SCAN-9606034

Constraints for solar neutrinos fluxes *

P. Quarati^{a,d}, A. Carbone^a, G. Gervino^{b,c}, G. Kaniadakis^{a,c}, A. Lavagno^{a,c} and E. Miraldi^a

^a*Dipartimento di Fisica - Politecnico di Torino - Italy*

^b*Dipartimento di Fisica Sperimentale - Università di Torino - Italy*

^c*Istituto Nazionale di Fisica Nucleare, Sezione di Torino*

^d*Istituto Nazionale di Fisica Nucleare, Sezione di Cagliari*

Constraints on the neutrino fluxes Φ_p , Φ_B , Φ_{CNO} and Φ_{Be} are derived from the available experimental results. Physical implications are discussed using the non-extensive statistics which takes into account many-body effects and long-range interactions. The different reduction factors for 7Be and 8B fluxes with respect to their standard solar model values are physically understood, on the basis of the shape of the electron and proton distribution functions.

swg624

Long-range interactions and many-body effects involving large numbers of particles influence the density of low-energy pairs in the central solar plasma. Stationary energy distributions different from the Maxwellian one, with depleted tails or vanishing at energies of some tenths of $k_B T$, can be justified on theoretical grounds by the Tsallis statistics by considering the sun as a stellar polytrope of index n . Nuclear rates, Gamow peaks and consequently neutrino fluxes can then be expressed as products of their standard solar model (SSM) values by a function of the parameter δ (or the q Tsallis parameter) characterizing the depletion of the Maxwellian tail. This has been recently discussed by us in order to make predictions of the solar neutrino fluxes and to compare these predictions with the detected fluxes. In this work some further progresses along these lines are presented.

We set four equations (constraints) for the four fluxes (variables) $\Phi_p(pp + p^*p)$, Φ_B , Φ_{CNO} , Φ_{Be} following the approach described, for instance, by Calabrese et al. [1].

These equations are relationships among the fluxes based on the well known value of the solar luminosity, on the Gallium signal, on the Chlorine experiment and on the result of the Kamiokande experiment, assumed by us to be correct, which gives the Boron neutrino flux $\Phi_B = (2.9 \pm 0.4) 10^6 \text{ cm}^{-2} \text{ s}^{-1}$.

Using the more recent experimental results [2-5] and the theoretical calculations of cross sections on Ga and Cl by Bahcall et al. [6], we can write the equations:

Luminosity

$$63.85 = 0.980 \Phi_p + 0.939 \Phi_{Be} + 0.937 \Phi_{CNO} + 0.000498 \Phi_B, \quad (1)$$

Gallium signal

$$79 \pm 11.7 = 1.23 (1 \pm 0.02) \Phi_p + 7.32 (1 \pm 0.03) \Phi_{Be} + 8.72 (1 \pm 0.05) \Phi_{CNO} + (2.46^{+2.1}_{-1.1}) \Phi_B, \quad (2)$$

Chlorine signal

$$2.55 \pm 0.25 = 0.0038 (1 \pm 0.02) \Phi_p + 0.24 (1 \pm 0.02) \Phi_{Be} + 0.41 (1 \pm 0.02) \Phi_{CNO} + (1.14 \pm 0.17) \Phi_B, \quad (3)$$

Kamiokande experiment

$$2.9 \pm 0.4 = \Phi_B. \quad (4)$$

All fluxes are in units of $10^9 \text{ cm}^{-2} \text{ s}^{-1}$ but the B flux which is in units of $10^6 \text{ cm}^{-2} \text{ s}^{-1}$ but . Of course we must consider that a quantity like $(a\Phi_{Be} + b\Phi_{CNO})$ (with a and b positive) cannot be negative, for physical reasons.

