New insights into the mechanism and character of core–collapse supernova explosions are transforming the approach of theorists to their subject. The universal realization that the direct hydrodynamic mechanism does not work and that a variety of hydrodynamic instabilities can influence the viability of theoretical explosions has ushered in a new era in supernova modeling. In this paper, I discuss the important physical and technical issues that remain. I review the neutrino–driven mechanism, the possible roles of Rayleigh–Taylor instabilities, questions in neutrino transport, and the various observational constraints within which theorists must operate. However, a consensus has yet to be achieved among active workers concerning many important details and some essential phenomenology. This synopsis is meant to accomplish two things: 1) to focus attention on the interesting problems whose resolution will bring needed progress, and 2) to assess the current status of the theoretical art.

1 Introduction

A new synthesis is emerging in the theory of core-collapse supernovae. This is not meant to imply that the basic mechanism has been found in their multi-dimensional character or that a compelling consensus has been amicably reached. Rather, new information and new ideas are accumulating at such a rapid rate on both the theoretical and observational fronts that questions and facts long ignored by “collapse” theorists can no longer be relegated to an indefinite future. The epiphany of SN1987A was a turning point, but new

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computational capabilities that emerged in the interim have also played a role, as has the exponentiation of astronomical data in the eighties and nineties. A complete theory of supernova explosions must explain a variety of observations and facts. Among the questions of relevance are:

1. What determines the scale of the supernova explosion energy ($E_{SN}$)? Why are $E_{SN}$ for SN1987A and SN1993J $\sim 1.5 \pm 0.5 \times 10^{51}$ ergs [1]?
2. What is the mass spectrum of the residual neutron stars and pulsars? Why is the average mass of the well-measured pulsars $\sim 1.35 M_\odot$ [2]?
3. Which progenitors leave black holes? What is their mass spectrum? What determines the masses of the X-ray Nova primaries, observed to range from $\geq 3.0 M_\odot$ to $16 M_\odot$ [3]?
4. What are the radioactive $^{56}$Ni, $^{57}$Ni, $^{26}$Al, $^{44}$Ti, $^{60}$Fe yields as a function of progenitor mass? Why are the $[^{57}Fe/^{56}Fe]$ and $[^{44}Ca/^{56}Fe]$ ratios what they are and what does this imply for the explosion mechanism and mass cut?
5. Similarly, what is the dependence of the iron-peak (e.g., $^{54,56,58}$Fe, $^{60,61,62}$Ni, $^{59}$Co, $^{55}$Mn, $^{45}$Sc, $^{52}$Cr, $^{48}$Ti, $^{64,66,68}$Zn, $^{51}$V, $^{63,65}$Cu) yields on progenitor star and mechanism?
6. When, how, and where are the r-process nuclei produced and ejected? Do different progenitors have different r-process yields?
7. What is the explanation for the intermediate mass, iron-peak, and r-process abundances observed in low-metallicity halo stars [4–6]?
8. Is there a range of intrinsic kicks or recoils imparted to the nascent neutron star that can explain the high proper motions observed [7]? Is there a bimodal distribution of natal kicks and, if so, why [8]? Is there a correlation between progenitor mass and kick magnitude and/or duplicity? What can be learned about the supernova explosion from pulsar spin–orbit angles [9]?
9. Do neutrinos play a significant role in the synthesis of $^{11}$B, $^{19}$F, etc. [10]?
11. Is the $\Delta Y/\Delta Z$ of galactic chemical evolution connected with the creation of stellar mass black holes [14,15]?
12. What do the abundance patterns and high metallicity in QSO absorption line systems and Lyman–α clouds at high redshifts say about the evolution and explosion of massive stars?
13. What determines the magnetic fields of radio and X-ray pulsars? With what multipolarity structure are pulsars born and why?
14. What is the spin period of pulsars at birth? Is it 2 milliseconds or 200 milliseconds? Is there a correlation between birth period, progenitor duplicity, and nascent magnetic field?

The mapping between progenitor mass and supernova characteristics (energy, mass cut, yields, recoils, B–fields, etc.) is the crucial goal of theoretical su-
pernova research. There is a solid, but evolving, literature on the relationship between progenitor mass, metallicity, and mass loss history and the density, entropy, and electron fraction structures of the progenitor cores that eventually implode [16]. Modern supernova theory seems posed to explain the influence of core structures on mass cut, post-bounce delay to explosion, explosion energy, and nickel and r–process, etc. yields, but has yet to do so. Theoretical questions that will first need to be addressed and answered are:

