We investigate the spectrum of stochastic gravitational wave background generated by hybrid topological defects: domain walls bounded by strings and monopoles connected by strings. Such defects typically decay early in the history of the universe, and their mass scale is not subject to the constraints imposed by microwave background and millisecond pulsar observations. Nonetheless, the intensity of the gravitational wave background from hybrid defects can be quite high at frequencies above $10^{-8}\text{Hz}$, and in particular in the frequency range of LIGO, VIRGO and LISA detectors.
The most interesting cosmological events, from the particle physics point of view, occurred during the first second after the big bang. However, the universe became transparent to electromagnetic waves only at the time of decoupling, \( t_{\text{dec}} \approx 6 \times 10^{12} (\Omega h^2)^{-1/2} \text{s} \), and our ability to observe these events is very limited. In fact, it appears that direct signals from sources at \( t < 1 \text{s} \) can arrive to us only in the form of gravitational waves. A new generation of gravitational-wave detectors will start operating early next century [1], and now is an appropriate time to think about the possible signals we can get from the very early universe.

A number of potential sources have already been suggested [2]: quantum fluctuations of the metric during inflation [3,4], oscillating loops of cosmic string [5], and colliding bubbles in a first-order phase transition [6]. In this paper we shall investigate the spectrum of gravitational waves generated by transient topological defects, monopoles connected by strings and domain walls bounded by strings.

Among the most significant cosmic events are the phase transitions corresponding to spontaneous symmetry breakings in the early universe. Such transitions are typically accompanied by the formation of topological defects [7,8]. Depending on the topology of the symmetry groups involved, the defects can be in the form of surfaces, lines, or points. They are called domain walls, strings, and monopoles, respectively. Hybrid defects can be formed in a sequence of phase transitions, e.g., the first transition produces monopoles, which get connected by strings at the second phase transition. Such defects are transient and eventually decay into relativistic particles. If this happens at a sufficiently early time, the decay products thermalize, and we can see no trace of the defects, except perhaps in the form of gravitational waves [9]. In the following, we have adopted units such that \( c = \hbar = 1 \).

**Walls bounded by strings.** The simplest sequence of phase transitions that results in walls bounded by strings is \( G \rightarrow H \times Z_2 \rightarrow H \). The first phase transition produces strings, and at the second each string gets attached to a domain wall.

The evolution of strings prior to wall formation is identical to that of “ordinary” strings (which do not get attached to walls). After a period of overdamped motion, the strings
start moving relativistically and approach a scaling regime where the characteristic scale of
the network is comparable to the horizon. A horizon-size volume at cosmic time \( t \) contains
a few long strings stretching across the volume and a large number of small closed loops.
The typical distance between long strings and their characteristic curvature radius are both
\( \sim t \), but in addition the strings have small-scale wiggles of wavelength down to \( l \sim \alpha t \), with
\( \alpha \ll 1 \). The typical size of the loops being chopped off the long strings is comparable to the
scale of the smallest wiggles, \( l \). The loops oscillate and lose their energy by gravitational
radiation. The lifetime of a loop of size \( l \) is \( \tau \sim l/G\mu \), where \( G \) is the Newton’s constant,
\( \mu \) is the mass per unit length of string, and \( \Gamma \sim 50 \) is a numerical coefficient.

The exact value of the parameter \( \alpha \) is not known. Numerical simulations of string
evolution [10] suggest that \( \alpha \lesssim 10^{-3} \), while the analysis of gravitational radiation back-
reaction indicates that \( \alpha \gtrsim \Gamma G \mu \). Thus, we expect \( \alpha \) to be in the range \( 10^{-3} > \alpha \gtrsim \Gamma G \mu \)
for \( G \mu \ll 10^{-5} \) and \( \alpha \sim \Gamma G \mu \) for \( G \mu \gtrsim 10^{-5} \). We note that for \( \alpha \sim \Gamma G \mu \) the loops decay in
about one Hubble time from their formation.

