THE ULTRA HIGH ENERGY NEUTRINO–NUCLEON CROSS SECTION ∗

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ABSTRACT

The ultra-high energy neutrino nucleon cross section grows at a surprising rate with energy due to QCD effects in the target. Recent electroproduction data allows an update of earlier predictions. We compare the results of our own calculations with those of other groups, and critically review the foundation and reliability of the calculations. The question of the “new physics” potential of neutrino telescopes sensitive to the total cross section in multi-PeV energy domain is considered. We point out a loophole in the arguments which might be an important consideration in extrapolating the cross section to extremely high energies.

1. Overview

The neutrino-nucleon interaction cross section illustrates different physics in different energy regimes. In the ultra-low energy region $\sigma_{\nu N}$ goes like $E_{\nu}^2$, an effect of the non-relativistic phase space of the final state electron. When the reaction products are relativistic, $\sigma_{\nu N}$ goes like $E_{\nu}$, which can be guessed from dimensional analysis of the 4-Fermi theory: $\sigma_{\nu N} \propto \sqrt{s} \propto G_F^2 s$, where $M_N$ is the target nucleon mass. In both of these regimes the cross section is exceedingly small: a parameterization

\[ \sigma_{\nu N}^{tot,4-Fermi} = 6.7 \times 10^{-39} \text{cm}^2(E_{\nu}/\text{GeV}). \]

Despite the smallness of the numbers, the linear increase of $\sigma_{\nu N}$ with energy indicates a problem with probability conservation, the well-known “unitarity violation” of the 4-Fermi model. The problem is fixed by including $W$-propagator effects present in the Standard Model, which kills the linear rise with energy at about $E_{\nu} = m_N/m_W^2$. If only the effects of the $W$-propagator on an elementary target were included, then the total cross section would only vary logarithmically with energy above this region.

The story becomes more interesting, especially for neutrino telescope purposes, in the ultra-high energy region. This is because the nucleon target is a complicated composite object, full of quarks, antiquarks and gluons. To the fast moving $W$ boson the proton looks sort of like a high-octane *grappa* \(^2\) of quark-antiquark pairs. More formally, the reaction is best described using the Bjorken variable \(x_{Bj} = Q^2/(2m_N E_W)\) and the momentum transfer \(Q^2\) of deeply inelastic scattering. With a spatial resolution set by \(Q^2\), the $W$ sees quarks (and antiquarks) which are carrying momentum fraction \(x = x_{Bj}\). The total cross section goes like each elementary cross section, times the proper number of quarks and antiquarks and their known weak charges-squared.

Recent experimental studies at HERA have confirmed that there are surprisingly many quark-antiquark pairs in the nucleon. This QCD phenomenon builds a significant enhancement of an electroweak cross section; it is a big effect that, if neglected, would lead to sizable errors in the energy region above $E_\nu = 100$ TeV. The effects have been incorporated \(^3\) with calculations in the Standard Model, resulting in handy, reliable formulas to be displayed below. From the neutrino telescope point of view, the QCD enhancement is very welcome, boosting the detection rates for interesting cosmic ray neutrinos from “out there” as we study them “down here” on planet Earth.

2. The Calculation

The effects of $W-$ and $Z-$ exchange in the Standard Model are described by well understood prefactors multiplying certain structure functions $F_{\nu N}$ that parameterize the target. The behavior of neutrino versus anti-neutrino beams and the $y$-dependence of differential cross sections are fixed by boson exchange kinematics and the known mixtures of vector and axial currents. Due to the special kinematic conditions, the differential cross sections at UHE can be written directly as

$$\frac{d\sigma}{dxdy} \bigg|_{UHE} = \frac{G_F^2 M_W^2 M_{W^2}}{2\pi} \frac{M_W^2}{s} (1 - y + y^2/2) F_2^{\nu N}(x,Q^2 = s x y)/(M_{W^2}/s + x y)^2 \quad (2)$$

A useful rule of thumb tells what the dominant $Q^2$ and $x$ regions in the integrations will be. One needs $Q^2 \ll M_W^2 \lesssim 6.6 \times 10^3$ GeV\(^2\). In addition, the important part of the reactions come when the squared center of mass energy of the struck–quark and neutrino subsystem is big:

$$2M_N E_\nu x \gtrsim M_W^2 \quad (3)$$

One therefore needs the quark and anti-quark number densities $(q_i(x), \bar{q}_i(x))$ in the region $x \gtrsim M_W^2/2E_\nu$, or in simpler form, $x = 3.7 \times 10^{-3}(E_\nu/100 TeV)^{-1})$. The first group to have noticed that small-$x$ is important seems to be Andreev, Berezinsky and Smirnov \(^5\). Those early calculations used parameterizations of the quark structure functions which were no good at small-$x$. Our contributions began with Ref \([4]\), where we put in more relevant QCD-inspired small–$x$ structure, also studied again by Ref. \([6]\).
The H1 and ZEUS experimental groups have recently found that the relevant number of partons involved in electroproduction, $e^2 x (\bar{q}_i(x) + q_i(x))$, increases like a fractional power of $1/x$. Previously data did not exist for the region $x \lesssim 10^{-2}$; HERA extends measurements down to $x = 10^{-4} - 10^{-5}$. However, the $Q^2$ values are mismatched for the cross section we need here: HERA has $Q^2$ of order 1-10 GeV$^2$ at smallest $x$. Moreover, the particular mix of flavors and charges measured in the experiment is not the same as the one involved in neutrino (or anti-neutrino) scattering.

We can overcome these problems. In the small-$x$ region of $x \lesssim 10^{-2}$, the proton loses almost all identity, becoming almost pure flavor singlet, with equal numbers of quarks and antiquarks. This is called the “sea dominance”. We know this is true by studying the evolution in $x$ and $Q^2$, which is dominated by gluons making gluons, and then coupling to quark and anti-quark pairs in the sea. Because of this, one can safely transcribe data measured by the photon into that measured by $W$’s and $Z$’s by suitable changes of coupling constants, as described in Refs.[3,4]. This approximation, also called “neglecting the valence”, is the right thing to do to understand the physics. It has been confirmed to be a good approximation at UHE.

The $Q^2$ evolution based on these ideas describes the data beautifully: our updated results for the QCD evolution subsumes several dozen data points with $\chi^2$/degree of freedom $=0.8$. The theoretical uncertainty from this procedure is typical of next-to-leading order QCD: it is about 10% relative error. This can be confirmed by examining the results of different fits which select different sets of data in global analyses of structure functions. Our own calculations are unique in using theoretical approximations which fit the small-$x$ region locally. They have recently been updated by new data at even smaller $x$, slightly changing the fits to give new results reported below. The fact that perturbation QCD reproduces the $Q^2$ dependence so well strongly indicates that the theory is reliable. As a point of caution, present theory remains limited to about 10% accuracy by a host of uncertainties, so that at this point citing 3-digit accuracy in numerical work seems unjustified.

After fitting the data at low $Q^2$, we use QCD evolution to predict the parton distributions measured by an UHE neutrino. The singular nature of the newly measured parton distributions makes our conservative estimates of some years ago obsolete. In addition, the objects known as dGLAP splitting functions must be employed at both leading and next-to-leading order. Since we did this, and all groups which have parton distributions on the market also do this, the procedure is not controversial, and this part of the physics can be considered reliable.

Let us note for later reference that the smallest $x$ values actually measured puts an upper limit on the region where the cross section can be predicted with high certainty: $x \geq 3 \times 10^{-5}$ translates into a region of $E_{\nu} \leq 10^4 TeV = 10 PeV$. This happens to include the region where radio detection is the best prospect, as discussed elsewhere.
3. Results and Reliability

In the region of energy from 1 TeV to 10 TeV the approximation of neglecting the valence is not wonderful, but tolerable with errors of 25–50%. The method begins to work well in the next decade of energy, and a single published data point at 70 TeV is nicely predicted. Note that a different number is used in Ref. [8], which cites unpublished values for the average used in its comparison. In the region of $E_{\nu} > 100$ TeV and on up, it is a useful approximation of our recent work to write simply

$$\sigma_{\text{tot}} = 1.2 \times 10^{-32} \text{cm}^2 \left(\frac{E_{\nu}}{10^{18} \text{eV}}\right)^{0.40}$$

(4)

Resonant $W$ production from electron antineutrinos on electron targets (the “Glashow resonance”) is not included in the formula above. The resonance occurs at $E_{\nu} = 6.7$ PeV, with an energy width of about $\pm 130$ TeV. The resonance is tall but narrow on the cosmic ray scale, and it is not a major effect in the absorption of a broad neutrino spectrum. However, under certain circumstances the resonance might be useful to calibrate detection methods (including radio) in this region.