(1) Is mass fallback a generic feature of supernova explosions?
(2) How does progenitor structure affect the delay to explosion, explosion energy, nucleosynthetic yields, and residual neutron star mass?
(3) Is convection exterior to the gain radius (Mayle [17]) crucial, useful, or merely of secondary importance to the explosion?
(4) What is the role of deeper core instabilities (salt–finger, semi–convection, or lepton–driven) in the explosion and in the subsequent evolution of the protoneutron star?
(5) What is the post–bounce delay to explosion?
(6) How will more accurate neutrino transport influence the mechanism and development of the explosion?
(7) How will a more accurate treatment of the neutrino opacities at high densities alter the neutrino light curves and spectra and, hence, the core–mantle coupling so crucial to the explosion?
(8) What are the remaining important issues surrounding the nuclear equation of state?
(9) What is the influence, if any, of neutrino viscosity on the character and nature of the explosion?
(10) Does rotation play an important role in the supernova explosion or in supernova observables?
(11) Does the asphericity and heterogeneity of the explosion itself influence the mixing of $^{56}$Ni into the outer stellar mantle, as observed in SN1987A?
(12) When does the protoneutron star wind that follows explosion turn on and upon what physics does its emergence depend?
(13) Can this wind be the site of the r–process? Which progenitor structures are more likely to yield r–process elements at infinity?
(14) What neutrino physics determines the entropy and $Y_e$ of this wind?
(15) Can black hole formation be accompanied by a supernova explosion?
(16) Are pulsar “kicks” in part a consequence of aspherical mass motions and/or neutrino emissions that accompany collapse, bounce, and explosion? What are the essential hydrodynamic ingredients? Is there a progenitor dependence?
(17) Do neutrino oscillations or exotic neutrino physics play a role?
(18) Do hydrodynamic instabilities at birth influence pulsar B–fields?
(19) What can one learn about the supernova mechanism from the detection of a galactic neutrino burst by SuperKamiokande, SNO, or any one of the Gran Sasso detectors (LVD, MACRO, Borexino, ICARUS)?
What can be learned about the internal dynamics of “collapse” from its gravitational radiation signature in LIGO [18] or VIRGO [19]?

The program implied by the above two lists of questions is daunting and will require the collective efforts of theorists and observers over many years. My purpose in assembling these lists is to focus theorists in particular on the variety of observational constraints to which their models must already conform and to suggest interesting, but neglected, theoretical topics that might be profitably explored. In this paper, I summarize or provide a commentary on a few of the most interesting of these facts, ideas, and outstanding issues. In §2, I describe the basics of the supernova explosion phenomenon. In §3, I summarize and analyze the effects of hydrodynamic overturn. Section 4 addresses new issues concerning neutrino transport and opacities. Section 5 discusses a few interesting constraints imposed by nucleosynthesis and the r–process. Various mechanisms of pulsar natal kicks are touched on in §6, which includes a short discussion of the gravitational wave signature of one of the kick mechanisms. In §7, I wrap up with a few general comments. Throughout, I assume that the reader is familiar with both the technical issues and the history of the subject. This allows me to concentrate on the interesting questions on the frontier of supernova theory, while avoiding diversionary minutiae.

2 Perspectives on the Mechanism

It is now generally accepted that the prompt shock [20] stalls at a radius between 80 and 150 kilometers, due to photodissociation, neutrino losses, and the accretion ram [21–25]. The focus of supernova theory is now on the subsequent behavior of this quasi-steady accretion shock on timescales of tens of milliseconds to seconds. The essence of a supernova explosion is the transfer of energy from the core to the mantle. The mantle is less bound than the core, whose binding energy can grow without penalty during the delay to explosion. The core is the protoneutron star that will evolve due to neutrino cooling and deleptonization over many seconds [26]. Bethe & Wilson [27] showed that it is possible and plausible that neutrino heating of the accreted material near the shock could, over time, lead to an explosion. That neutrinos mediate this energy transfer and are the agents of explosion seems compelling [28–32]. However, this said, the roles of many important phenomena and processes still need to be delineated and refined. Foremost among these are the multi-dimensional hydrodynamic effects [23,33–35] and the neutrino opacites that regulate the driving neutrino luminosities. Before I address those topics, a few words on the explosion condition itself may prove useful.