The evolution of the network after domain walls form, having the strings as their bound-
daries, depends on the relative magnitude of the wall tension \( \sigma \), and the force per unit length
on a curved string due to the string tension \( \mu \), \( F \sim \mu/t \), where we have assumed that the
curvature radius of string is \( R \sim t \). If the walls are formed at \( t_w \ll \mu/\sigma \), then initially they
have little effect on string dynamics. The walls become dynamically important at \( t \sim \mu/\sigma \),
when they pull the strings towards one another, and the network breaks into pieces of wall
bounded by string. Alternatively, if the walls form at \( t_w \gtrsim \mu/\sigma \), then the breakup of the
network occurs immediately after the wall formation. We shall use the notation \( t_s \) for the
time when the strings start moving relativistically and \( t_s = \max\{\mu/\sigma, t_w\} \) for the time of
the network breakup. The typical size of the pieces is expected to be \( \sim \alpha t_s \).

Gravitational waves emitted by oscillating loops of string during the period \( t_s < t < t_s \)
add up to a stochastic gravitational-wave background with a nearly flat spectrum [5,11–13],

\[
\Omega_g(\nu) \equiv \frac{\nu}{\rho_c} \frac{d\rho_g}{d\nu} \sim 300 \left( \frac{\alpha G \mu}{\Gamma} \right)^{1/2} \left( \frac{N'}{N_c} \right)^{1/3} \Omega_r. \quad (1)
\]
Here, $\rho_g$ is the energy density of gravitational waves, $\rho_c$ is the critical density, $\nu$ is the frequency, $N(t)$ is the effective number of spin degrees of freedom in the cosmic plasma at time $t$, $N_e$ is the value of $N$ when waves of frequency $\nu$ were emitted, and $\Omega_r = \rho_r/\rho_c$, where $\rho_r$ is the energy density in radiation and relativistic particles. The spectrum (1) extends over a range of frequencies, $\nu_{\text{min}} < \nu < \nu_{\text{max}}$, which is determined by the times $t_s$ and $t_*$. For a rough estimate, we can use a simple model which assumes that the loops emit most of their energy in the first few harmonics with frequencies $\nu \sim 2/l$, where $l$ is the loop’s length.

To simplify the discussion, we shall assume also that $t_*$ is in the radiation era and that $\alpha \sim \Gamma G\mu$ (which should be reasonably accurate for superheavy strings with $G\mu \gtrsim 10^{-6}$). Then, gravitational waves of present frequency $\nu$ were emitted at

$$t_\nu \sim 10^{-10} \mu_{-6}^{-2} \left(\frac{\nu}{1 \text{Hz}}\right)^{-2} \text{s},$$

where $\mu_{-6} = G\mu/10^{-6}$ and we have used the approximate form, $a \propto t^{1/2}$, for the scale factor during the radiation era. The cutoff frequencies $\nu_{\text{min}}$ and $\nu_{\text{max}}$ correspond to the times $t_*$ and $t_s$, respectively. In a more realistic model, with a full account taken of the radiation spectrum by the loops, the cutoffs would be spread over about two orders of magnitude in frequency, while the middle part of the spectrum (1) would be unaffected.

According to Eq.(2), gravitational waves in the frequency range of the LIGO and VIRGO detectors ($\sim 100$ Hz) were emitted at $t_{LV} \sim 10^{-14} \mu_{-6}^{-2}$s, while waves in the sensitivity range of the millisecond pulsar ($\sim 10^{-8}$ Hz) [14] were emitted at $t_{MP} \sim 10^6 \mu_{-6}^{-2}$s. For $t_{LV} < t_* < t_{MP}$, the gravitational wave background is in the frequency range accessible to LIGO/VIRGO, but not to the pulsar. Since in this case the strings decay before decoupling, they leave no trace on the microwave radiation, and thus the only constraint on the possible values of $G\mu$ comes from the nucleosynthesis considerations [15].

At the time of nucleosynthesis, the total energy density in gravitational waves, in terms of the critical density, is given by

$$\Omega_g(\text{nucl}) \sim 150 G\mu \left(\frac{N_{\text{nucl}}}{N_*}\right)^{1/3} \ln \left(\frac{t_*}{t_s}\right).$$

(3)
Here, $N_*$ and $N_{\text{nucl}} \sim 10$ are, respectively, the values of $N$ at $t_*$ and at the time of nucleosynthesis, $t_{\text{nucl}} \sim 1$ s, and we have assumed for simplicity that the value of $N$ does not change through the whole period from $t_s$ to $t_*$. For the standard nucleosynthesis scenario to work, $\Omega_g(\text{nucl})$ should not exceed 0.054. With $N_* \sim 100$, the resulting bound on $G\mu$ is

$$G\mu \lesssim \frac{8 \times 10^{-4}}{\ln(t_*/t_s)}.$$  \hspace{1cm} (4)