The SSM predictions are well known and we may choose the values obtained by Bahcall et al. [7] which do not differ sensibly by other standard model evaluations, like that of Dar and Shaviv [8], we have

*To be presented to the IV International Conference on NUCLEI IN THE COSMOS, University of Notre Dame, Notre Dame, Indiana, USA, June 20-27, 1996
e-mail: Quarati@polito.it

$$\begin{aligned}
\Phi_p^{SSM} &= 59.24 (1 \pm 0.01) \\
\Phi_{Be}^{SSM} &= 5.15 (1 \pm 0.06) \\
\Phi_{CNO}^{SSM} &= 1.17 (1 \pm 0.2) \\
\Phi_B^{SSM} &= 6.62 (1^{+0.14}_{-0.17})
\end{aligned} \tag{5}$$

The several coefficients of the above four equations (1)÷(4) depend on different quantities, among which the position of the Gamow peak of the different reactions and the maximum neutrino energies [9]. These quantities are functions of the parameter δ . However the corrections to be introduced for this dependence on δ are small and negligible; we do not discuss more extensively on this argument here.

After algebraic manipulation of the equations we find that the following values are consistent with the four constraints (1)÷(4)

$$\begin{aligned}
\Phi_p &= 60 & \Phi_{Be} &< 0.65 \\
\Phi_{CNO} &< 0.4 & \Phi_B &= 2.9 \pm 0.4
\end{aligned} \tag{6}$$

These quantities are not too different from the constraints indicated by Castellani et al. [10]. Therefore we must expect that the SSM values be reduced for some physical reasons of a factor $\simeq 8$ for Φ_{Be} , of $\simeq 3$ for Φ_{CNO} , 2.35 ± 0.35 for Φ_B ; on the contrary for Φ_p there is no reduction, rather we must impose a very small enhancement. The Φ_p can weakly differ from the SSM value increasing its value, and still satisfying the constraints imposed by Eqs. (1)÷(4).

An explanation of the behavior, just described, of the different fluxes by means of the approach recently introduced by us [11], based on the non-extensive Tsallis statistics [12] will be here proposed.

By means of it we can handle easily the long-range gravitational interaction and the many-body effects among the particles of the different systems (H , $H\epsilon$, Z and ϵ^-) constituting the solar core. Moreover by using the Clayton and coll. [13] intuition, the physical meaning of the depletion of the Maxwellian tail of the ion and electron distribution functions can be understood.

Starting from the Tsallis entropy, suitable distribution functions for the stellar polytropes can be derived. The internal structure of the sun can be considered polytropic with index $n = 3.5$ [14]. The Clayton et al. distribution belongs to Tsallis class family and is certainly due to long-range gravitational interactions, many-body effects and, in particular for the electrons, to strong electromagnetic field. We obtained a relation between n and the Tsallis parameter q linked to the Clayton parameter δ ($\delta = (1 - q)/2$) (see Eq.(13) of Ref. [11] and Eq.(9) below). The distribution functions expected by the Tsallis the statistics vanish at some tenths of $k_B T$, depending on the value of q . The depleted Maxwellian distribution represents a good approximation (see also Ref. [15] for a general discussion on non-Maxwellian distributions).

The correction to the SSM predictions can be factorized following the Clayton approach independently of the initial SSM values and can be calculated as a function of the parameter δ .

The corrections of the four SSM fluxes are reported in the Figure. For the sake of utility we report the reduction factor f_{red} as a function of δ ($\Phi_{B,CNO,Be} = \Phi_{B,CNO,Be}^{SSM} / f_{red}$). All fluxes are normalized to one at $\delta = 0$ (SSM). The factor of enhancement f_{incr} concerns the proton flux which changes very smoothly as a function of δ , compared to the corresponding reduction of Φ_B , Φ_{CNO} and Φ_{Be} ($\Phi_p = \Phi_p^{SSM} f_{incr}$). Analytical expressions for both $f_{red}(B)$ and $f_{red}(Be)$ are also given.