It may be surprising that particle physics can make reliable predictions for energies that are orders or magnitude higher than those measured in the laboratory. It can done because “new physics” is severely restricted by precision electroweak measurements. There is next to no room for bosons lighter than the known $W$ and $Z$, that couple leptons to quarks in the t-channel. If one made the hypothesis anyway, the couplings would have to be so weak that the processes could not compete in the total cross section. Of course there might be weak bosons that are much, much heavier than the $W$. This has caused colleagues to ask us whether the puzzles recently seen in Fermilab jet cross sections, which may indicate new glue-type physics, might not be replayed in some form for neutrinos. The practical answer is no: the total cross section is quite insensitive to such effects which occur at kinematic boundaries. For example, the proton total cross section is practically unaffected by those anomalies: $\sigma_{\text{tot}}$ is still determined by soft physics and pion exchanges even at the gigantic energies of colliding Tevatron beams. Similarly, new physics for neutrinos might show up in a rare process with very large momentum transfer, but such events are very unusual and require instruments specialized to see them. Again, the Fermilab events may be explainable by small adjustments in the parton distributions near $x \approx 1$. The effects of these on $\sigma_{\text{tot}}^{\nu N}$ at UHE are very small. For the total cross section we cannot beat the vector bosons we already know; it is a general rule of scattering theory that total cross sections are dominated by the lightest t-channel exchange. As for something new happening in the s-channel, leptons do not annihilate with quarks, up to exceptionally good limits from proton (non-) decay, etc. Speculative $s$-channel production of heavy bosons on atomic electrons can be contemplated. This might make a big
effect, but only in a limited resonance region. Could a neutrino telescope see this? Perhaps the question merits investigation.

Another possibility for finding new physics has been suggested by Learned and Pakvasa\textsuperscript{13}. They point out that tau neutrinos, if any, would be an indication of neutrino oscillations, with a signal of a “double–bang” at PeV energies from the charged current event and later decay of the tau lepton.

Our group originally got interested in the total cross section problem via a long series of efforts to evade our own logic. The motivation was the Cygnus X3 problem, which we hoped new neutrino physics might explain.\textsuperscript{4,20} We only discovered the small-$x$ QCD effect by going through all possibilities we could imagine, convincing ourselves that it is the biggest non-resonant thing that actually can happen, and finding Ref. [5] after working things out our own way.

### 4. Third Order Second Thoughts

Having often reconsidered our second thoughts, we were asked\textsuperscript{15} to think more deeply on these questions: is it really rock-hard-reliable that QCD and the Standard Model can predict the cross section? There is a loophole. It is in the extrapolations presently used to go to smaller-$x$ than has been measured. As mentioned earlier, we know the $x$-distributions down to about $10^{-5}$, corresponding to laboratory-based predictions up to neutrino energies of about 10 PeV. Extrapolations above this energy are really based on theoretical expectations, which might be right or wrong.

Two approaches exist. One is to apply the criteria of Gribov, Levin and Ryskin\textsuperscript{16}, which allows parton densities at small-$x$ to continue to grow until unitarity is confronted at the $Q^2$ of the process. Theorists who try to predict the $x$-dependence\textsuperscript{17} tend to predict a power-law rise (which is an eigenvector of dGLAP evolution). In this sense the theory is under control. For $Q^2 = M_W^2$, the results of this approach is continued growth of the cross section up to an extraordinarily high energy, of order $E_\nu \approx 10^{20}$ eV. The point of leveling–off occurs where the $\nu p$ cross section approaches the $pp$ total cross section! Since we first applied this criterion to the neutrino problem, we are fond of it and have promoted it accordingly.

On the other hand, hadrons are complicated beasts that nobody understands well. It would not violate any known laws of Nature for the observed small-$x$ growth simply to stop. This might happen at a point earlier than current estimates indicate: experimental measurements are needed to know for sure. A guesstimate might be $(\alpha_s/2\pi) \log(1/x) \cong 1$ indicating saturation at $x$ below the $10^{-5}$ region. If this is correct, then one cannot absolutely rely on having a gigantically rising cross section forever, and $\sigma_{tot}^\nu N$ might round off above 10 PeV. As responsible theorists, we are obliged to note this disconcerting possibility. We believe that the value at 10 PeV
serves as a lower bound on $\sigma_{\nu N}^{\nu N}$ if this phenomenon should occur.

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6. References

2) Probably Nebbiolo
3) G. M. Frichter, D. W. McKay, and J. P. Ralston, *Phys. Rev. Lett.* **74** (1995) 1508; ibid *Erratum*, (in press). The erratum corrects a typographic error in the symbol A which should read $A = 0.051 \pm 0.010$, and in $\alpha_0$ which should read $\alpha_0(Q^2)/2\pi = 6/(11N_c - 2N_f)\ln(Q^2/\Lambda_{QCD}^2)$.
11) We thank Buford Price for suggestions and discussions on this point.
15) Special thanks to Al Mann.


19) See the contribution “Toward Radio Detection of PeV Neutrinos on the Cubic Kilometer Scale” by G. Frichter *et al.* (these proceedings).