If there were no post-bounce accretion of the outer mantle, it can be shown that due to neutrino heating the material exterior to the inner core would
be unstable to outflow. This follows directly from simple arguments, analogous to those employed by Parker [36] in his early ruminations concerning the solar wind, that the atmosphere above the neutrinosphere cannot be simultaneously in hydrostatic and “thermal” equilibrium. In the solar wind context, Parker showed that, since the thermal conductivity of the plasma is almost independent of density and depends on the temperature to a stiff power ($\propto T^{5/2}$), the equilibrium temperature profile would be a shallow function of radius ($\propto r^{-2/7}$). He then went on to show that since the power $2/7$ was less than 1, hydrostatic equilibrium of an ideal gas mantle would require a finite pressure at infinity. Without such a pressure, a wind would be driven by electron conduction heating. While this is no longer a viable model for the solar wind itself, these physical arguments translate directly to the neutrino heating/cooling context of the post–bounce protoneutron star. Since the neutrino heating rate per baryon goes as $1/r^2$ and the neutrino cooling rate per baryon goes as $T^6$, the equilibrium temperature profile in a static atmosphere would go as $1/r^{1/3}$. Since $1/3$ is below 1, a finite pressure at large distances is required in order to thwart the spontaneous excitation of a vigorous outflow. Protoneutron star atmospheres are unstable to neutrino-driven ejection. It is continuing accretion (and the large pressures it affords) that suppresses the “explosion” of the mantle. However, as is clear from hydrodynamic simulations, the accretion rate need not decay to nothing before the mantle lifts off. Deriving the precise time and circumstances of the transition from quasi-steady accretion to explosion requires some sophistication. The presence, due to accretion, of more neutrino-energy-absorbing mass than would be available in the thin wind context ultimately results in a more energetic explosion [35]. However, the outflow will always eventually make the transition to a thin stable wind [28,23,37,38].

Another way to look at the condition for explosion is to note that, in order to eject matter, neutrino heating must result in a steady matter temperature in the shocked mantle that is above the “escape temperature,” crudely derived by setting the specific internal energy equal to $GM/r$. This is the so-called “coronal” condition, familiar in many other astronomical contexts. Because heating occurs predominantly behind a shock stalled at finite radius ($R_s$), there may not be enough matter or volume exterior to the gain radius that satisfies the coronal condition and the mantle will not explode. Anything that enlarges the region in which the coronal condition is satisfied pushes the object closer to the supernova threshold. Hence, it is advantageous to increase $R_s$. Two agencies that can do this are an increased neutrino luminosity and overturning convection near the shock [39,30,23,34].

To discover the critical conditions for explosion and its subsequent development requires a full hydrodynamic code, but insight into the pre-supernova structure and its stability can be gained by studying it as a quasi-static structure in equilibrium [40]. This allows one to convert the partial differential
equations of hydrodynamics into simple ordinary differential equations and the problem into an eigenvalue problem. For a given core mass, equation of state, neutrino transport algorithm, neutrino luminosity \(L_\nu\), and mass accretion rate \(\dot{M}\), the shock radius (the eigenvalue) and the stellar profiles (the eigenfunctions) can be derived. For a given core mass, \(R_s\) as a function of the control parameters \(L_\nu\) and \(\dot{M}\) can be obtained. What Burrows & Goshy [40] showed was that for a given \(\dot{M}\), there was a critical \(L_\nu\) above which there was no solution for \(R_s\). They identified this critical \(L_\nu\) versus \(\dot{M}\) curve with the approximate condition for the onset of explosion. After the expansion commences, the problem must be handled hydrodynamically. However, expansion decreases the matter temperature and, hence, the cooling rate faster than it decreases the heating rate and the instability should run away for a given core \(L_\nu\). Since expansion cuts accretion and accretion contributes in part to \(L_\nu\), a concommitant decrease in \(L_\nu\) might be of concern. However, there is a time delay in the decrease of \(L_\nu\) due to expansion equal to the matter settling time from the shock to the core. This can be a comfortable \(\sim 30\) milliseconds and is larger for larger pre-explosion shock radii. Again, anything that increases \(R_s\) brings the star closer to the explosion threshold.

The calculations and assumptions of Burrows & Goshy [40] were crude, particularly in the transport sector, but illuminate semi-quantitatively the basics of the phenomenon. A similar analysis for quite a different system, AM Her objects, was performed by Chevalier & Imamura [41] and those who have difficulty understanding Burrows & Goshy [40] are heartily referred to that paper. Note that it is the essence of equilibrium that timescales of the relevant processes are comparable: hydrostatic equilibrium is “equivalent” to the equality of sound–travel and free–fall times. In the quasi-static pre-explosion phase of the protoneutron star bounded by an accretion shock, equilibrium is equivalent to the equality of the heating/cooling time \(\tau_\nu\) and the settling time \(\tau_s\) of matter as it sinks from the shock to the core \((\lesssim R_s/v)\). The shock radius, as the eigenvalue of the problem, adjusts to accomplish this equality of timescales. Hence, that \(\tau_\nu\) and \(\tau_s\) are comparable is a requirement of equilibrium, and is not a problem of the quasi-static assumption. It is only when the characteristic timescale for the change of \(\dot{M}\) becomes comparable to the other relevant times that the quasi-static assumption is dubious. Indeed, that timescale can at times approach \(\tau_\nu\) and \(\tau_s\), but is often significantly longer. When it is short, the core plus shock must be handled hydrodynamically. However, when it is long, the core plus mantle can also be subjected to a pulsation (perturbation) analysis around an equilibrium structure. In direct analogy with the standard stellar pulsation problem, there are driving regions due to neutrino heating, damping regions due to neutrino cooling and damping due to shock motion [42]. A mode stability analysis of a protoneutron star bounded by an accretion shock may prove illuminating.
That in the protoneutron star and supernova contexts there should be hydrodynamic instabilities (Rayleigh–Taylor, salt–finger, semi–convection) has been known and studied since the work of Epstein [43]. A review of this literature can be found in [23, hereafter BHF]. However, the role of convection and overturn has been controversial and ambiguous from the outset. Many, evoking Ockam’s Ravor, have opted to ignore it. This should no longer be possible.