We can observe that the reduction needed to explain the value of the Boron flux implies $\delta(\text{Boron})=0.003$, for the CNO flux $\delta(CNO) > 0.0025$ but smaller than 0.03 (as it will be explained later), for the Be flux $\delta(Be) > 0.02$, but smaller than 0.06. The proton flux depends very smoothly on δ and within the errors we can accept any values of δ between 0 and 0.01 because also a value little more greater than 60 (let us say 64) satisfies the constraints.

We know that the flux Φ_{Be} is proportional to the rate $R(\epsilon^- \rightarrow Be)$ [16]

$$\Phi_{Be} \propto N(\epsilon^-) N(Be) \langle \epsilon^- \rightarrow Be \rangle, \tag{7}$$

where $N(\epsilon^-)$ is the electron density, $N(Be)$ the Be density, $\langle \epsilon^- \rightarrow Be \rangle$ is the reaction cross section times electron velocity averaged over a certain distribution (Maxwellian or non) and that

$$\Phi_B \propto N(\epsilon^-) N(p) \langle p \rightarrow B \rangle. \tag{8}$$

Therefore Φ_{Be} depends on the electron distribution while Φ_B on the proton distribution. Also Φ_{CNO} depends on the proton distribution because all nuclei involved in this cycle react with protons. Considering the Kamiokande experiment correct, it is just its result on Φ_B that fixes the value of δ_B , δ_{CNO} and δ_p .

The consequence is that $\delta_p = \delta_B = \delta_{CNO}$ and we may select the value 0.003, indicated above. Very different is the value of the depletion parameter to be used for Φ_{Be} . It can be fixed to $\delta_{Be} = 0.024 \div 0.06$ (the upper value holds if the Be flux nearly vanishes). Without loosing generality we select 0.024.

The shape of the distribution of the Helium (three and four) and the parameter δ_{He} do not enter directly and explicitly in the derivation of the fluxes. Rather these quantities are contained in the percentage of the nuclei taking part to the reactions, responsible of the neutrino emission, through the density $N(^7Be)$.

The presence of a resonance in helium channels seems to be excluded [17], our results are independent of it.

An improvement of this phenomenological analysis can be accomplished using the relation between δ and n

$$\delta(n+1) = \frac{1}{2} \frac{k_B T N}{\mathcal{P}} \quad , \quad (9)$$

derived in Ref. [11] ($k_B T$ is the temperature, $\mathcal{P}$ the pressure and N the number of particles per unit volume $N = \mathcal{N}_{Av} \rho / \mu$, ρ is the density and μ the molecular weight).

In order to simplify our discussion, we do not consider the dependence of $\mathcal{P}$ on δ and the very small variation, within the Tsallis statistics, of the perfect gas law [18]. Therefore for a system of particles of average molecular weight μ , we have:

$$\delta = \frac{1}{2} \frac{1}{n+1} \quad , \quad (10)$$

($n = 3.5 \Rightarrow \delta = 0.111$, $n = 5 \Rightarrow \delta = 0.0083$).

The quantity N can be written as $N = \sum_i N_i$ where i refers to H , 3He and 4He , Z (nuclei heavier than helium) and e^- . The parameter δ is therefore the sum of the single parameters δ_i , given by

$$\delta_i = \frac{1}{2} \frac{1}{n+1} \frac{N_i}{N} \quad . \quad (11)$$

We can also write δ as the sum of the three depletion parameters of the partial proton distributions responsible of the reactions allowing the emission of B neutrinos, $CNO - \nu$'s and $p - \nu$'s plus the depletion parameter of the electron distribution responsible for the emission of the $Be - \nu$'s.

We still have two choices: a) we take for all fluxes the same polytropic index n , b) we take two different n (n_I , n_{II}) for the core and for the envelope [19]. Let us limit the following discussion to the case a). We know that $n = 3.5$ is an appropriate value for a radiative sun [14] (the constraint on n is: $n < 5$).