With the same assumptions, Eq. (1) gives

$$\Omega_g(\nu) \sim 6 \times 10^{-3} G\mu h^{-2}.$$ \hspace{1cm} (5)

The logarithm in Eq. (4) can take values from $\sim 2$ to $\sim 100$. Consider, for example, strings with $G\mu \sim 10^{-5}$ and walls with symmetry breaking scale $\eta_w \sim 100$ GeV. If the symmetry breaking potential for the walls does not have very small couplings, then $\sigma \sim \eta_w^2$ and $t_* \sim 1000$ s. The time $t_*$ when the strings start moving relativistically depends on the cosmological scenario. In some models this will be the time when the damping force due to the interaction of strings with plasma becomes smaller than the force of tension in the strings, $t_s \sim (G\mu)^{-2} t_p \sim 10^{-33}$ s, where $t_p$ is the Planck time. Then, $\ln(t_*/t_s) \approx 80$, and the constraint (4) is marginally satisfied.

The time $t_s$ can be greatly increased in inflationary models where the string-forming phase transition occurs during inflation, but sufficiently close to its end, so that the strings are not completely inflated away. In this case, $t_s$ is the time when the typical distance between the strings becomes smaller than the Hubble radius. Strings can also be produced in a “pre-heating” transition after inflation [16]. The amplitude of scalar field fluctuations at pre-heating can reach Planckian values, and superheavy defects can be formed even if the thermalization temperature after inflation is very low. The time $t_s$ is then shortly after the end of inflation. In some supersymmetric models, certain couplings can be naturally small, and superheavy strings can be formed as late as the electroweak phase transition [17]. Then, $t_s \sim t_{\text{ew}} \sim 10^{-11}$ s. Finally, the effective value of $t_s$ can be increased by massive particle annihilations (that is, by a decrease of $N$ between $t_s$ and $t_*$), resulting in a dilution of gravitational waves from earlier epochs.
The time $t_*$ is, of course, also model-dependent (through the parameters $\mu$ and $\sigma$), and in general $t_s$ and $t_*$ do not have to be separated by many orders of magnitude. With $\ln(t_*/t_s) \sim 1$, the constraint (4) requires only that $G\mu \lesssim 8 \times 10^{-4}$, and Eq. (5) gives $\Omega_g(\nu) \gtrsim 5 \times 10^{-6}h^{-2}$. The first version of LIGO interferometer will be able to detect a stochastic background with $\Omega_g(\nu) \gtrsim 2 \times 10^{-6}h^{-2}$ after two years of observations, at frequencies $\sim 100$ Hz and with a 90% confidence [2], and thus a detection of gravitational waves from a string-wall network is only marginally possible. Advanced LIGO and VIRGO will be sensitive to $\Omega_g(\nu) \gtrsim 5 \times 10^{-11}h^{-2}$, which corresponds to $G\mu \gtrsim \times 10^{-8}$. A similar sensitivity will be achieved by LISA space-based interferometer at $\nu \sim 10^{-3}Hz$.

Monopoles connected by strings. The prototypical sequence of symmetry breakings resulting in monopoles connected by strings is $G \to H \times U(1) \to H$. Monopoles formed at the first phase transition get connected by strings at the second phase transition. If the monopole-forming phase transition occurs after inflation, then the average monopole separation is always smaller than the Hubble radius, and when monopole-antimonopole ($M\bar{M}$) pairs get connected by strings and start oscillating, they typically dissipate the bulk of their energy to friction in less than a Hubble time [8,18]. This does not result in any appreciable gravitational-wave background.