There are three classes of instabilities to address: those in the core near and below the neutrinospheres, overturning and boiling motions due to heating from below between the gain radius and the shock [30,39,23,34], and Rayleigh–Taylor and Richtmyer–Meshkov instabilities in the outer stellar mantle far beyond the “iron” core. In this paper, I will ignore the latter and concentrate on the former, since they are more germane to the explosion mechanism.

Convection below and near the neutrinospheres has been invoked to boost the driving neutrino luminosities. Mayle & Wilson [44,45] suggested that “neutron–fingers”, akin to salt–fingers in the Earth’s oceans, advect sufficient heat outward to enhance the emergent neutrino luminosities by twenty or more percent and, thereby, to turn a fizzle into an explosion. Without such a boost, they did not obtain explosions and handled this convection with a mixing–length prescription. Bruenn & Dineva [46] have recently challenged the existence of such a finger instability in this context with a compelling analysis of the details of energy and lepton transport in the protoneutron star core. Burrows [28] suggested that standard lepton– or entropy–driven convection beneath the neutrinospheres could provide a similar boost and BHF do obtain such an enhancement during the hundreds of milliseconds after bounce. However, in those calculations, while such convective motions are definitely present, it is difficult to disentangle this effect from everything else going on. An enhancement of 5% to 20% may be inferred, but the jury is still out on the magnitude and importance of such convective motions, driven in part by negative lepton gradients maintained as the protoneutron star deleptonizes from without.

It is neutrino–driven convection (overturn) near the stalled shock that has recently achieved prominence. Following the suggestion by Bethe [30], Herant et al. [39, hereafter HBC] [33, hereafter HBHFC], BHF, and Janka & Müller [34, hereafter JM] demonstrated the positive effect of such convective motions in aiding (perhaps enabling) the supernova explosion. However, the different groups interpret the specific role of these multi–dimensional motions differently. Here, I will briefly lay out the issues, but with the obvious bias.

The stalling shock dynamically creates a negative entropy gradient that is unstable to overturn on slightly longer timescales (5–15 milliseconds). This con-
vection is the first multi-dimensional effect of note after bounce [47]. However, as shown by BHF, Wilson [44], and Bruenn [48], neutrino transport smooths out this gradient within 25 milliseconds. Importantly, as demonstrated by BHF and HBC another maximum in entropy develops at the gain radius, exterior to which the matter is unstable to overturn, in a matter akin to the boiling of water on a stove. A new convective zone is established between the gain radius and the shock, but the mantle does not yet explode. Instead, the protoneutron star, now with a convection zone, evolves quasi-statically for fifty to hundreds of milliseconds as it continues to accrete matter through the shock. BHF claim that most of the accreted matter eventually settles onto the core and does not dwell more than a few convective cycles in the gain region. However, while it does cycle it has longer to absorb neutrino energy. The net effect is a larger average entropy in the convective gain region than can be attained in one-dimension. In 1-D, matter moves through the gain region quickly and deliberately [39]. Entropies of 10–20 are achieved. In the two-dimensional calculations of BHF, average entropies in the gain region reach 25–35. At these entropies, the matter is not radiation-dominated and the term “hot bubble” is inappropriate. Since convective motions smooth out the average entropy distribution in the gain region, and the average entropies themselves are larger, the entropies near the shock are larger than in 1-D. Atmospheres with larger entropies have larger radii, all else being equal. These larger radii are just what the discussion in §2 claimed were advantageous for explosion and are the major consequence of multi-dimensional effects. However, this does not guarantee an explosion. It merely facilitates it by lowering the explosion threshold. In sum, the critical luminosity is lower in 2–(3–)D than in 1–D, but neutrinos are still in the driver’s seat.