At a central density $\rho = 148 \text{ gr cm}^{-3}$, the following values (in units of 10^{-13} fm^{-3}), are obtained

$$\begin{aligned} N_H &= 0.3050 & N_{He^3+He^4} &= 0.1435 \\ N_{CNO} &= 0.0015 & N(e^-) &= 0.59 & N &= 1.04 \quad . \end{aligned}$$

With the "experimental" values of δ_i derived above ($\delta_p = \delta_B = \delta_{CNO} = 0.003$, $\delta_{Be} = 0.024$) we may obtain, from Eq. (11), the densities N_i of the different nuclei entering the neutrino production reactions with the constraints

$$N_{pp+pep}^p + N_B^p + N_{CNO}^p + N_{other}^p = N_H ,$$

$$N_{Be}^{e^-} + N_{other}^{e^-} = N(e^-) \text{ and}$$

$$N_H + N_{He} + N_Z + N(e^-) = N .$$

These relations fix the maximum values of δ_{Be} and of δ_{CNO} indicated above.

We report the following results:

$$\text{a) } N_B^p = 0.03, \quad \text{b) } N_{CNO}^p = 0.02,$$

$$\text{c) } N_p^p = 0.03, \quad \text{d) } N_{Be}^{e^-} = 0.18,$$

e) the depletion parameter of helium distribution is, at maximum, $\delta_{He} = 0.015$ (a value of 0.003 is suitable for calculations), the one for Z is $\delta_Z = 0.00016$ (distribution very close to the Maxwellian).

$$\text{f) } \delta_p + \delta_{He} + \delta_Z + \delta_{e^-} = \delta = 0.11$$

Assuming that the Kamiokande experiment is correct we can fix the depletion or Tsallis parameter δ_B and consequently δ_{CNO} and δ_p with the same value of δ_B . The three fluxes Φ_B , Φ_{CNO} and Φ_p depend on the distribution function of protons which is non-Maxwellian with a smoothly depleted tail. Contrarily to the other fluxes, the magnitude of Φ_p increase slightly with δ .

Berillium flux behaves very differently since it depends on the electron distribution which is much more depleted than the ions distribution. This means that electrons are submitted to more intense interactions than ions, probably due to the strong electromagnetic fields in which they are immersed in addition to the long-range gravitational potential.

We recall that the corrected fusion nuclear rates can be found in Refs. [11,13]; the electron capture on ${}^7\text{Be}$ rate is the SSM one [20] corrected by the factor $(1 - 2\delta + 12\delta^2)$.

We calculate the dependence of the four neutrino fluxes upon the central temperature T_c when all the SSM calculations are accomplished at the central SSM temperature T_c^{SSM} . Recently both Bahcall and Ulmer [16] and Calabresu et al. [21] have analyzed this dependence. We use the results of Ref.(16) for the one-zone model and the dependence of T_c upon δ which can be approximately given by (13):

$$T_c = T_c^{SSM}(1 + 3.12 \delta) \quad \delta < 0.1 \quad (12)$$

we obtain

$$\Phi_{pp} \propto \left[1 - 0.08 \left(\frac{T_c}{T_c^{SSM}} \right)^{11} \right] \left[1 + 0.44 \left(\frac{T_c}{T_c^{SSM}} \right)^{0.62} \right] \quad (13)$$

$$\Phi_B \propto T_c^{25} \frac{1}{1 + 1.0417 \cdot 10^4 \left(\frac{T_c}{T_c^{SSM}} - 1 \right)^{1.92}} \quad (14)$$

$$\Phi_{CNO} \propto [x T_c^{20} + (1-x) T_c^{23}] \frac{1}{1 + 13.66 \left(\frac{T_c}{T_c^{SSM}} - 1 \right)^{0.43}} \quad (15)$$

(where x : percentage of the contribution of ${}^{13}\text{N}$ and ${}^{15}\text{O}$; $(1-x)$: ${}^{17}\text{F}$)
and

$$\phi_{Be} \propto T_c^{11} \frac{1}{1 + 196.26 \left(\frac{T_c}{T_c^{SSM}} - 1 \right)^{1.29}} \quad (16)$$

The "experimental" values of δ we have selected from the measured fluxes imply that the central temperature T_c is 1% greater than T_c^{SSM} for p , B , CNO fluxes and 7% greater than T_c^{SSM} for Be flux. The first lower temperature must correspond to the ionic plasma temperature, the second greater to the electronic plasma temperature.