The most interesting scenario, for our purposes, is when monopoles are formed during inflation (but are not completely inflated away). Strings can either be formed later during inflation, or in the post-inflationary epoch. The characteristic length scale of strings at formation is then much smaller than the monopole separation; the strings connecting monopoles have Brownian shapes, and there is also a large number of closed loops. The evolution of strings is initially identical to that of topologically stable strings, without monopoles. At $t \sim t_*$, the strings start moving relativistically and generate a gravitational-wave background of intensity (1). In the course of the evolution, the characteristic length of strings grows like $t$ and eventually becomes comparable to the monopole separation, so that we are left with $M\bar{M}$ pairs connected by more or less straight strings. We shall call the time when this happens $t_m$ and the monopole separation at that time, $d$ (note that $d \sim t_m$). At $t > t_m$,
$M\bar{M}$ pairs oscillate and gradually lose their energy by gravitational radiation.

In the above discussion we have assumed implicitly that $t_m > t_s$. In this case, using the inequalities $d \sim t_m > t_s \gtrsim (G\mu)^{-2}t_p$, it is easily verified that

$$\mu d \gg m,$$

(6)

where $m$ is the monopole mass. Then the motion of monopoles is relativistic, with a typical Lorentz factor

$$\gamma \sim \mu d/m \gg 1.$$

(7)

If strings are formed during inflation, soon after the monopoles, then the strings connecting $M\bar{M}$ pairs can be nearly straight. In this case, the strings do not go through a period of relativistic evolution, and no gravitational radiation is produced prior to $t_m$. But unless these defects are formed very close to the end of inflation, the inequality (7) will still be satisfied. We shall assume this to be the case.

Oscillating $M\bar{M}$ pairs lose their energy by gravitational radiation and by friction due to scattering of plasma particles off the monopoles [19]. To estimate the rate of gravitational energy loss, we calculated the power and the spectrum of gravitational waves radiated by an oscillating pair connected by a straight string. Details of this calculation will be given elsewhere [20], while here we shall only summarize the relevant features of the radiation spectrum. Most of the energy of an oscillating $M\bar{M}$ pair is radiated in the frequency range

$$d^{-1} \lesssim \nu \lesssim \gamma^2 d^{-1},$$

(8)

with a spectrum

$$d\mathcal{E}_g/dtd\nu \approx 4G\mu^2\nu^{-1}.$$  

(9)

At higher frequencies, $d\mathcal{E}_g/dtd\nu \propto \nu^{-2}$. The total radiation power is [21]

$$\dot{\mathcal{E}}_g = \tilde{\Gamma}G\mu^2,$$

(10)

where
\[ \tilde{\Gamma} \approx 8 \ln \gamma \]  

is a numerical coefficient taking values in the range \(10 \lesssim \tilde{\Gamma} \lesssim 10^3\), depending on the Lorentz factor \(\gamma\).

A monopole-antimonopole pair connected by a straight string is, of course, a very special configuration, and one could be concerned that the radiation spectrum in this case may be very different from that for a generic configuration. To address this issue, we calculated the spectrum of electromagnetic radiation emitted by a pair of oscillating, equal and opposite charges connected by a straight string. We found that the resulting spectrum is qualitatively similar to that in the generic case \([22]\). This suggests that the same may be true in the case of gravitational radiation, but of course a detailed analysis is required before any definite conclusion can be drawn. Here, we shall assume that the spectrum \((9)\) is representative of the generic case \([23]\).

The frictional energy loss is of the order

\[ \dot{E}_f \sim T^2, \]  

where \(T\) is the temperature, and it is easily verified that \(\dot{E}_f \ll \dot{E}_g\) for \(t_m \gg t_s\). Hence, the lifetime of a pair is

\[ \tau \sim \frac{\mu d}{\dot{E}_g} \sim \frac{d}{\tilde{\Gamma} G \mu}. \]  

The number density of \(M\bar{M}\) pairs at \(t \sim t_m \sim d\) is \(n_M(t_m) \sim d^{-3}\), and at later times \(n_M(t) \sim (td)^{-3/2} \quad (t_m < t < \tau)\). The spectral power of the gravitational wave background emitted by the oscillating pairs per Hubble time at time \(t\) is

\[ \Omega_g(t) \sim 4G\mu^2 t n_M(t)/\rho_c(t) \sim 120(G\mu)^2(t/d)^{3/2}, \]  

where \(\rho_c(t) \sim 1/30Gt^2\) is the critical density. The highest power comes from the radiation emitted at \(t \sim \tau\). The frequency range \((8)\) at that time corresponds to the present range

\[ \nu_* \lesssim \nu \lesssim \gamma^2 \nu_*, \]  