Not all workers agree with this description. Bethe in particular claims that soon after the establishment of the convective zone, matter and energy “accumulate” in the gain region and matter does not leak onto the core (see also HBC). The matter accreted through the shock and its energy “build” until the total energy in the gain region, including that due to nuclear recombination, reaches approximately $10^{51}$ ergs, at which time the mantle explodes. Bethe points out that overshoot in three dimensions is weaker than in two dimensions, so that the leak of matter onto the core and out of the gain region might be plugged. This must be explored. Nevertheless, a few of the ingredients of the paradigm are problematic. First, neither BHF nor JM see such a building, until the explosion commences. The mass and energy in the region actually decrease prior to the instability. In addition, in the calculations of BHF, the net total enthalpy (read energy) flux is inwards, not outwards, despite the rising plumes. Such plumes must contend with inward accretion and are balanced by downward moving plumes. Second, due to electron capture, the 0.1–0.2 $M_\odot$ that would reside in the gain region would be very neutron-rich and, to be consistent with severe nucleosynthetic constraints, must not be ejected. This requires that a lot of matter must fall back during the latter
stages of the explosion. While this is not implausible for the most bound stellar progenitors ($\gtrsim 20 \, M_\odot$) [23,49], it is difficult to understand for the lighter progenitors (9–15 $M_\odot$), whose binding energies are quite modest, but which dominate the IMF. Furthermore, without fallback the gravitational masses of the neutron star residues would be too small (1.1–1.2 $M_\odot$) to explain the observations (which, however, could indeed be selection–biased). Until other groups weigh in on this and credible 3–D calculations are performed, these questions will remain open, but intriguing. The resolution of these questions will be intimately coupled to the systematics of explosion energy with progenitor mass and structure and there may yet be some surprises. The progenitor structures themselves have not converged [50–52]. The binding energies of the mantles may well regulate the explosion and/or determine its energy [23] and mass cut.

Whatever the final word, that the character of the explosion is multi-dimensional seems robust. Supernovae do not explode as spheres, but aspherically in plumes, resembling cauliflower and broccoli more than oranges. Sato and collaborators [53–56] even suggest that rapid rotation leads to aspherical neutrino emissions that translate into jet–like neutrino–driven explosions. Be that as it may, the exploration of many of the major questions of supernova theory and supernova explosions may now require multi–dimensional treatment.

4 Neutrino Transport and Neutrino Opacities

Though the new hydrodynamic issues are important, equal weight should be given to the whole subject of neutrino transport and neutrino/matter interactions. Core–collapse supernovae are the only context, apart from the big bang, in which neutrinos are pivotal agents of dynamics and evolution. Rather than present a comprehensive discussion of neutrino transport, I will touch on a few important topics that should be the focus of future investigations.

There has been a lot of effort in the last decade to understand the character of lepton trapping and to derive the trapped lepton fraction ($Y_l$) and the entropy generated during infall [24,25,57]. A larger $Y_l$ makes for a more energetic bounce and generates the shock wave further out in interior mass [58]. This leaves less mass through which the shock must fight to emerge. With the viability of the direct mechanism at stake, this focus was understandable and a great deal of insight was gained [48]. However, the best transport calculations reveal that $Y_l$ is far too low, neutrino losses when the shock breaks out of the neutrinospheres are far too large, and nuclear dissociation is far too debilitating for the prompt mechanism to be salvaged [23,57]. The shock stalls into accretion, as stated in §2, and must be revitalized. As Gerry Brown has mused, this is the “pause that refreshes,” yet the duration ($T_d$) of this
delay to explosion (one hundred milliseconds, seconds?) has not been established. The value of $T_d$ must determine in part the residual neutron star mass, whether a black hole forms, the ejected nucleosynthesis, the explosion energy, and in fact most of the interesting questions that surround the supernova event. There are many indications that the delays in the calculations of BHF and HBHFC are too short. Not only is $T_d$ a function of progenitor structure, but it hinges on the character of the emergent neutrino luminosity ($L_\nu$) and spectra as well. Therefore, what determines them and their evolution determines the outcome of collapse. However, because of the numerous feedbacks in the radiation hydrodynamics of collapse and bounce, the consequences of various interesting neutrino processes have at times been exaggerated. One thinks immediately of $\nu - \bar{\nu}$ annihilation into $e^+ - e^-$ pairs, which is effective only near the neutrinospheres, where heating and cooling processes are always dominated by the more mundane charged–current absorptions on nucleons. The process $\nu + \bar{\nu} \rightarrow e^+ + e^-$ can not be pivotal in reigniting the supernova explosion [59], but should be included for completeness. Let us proceed to highlight various neutrino processes whose study might indeed be profitable and whose character has yet to be fully delineated.

Mu and tau neutrinos ($\nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$, hereafter “$\nu_\mu$’s”) collectively carry away most of the binding energy (50%–60%) of the neutron star. Hence, their effect on its thermal evolution is crucial. It is thought that neutrino–electron scattering and inverse pair annihilation are the processes most responsible for the energy equilibration of the $\nu_\mu$’s and their emergent spectra. However, credible calculations imply that the inverse of nucleon–nucleon bremsstrahlung (e.g., $n + n \rightarrow n + n + \nu \bar{\nu}$) is more important [60] in equilibrating the $\nu_\mu$’s. This process has not heretofore been incorporated in supernova simulations. Preliminary calculations imply that the emergent $\nu_\mu$ spectra are softened by this effect. This has consequences for neutrino nucleosynthesis, in particular, since the relevant inelastic neutral–current processes are stiff functions of neutrino energy [61,10].

