqrt{\beta\epsilon^{min}}. \quad (11)$$

To summarize, we 'scrape off' the outer part of the distribution and we 'remove' the core of the beam to speed up the tracking procedures. Therefore, our initial phase-space configuration is obtained from the original bi-

Gaussian distribution by retaining only the annular elliptical region bound by two invariant ellipses defined by two critical transverse emittances, ϵ^{min} and ϵ^{max} , discussed above. This procedure reduces computing time without adversely affecting physics of our model simulation.

4.2 Crystal

The interaction of the beam with the crystal is implemented in our simulation as follows.

Each particle arriving at the crystal location is represented to be a point in the transverse phase-space (four coordinates). First, we select an ensemble of particles interacting with the crystal, simply by applying an obvious geometric criterion $x > x_{crystal}$ for the particle to meet the horizontal dimension of the crystal. Then a channeling criterion is applied to the interacting ensemble – all particles with $y' < \theta_{crit}$, 'successfully' enter a crystal channel. We assume no dechanneling effects [3] due to the centrifugal force, since the vertical bending angle of 640 micro radians is more than order of magnitude smaller than the critical bending angle corresponding to the Tsyanov radius of curvature [10]. We also disregard the so called partial channeling [3], therefore all those particles are assumed to channel through the entire length of the crystal – they are being extracted from the beam (in our simulation they are removed from the distribution and from further tracking).

All remaining particles traverse the bulk of the crystal (without being channeled), therefore they interact with an amorphous material rather than a crystal. As illustrated in Figure 2, knowing the phase-space coordinate, x, x' , of a given particle at the crystal face and the crystal geometry one can easily calculate the effective length of the beam trajectory within

Figure 2: Effective length of a particle traversing the crystal (with no channeling).

the crystal, l .

$$l = \begin{cases} \frac{(x_{cryst}-x)}{\sin x'} & x' < \alpha \\ \frac{L}{\cos x'} & \alpha < x' < \beta \\ \frac{(X-x)}{\sin x'} & x' > \beta \end{cases}$$

where

$$\alpha = -\tan^{-1}\left(\frac{x - x_{cryst}}{L}\right) \quad (12)$$

$$\beta = \tan^{-1}\left(\frac{X - x}{L}\right)$$

Here, $L = 4$ cm is the length of the crystal and $X = 1$ cm is its horizontal width (see Figure 2). The effective path, l , is used to calculate the probability for the incident particle to undergo inelastic nuclear interaction given by the following distribution

$$P(l) = 1 - e^{-l/\lambda_T}. \quad (13)$$

where λ_T is the nuclear collision length. Using this distribution, one can select a fraction of the initial ensemble (interacting with the crystal) to undergo wide angle nuclear scattering [4]. In our simulation these particles are considered moved outside the angular aperture of the machine – they are removed from the ensemble and they are no longer being tracked by the simulation.

Now, all remaining particles in the ensemble undergo multiple scattering traversing the crystal. This interaction is implemented in our simulation by

adding an angular kick (in x and y plane) to the the original phase-space coordinate of each particle of the interacting ensemble. Those angular kicks are generated according to the Gaussian distribution, where the sigmas are given by the following expression

$$\sigma_x = \sigma_y \quad (14)$$

$$\sigma_x = \frac{13.6 MeV}{\beta c p} \sqrt{\frac{l}{X_0}} [1 + 0.038 \ln(\frac{l}{X_0})]. \quad (15)$$

Finally, the resulting phase-space distribution (leaving the crystal) is ‘propagated’ through the entire Tevatron ring – it is mapped via the one-turn-transfer map, R_1 , which includes all the details of the beam optics. This step closes one full loop of our tracking routine.

Some of the relevant material properties of Silicon crystal are listed for completeness in Table 3.

Table 3: Crystal Characteristics

Crystal	Silicon
Channeling plane	[1 0 0]
Z	14
A	28.09
density	2.33 g/cm ³
dimensions	4cm × 1cm × 1cm
nuclear collision length	30.3 cm
radiation length	9.36 cm

5 Results and Conclusions

One of the goals of our simulation was to gain insight into the transverse phase-space evolution under extraction conditions (kick mode, beam – crystal interaction). To illustrate that a snap-shot of the horizontal phase-space is displayed every turn 'just after' the beam interacts with the crystal.

Figures 3 and 4 illustrate these snap-shots over many turns after the kick is applied. They are consistent with the motion of the beam centroid induced by the initial kick (see Figure 1). As one can see from the phase-space snap-shots, turns 4, 7 and 9 are the most proliferous in terms of the beam halo – crystal interaction. This conclusion also agrees very well with measured intensities of the extracted beam on those turns, which are reported elsewhere [6]. The phase space previously distorted via crystal scattering arrives to the face of the crystal, providing enriched multiple-turn interaction, which leads to increased extraction efficiency (see Figure 5). Using our model simulation one can extract in principle about 18 percent of the interacting beam. This needs to be compared with future measurement [13] of the extraction efficiency, planned for E853.

Another interesting result is illustrated in the last snap-shot of Figure 4 (turn 50). It shows uneven phase space depletion (to be confirmed by the experimental observation in the future runs). This can be explained by pure beam dynamics consideration. It is a consequence of the horizontal tune 'sitting on top' of the 7/12 resonance. The relaxing (after the kick) phase-space of the beam undergoes rotation every turn, while the effective interaction with the crystal happens every other turn – a 'chopping wall' illustrated in Figure 6. Simple numerology shows that roughly half of the

angular component of the phase-space is exposed to the interaction with the crystal (depletion), while the other part remains unaffected by the interactions with the crystal.

The remaining questions are related to possible diffusion mechanisms like that of using a thin pre-scatterer [3] to enhance the initial step size of particles approaching the 'channeling window' in the transverse phase-space (entering a crystal channel with a 'pitch' angle smaller than the critical angle for the planar channeling).

Similar simulation will be conducted to examine more exotic schemes of crystal extraction, e.g. to employ noise induced perturbations or multiple resonance crossing to drive large amplitude phase-space diffusion, which would enhance the beam halo formation and it would eventually boost the overall parasitic extraction efficiency without 'disturbing' the core of the beam – no luminosity degradation for the collider operation.

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Figure 3: Horizontal phase-space evolution under extraction condition in the kick mode – initial set of snap-shots.

Figure 4: Horizontal phase-space evolution under extraction condition in the kick mode – further continuation of the phase-space snap-shots.

Figure 5: Predicted turn-by-turn extraction efficiency including a budget of specific interactions with the crystal.

"Moving Wall" - 7/12 Resonance

Figure 6: Uneven phase-space exposure – 7/12 resonance 'chopping wall' effect.

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