8
\[ \nu_\ast \sim d^{-1} (\tau/t_{\text{eq}})^{1/2} z_{\text{eq}}^{-1}, \]  

(16)

t_{\text{eq}} is the time of equal matter and radiation densities, and \( z_{\text{eq}} \) is the corresponding redshift. The spectral power in the range (15) is [compare with Eq.(1)]

\[ \Omega_g(\nu) \sim 120 \tilde{\Gamma}^{-3/2} (G\mu)^{1/2} (N/N_\tau)^{1/3} \Omega_r. \]  

(17)

For \( \nu < \nu_\ast \), the main contribution to the spectrum is made at the time \( t_\nu \sim (\nu/\nu_\ast)^2 \tau \), when the frequency corresponding to \( \nu \) at the present time is \( \sim d^{-1} \). The resulting spectral power is given by Eq.(17) with an additional factor \((\nu/\nu_\ast)^3\). This part of the spectrum extends to the minimum frequency \( \nu_{\text{min}} \sim (\tilde{\Gamma} G\mu)^{1/2} \nu_\ast \).

Radiation emitted prior to \( t_{\text{nucl}} \sim 1\,\text{s} \) should satisfy the nucleosynthesis constraint. Assuming that \( \tau < t_{\text{nucl}} \), the total energy density in gravitational waves at \( t_{\text{nucl}} \) is

\[ \Omega_g(\text{nucl}) \sim 30(G\mu/\tilde{\Gamma})^{1/2} (N_{\text{nucl}}/N_\tau)^{1/3} \lesssim 0.05, \]  

(18)

where we have used Eq.(11). Combining this with (17), we have

\[ \Omega_g(\nu) \lesssim 10^{-5} \tilde{\Gamma}^{-1} h^{-2} \]  

(19)

For the frequency range (15) to overlap with the sensitivity range of LIGO/VIRGO, it is necessary that \( d \gtrsim 2 \times 10^{-13} (\tilde{\Gamma} G\mu)^{-1} \, \text{cm} \), which implies \( \tilde{\Gamma} \lesssim 400 \) (assuming that \( m < m_p \), where \( m_p \) is the Planck mass), and thus \( \Omega_g(\nu) \lesssim 2 \times 10^{-8} h^{-2} \). This is below the sensitivity of the first version of LIGO but may be within the reach of the advanced detectors.

If the strings did have a period of relativistic evolution, then the spectrum includes another flat region with spectral power (5) at \( \nu > (\tilde{\Gamma} G\mu)^{-1/2} \nu_\ast \). This power is greater than (17) if \( G\mu > \tilde{\Gamma}^{-3} \) which, for \( \tilde{\Gamma} \sim 10^3 \), includes all interesting values of \( G\mu \). The analysis of this part of the spectrum is identical to that for walls bounded by strings, with \( t_\ast \) replaced by \( t_m \). As before, the advanced detectors will have sufficient sensitivity in the case of strings with \( G\mu \lesssim 10^{-8} \), provided that the flat portion of the spectrum includes \( \nu \sim 100 H_\text{z} \).

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REFERENCES


[9] There are some notable exceptions to this rule. If the constituent fields of the defects have baryon-number-nonconserving couplings, then the decay of transient defects can result in a net baryon asymmetry of the universe. Also, in the case of global defects, a background of very weakly interacting goldstone or pseudo-goldstone bosons can be generated.


[19] Here we assume that monopoles do not have unconfined magnetic charges (that is, the only magnetic charge they have is the one whose flux is squeezed into the strings). Otherwise, monopoles would also lose energy by radiating gauge quanta.


[21] Note that Eq.(10) could be obtained as a rough estimate using the quadrupole formula, 
\[ \dot{E}_g \sim G\mathcal{E}^2 d^4\nu^6 \sim G\mu^2, \]
where \( \mathcal{E} = \gamma m \sim \mu d \) is the monopole energy and \( \nu \sim d^{-1} \) is the characteristic frequency of oscillation. However, the validity of the quadrupole formula for ultrarelativistic monopoles is rather dubious.


[23] At large frequencies, the \( \nu^{-2} \) behaviour comes from the discontinuity in the monopoles acceleration when they pass one another and the string tension is reversed. In a more general case, the acceleration should vary smoothly and the spectrum will fall exponentially.