The “pinching” of the neutrino spectra [57] in flux–limited, multi–group calculations has a similar effect, but calculations of Mezzacappa & Bruenn [62,63] and Burrows, Hayes & Pinto [64], solving in the former case the Boltzmann equation and in the latter case the full velocity–dependent transport equation (without flux–limiter) indicate that the flux at higher neutrino energies may be higher than seen in flux–limited calculations. The hardness of the neutrino spectrum is crucial not only in neutrino nucleosynthesis calculations, but in the supernova mechanism itself, since the neutrino–matter coupling (heating) rate is an increasing function of $\nu_e$ and $\bar{\nu}_e$ energy. For a given $\nu_e$ and $\bar{\nu}_e$ luminosity and average neutrino energy, the heating rate may indeed be higher when calculated accurately. All else being equal, a 10% – 30% increase in this heating rate (due to a more accurate treatment of neutrino transport) may facilitate explosion.
There has been sporadic interest over the years in corrections to the neutrino/matter cross sections due to collective effects at modest densities ($\gtrsim 10^{12}$ gm cm$^{-3}$). Corrections to neutrino–nucleus Freedman scattering due to finite nuclear size (form factor) [65,66], ion–ion correlation [67], and electron screening [68,69] have been and continue to be the subject of study. (Note that the screening correction for single nucleon scattering has yet to be estimated.) All these effects decrease the $\nu - A$ cross section, the form factor at high energies and the other effects at low energies. A preliminary estimate of the cumulative effect of these corrections on protoneutron star cooling [67] indicates that the neutrino luminosities in the first seconds are higher as a result and this is germane to the neutrino–driven mechanism [28,29]. However, equally interesting are the nucleon–blocking and Fermi–liquid corrections to neutrino–nucleon scattering and absorption at high densities ($\gtrsim 10^{14}$ gm cm$^{-3}$) [70,71]. In particular, the axial–vector coupling constant, $g_A$, is renormalized to a lower value due, among other things, to the $\Delta$ resonance [72]. This decreases the dominant neutrino–matter cross section by of order a factor of two [26] and, hence, increases the neutrino luminosity from the core on timescales of not tens of milliseconds, but hundreds of milliseconds to seconds. With the prospects of higher long–term luminosities (due to weaker neutrino/matter coupling at high densities) and harder emergent electron neutrino spectra (with the resulting greater neutrino heating rates exterior to the neutrinospheres), many of the anticipated improvements in the neutrino transport sector of supernova modeling favor explosion. It is only when the neutrino physics is well in hand that the ultimate role of hydrodynamic instabilities can be assessed properly.

5 Nucleosynthetic Constraints: The Mass Cut and the r–process

The study of the production and ejection of heavy elements in supernova explosions has a long pedigree and is too large a subject to be more than superficially addressed here [73,74]. It involves the proper calculation of the pre-supernova nested “onion–skin” structure of freshly synthesized elements (with its dependence on convective burning algorithms, thermonuclear rates, and electron capture rates), the explosive processing of the inner zones of the ejecta, and the hydrodynamics of the explosion itself. The latter is poorly handled by those who take great pains with the former. Those who have focussed on the mechanism have paid insufficient attention to the nucleosynthetic consequences. Clearly, the two theoretical domains should be fused in future investigations.

SN1987A is a treasure trove of information on all aspects of the core–collapse supernova phenomenon, yet its bounty is still underutilized by supernova modelers. I will not illustrate this statement in detail within the narrow confines of this brief report, but will summarize a few useful derived constraints on the
hydrodynamics of supernova explosions.

As pointed out by Thielemann, Nomoto, & Hashimoto [1], the observation of $^{57}$Co and $^{57}$Fe in SN1987A at $\sim 1.5$ times the solar ratio with $^{56}$Fe [75–77] and the stiff dependence of $[^{57}Fe/^{56}Fe]$ on $Y_e$ force the eventual mass cut separating neutron star (or black hole) from the ejecta to be near $Y_e \sim 0.496 - 0.498$