The meaning of these relations is very different from the one coming from the relations proposed by Bahcall et al. [16] and Calabresu et al. [21]. While, according to these authors, the central temperature must decrease respect to the SSM value to reproduce the behavior of the ratio, in our approach the central temperature must increase slightly with δ and the fluxes (except Φ_p) decrease because of the depleted Maxwellian tail and of the consequent rearrangement of the particle distribution.

- [1] E. Calabresu, G. Fiorentini, M. Lissia, B. Ricci, Nucl. Phys. B (P.S.) (1996) in press;
see also B. Ricci, Nucleosintesi e produzione di neutrini nel sole, Tesi di Dottorato, Università di Padova, 1995.
- [2] J.N. Abdurashitov et al., Nucl. Phys B (P.S.) 38 (1995) 60.
- [3] P. Anselmann et al., Phys. Lett. B 342 (1995) 440; Nucl. Phys. B (P.S.) 38 (1995) 68.
- [4] B.T. Cleveland et al., Nucl. Phys. B (P.S.) 38 (1995) 47.
- [5] Y. Suzuki et al., Nucl. Phys. B (P.S.) 38 (1995) 54.
- [6] J.N. Bahcall et al., Phys. Rev. C (1996) in press.
- [7] J.N. Bahcall, M. Pinsonneault and G. Wasserburg, Rev. Mod. Phys. 67 (1995) 781.
- [8] A. Dar and G. Shavir, *Standar solar neutrinos*, preprint: astro-ph/9604009.
- [9] J.N. Bahcall, Phys. Rev. D 44 (1991) 1644.
- [10] V. Castellani et al., Phys. Rev. D 50 (1994) 4749.
- [11] G. Kaniadakis, A. Lavagno and P. Quarati, Phys. Lett. B 369 (1996) 308.
- [12] C. Tsallis, J. Stat. Phys. 52 (1988) 479.
- [13] D. Clayton et al., Ap. J. 199 (1975) 494.

- [14] W.A. Fowler, Nature 238 (1972) 24.
- [15] W. Anderson, H. Haubold, A. Mathai, Astrophys. and Sp. Sci. 214 (1994) 49.
- [16] J.N. Bahcall, A. Ulmer, *The temperature dependence of solar neutrino fluxes*, preprint astro-ph/9602012.
- [17] LUNA experiment, private communication.
- [18] A.R. Plastino, A. Plastino, C. Tsallis, J. Phys. A 27 (1994) 5707.
- [19] S. Chandrasekhar, *An introduction to the study of stellar structure*. Chapt. IV, Dover, New York, 1958.
- [20] J. Bahcall et al., Rev. Mod. Phys. 54 (1982) 767, see Eq.(9).
- [21] E. Calabresu, G. Fiorentini, M. Lissia, Astropart. Phys. (1996) in press.

Figure Caption The reduction factors f_{red} for the fluxes Φ_B , ϕ_{CNO} and Φ_{Be} and the increasing factor f_{incr} for the flux ϕ_p as functions of δ are reported. The curves can be fitted by

$$\begin{aligned}
 f_{red}^B &= \Phi_B^{SSM}/\Phi_B = 1 + 92581.75 \delta^{1.92}, \\
 f_{red}^{CNO} &= \Phi_{CNO}^{SSM}/\Phi_{CNO} = 1 + 22.28 \delta^{0.43}, \\
 f_{red}^{Be} &= \Phi_{Be}^{SSM}/\Phi_{Be} = 1 + 851.71 \delta^{1.287}, \\
 f_{incr}^p &= \Phi_p/\Phi_p^{SSM} = 1 + 0.89 \delta^{0.62}.
 \end{aligned}$$