Similar conclusions can be drawn from the solar and SN1987A abundance ratios of $^{56}$Fe with the neutron-rich isotopes $^{58}$Fe, $^{58}$Ni, $^{60}$Ni, and $^{61,62}$Ni. (Stable nickel was indeed detected in SN1987A via the infrared 6.634 $\mu$m line [78]). In the progenitor models of Nomoto & Hashimoto [51] and Woosley & Weaver [50], this cut is almost always exterior to the iron core and is often close to the inner oxygen zones. For the more massive progenitor models of Woosley & Weaver [50] ($\gtrsim 19 M_\odot$), this demarcation line is exterior to $2 \times 10^3$ kilometers and $1.7 M_\odot$, while for those of Nomoto & Hashimoto [51], it is closer in in mass. Be that as it may, all the extant successful supernova calculations (BHF, HBHFC, JM) eject too much material whose neutron–richness is inconsistent with observed nucleosynthesis (including the “$N = 50$” (e.g., Sc, Y, Zr) abundances). This implies either that the delay to explosion is longer than calculated, that generically there is fallback of 0.1–0.3 $M_\odot$ of envelope, that the progenitor models are inaccurate, or some combination of all three. In addition, the yields of $Si$, $S$, and $Ca$ in the models of Woosley & Weaver [50] are too large to explain halo star [4,5] and SN1987A abundances, unless the mass cut is further out than heretofore assumed. The models of Nomoto & Hashimoto [51] don’t have this problem to the same degree. There is also a hint in the SN1987A Ginga X–ray data that in order to avoid excessive photoelectric absorption in the SN1987A debris cloud, the $Si$, $S$, and $Ca$ yields must be smaller than in the Woosley & Weaver [50] models [79].

What the nucleosynthetic constraints are collectively telling us about the explosion is not yet clear in its entirety, but they embolden one to speculate nevertheless. One conclusion to be drawn is that if the prompt mechanism obtains, there must be appreciable fallback. Since fallback is a function of mantle binding energy, which increases with progenitor mass [23,49], the low–mass massive stars ($\lesssim 15 M_\odot$?) can not explode by this mechanism. Since this binding energy is correlated with the other shock killers (breakout neutrino losses, photodissociation, accretion ram), the more massive progenitors can not explode directly either. The bounce shock must stall, as all the best hydrodynamic models imply, leaving the delayed mechanism by default.

How long is the delay, $T_d$? For “low–mass” progenitors, since there may not be appreciable fallback, the delay must be sufficient that the shocked material does not reach densities large enough to result in appreciable electron capture and the ejection of matter with anomalous $Y_e$’s. Furthermore, since the iron and ONeMg cores of such progenitors have the lowest $Y_e$’s ($\lesssim 0.43$), none of this core material can be ejected. It must be accreted. How long this takes depends
upon the progenitor radial structure. (Care must be taken in estimating and quoting these delays to distinguish between total collapse time and time since bounce.) For the highest mass progenitors that still explode, fallback and/or a long delay due to higher accretion $\dot{M}$’s may naturally bury neutron–rich matter. Furthermore, it may be that the explosion energy is set in some way by the envelope binding energy, in which case the lighter progenitors may explode with a lower energy that could be inadequate to forestall fallback. Hence, there may be feedback that necessitates fallback for both light and heavy progenitors. Whether this is true remains to be investigated.

It is thought that the r–process nuclei are produced in the winds that emerge from the protoneutron star after the explosion commences [80,37,81,82]. The onset of this supersonic wind phase is clearly manifest by the emergence of a second shock wave and a contact discontinuity [23] and is suppressed until the large pressures in the inner supernova ejecta abate due to expansion. How long the wind is suppressed depends upon neutrino transport and neutrino luminosities, the delay to explosion, and the progenitor structure. It is possible that the more massive progenitors, with their denser and more bound envelopes, suppress the wind long enough and/or smoother it with fallback that they do not yield r–process nuclei. This would be consistent with the suggestion by Mathews, Bazan, & Cowan [83] that only low–mass massive stars ($\sim 10 M_\odot$) produce the r–process isotopes seen in halo stars. This idea should be reinvestigated using the new data of Sneden et al. [6], Cowan et al. [84], and McWilliam et al. [4,5] and a better $[\text{Fe}/\text{H}]$ versus age relation [85]. Something very important concerning supernova explosions lurks in the halo abundance data. However, extracting it may require a more sophisticated galactic chemical evolution model than employed to date.

For those supernovae that yield r–process nuclei, a simple (simplistic ?) scenario suggests itself. 1) The stalled shock is reenergized after material with $Y_e$’s near 0.497–0.5 is accreted. 2) The mass cut is near the mass interior to the shock when it is relaunched. 3) After the material interior to the mass cut is accreted onto the protoneutron star and the pressure around it abates, a neutrino–driven wind with lower $Y_e$’s, determined in part by $\nu_e$ and $\bar{\nu}_e$ absorption, emerges. This must happen when the driving neutrino luminosities have decayed enough so that $\dot{M}_w$’s, and, hence, $\Delta M_w (= \int_{t_i}^{\infty} \dot{M}_w dt)$ are small. In particular, $\Delta M_w$ should be below $\sim 10^{-4}$ to $\sim 10^{-3} M_\odot$. A large driving $L_\nu$ might result in the wind ejection of too much neutron–rich material. However, there may be enough leeway in $\Delta M_w$ to accommodate the production and ejection of “$N = 50$” isotopes and perhaps p–process nuclei in the wind’s first phases ($Y_e \leq 0.49$) (see also Hoffman et al. [86]). The wind’s last, high–entropy, phases eject the r–process nuclei. Whether this scenario or similar scenarios holds up hopefully will be tested by the next generation of supernova models. Whatever scenario obtains, the iron–peak abundances require a mass cut above $Y_e = 0.497$, while the “$N = 50$,” p–process, and r–process
nucleosynthesis require a $Y_e$ below this value in less than a few times $10^{-3} \, M_\odot$.

6 Pulsar Proper Motions, Natal Kicks and Gravitational Radiation

A new pulsar distance scale [87], recent pulsar proper motion data [7], and the recognition that pulsar surveys are biased towards low speeds [88] imply that radio pulsars are a high-speed population. Mean three-dimensional galactic speeds of $450 \pm 90 \, \text{km s}^{-1}$ have been estimated [88], with measured transverse speeds of individual pulsars ranging from zero to $\sim 1500 \, \text{km s}^{-1}$. Impulsive mass loss in a spherical supernova explosion that occurs in a binary can impart to the nascent neutron star a substantial kick that reflects its progenitor’s orbital speed [89,90]. However, theoretical studies of binary evolution through the supernova phase have difficulty reproducing velocity distributions with the required mean and dispersion [91,92]. This implies that neutron stars receive an extra kick at or after birth.

In the past, an off-center (and rapidly rotating) magnetic dipole [93] and anisotropic neutrino radiation [94,56,95,96] have been invoked to accelerate neutron stars. A 1% net asymmetry in the neutrino radiation of a neutron star’s binding energy results in a $\sim 300 \, \text{km s}^{-1}$ kick. However, Burrows & Hayes [97] have recently demonstrated that if the collapsing Chandrasekhar core is mildly asymmetrical, the young neutron star can receive a large impulse during the explosion in which it is born. In those calculations, rocket-like mass motions, not neutrinos, dominated the recoil, which reached $\sim 530 \, \text{km s}^{-1}$. Such a speed is large, but is only $\sim 2\%$ of that of the supernova ejecta. This asymmetry/recoil correlation seems generic. However, whether such asymmetries are themselves generic has yet to be demonstrated. Recent calculations of convection during shell oxygen and silicon burning [52] and theoretical arguments [98] suggest that the initial density, velocity, and composition asymmetries might indeed be interesting.

The impulse delivered to the core depends upon the dipole moments of the angular distribution of both the envelope momentum and the neutrino luminosity. The gravitational waveform depends upon the corresponding quadrupole moments. Curiously, using the standard quadrupole formula, Burrows & Hayes [97] derived that due to the intense and anisotropic early neutrino burst, the neutrino contribution to the metric strain, $h_{zz}^{TT}$, can dominate during the early post-bounce epoch. This is true despite the fact that the neutrinos do not dominate the recoil and is a consequence of their relativistic nature. The gravitational waves are radiated between 10 and 500 Hz. and $h_{zz}^{TT}$ does not go to zero with time. Hence, there may be “memory” [99] in the gravitational waveform from a protoneutron star that is correlated with its recoil and neutrino emis-
sions. This memory is a distinctive characteristic of asymmetric collapse and explosion. Using [18], one finds that the 2’nd-generation LIGO might be able to detect a signal from a core collapse anywhere in our galaxy. Even without rotation [100], an asymmetric collapse can result in appreciable gravitational wave emission. The simultaneous detection of the neutrino and gravitational radiation signatures and of the recoil would provide direct information concerning supernova dynamics.

The kick mechanism suggested by Burrows & Hayes [97] is but one of several that researchers are now exploring. The group of W. Benz, M. Herant, C. Fryer, and S. Colgate is exploring the possibility that asymmetrical Bondi-Hoyle accretion of fallback by the young neutron star could result is asymmetrical neutrino emission and a “neutrino rocket.” In order for this mechanism to work, on the order of 0.1–0.2 $M_\odot$ must fall back and the resulting angular asymmetry in the mass accretion (and, hence, the neutrino emission) must be large. One of the virtues of this mechanism is that the motion of the young neutron star might thereby be unstable to acceleration, until the ram pressure balances the neutrino term at some hundreds of kilometers per second.

7 Conclusion

What I have presented here is a collection of thoughts on the current status of core–collapse supernova theory and its components. I have not answered the questions posed in the Introduction because they are the subject of ongoing investigations, or should be. Most are as yet unresolved. However, I have attempted to map out a viable research strategy for the field that will resolve many of the ambiguities that remain. It is important not only to obtain credible computer explosions, but to discover the systematics with progenitor mass and composition of the explosion energies, ejecta nucleosynthesis, neutron star masses, explosion morphologies, and natal kicks, among other things. Such has been the recent progress that this truly comprehensive theory seems within our grasp in the next few years.

References