SCAN-9606068

Loop Diagrams in Cavity QCD and the Properties of the Nucleon

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Abstract

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November, 1995

Invited lecture at the Erice School on Nuclear Physics, 17th Course: "Quarks in Hadrons and Nuclei", 19-27 September 1995, Erice, Italy (to appear in *"Progress in Particle and Nuclear Physics"*, Vol.36, Elsevier, Oxford)

Loop Diagrams in Cavity QCD and the Properties of the Nucleon

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ABSTRACT

Using a technique which is based on dimensional regularization, loop corrections to vertex diagrams of a massless quark moving in a spherical cavity are calculated in an arbitrary covariant gauge. Including the finite one-gluon-exchange diagrams, the corrections to the magnetic moments, vector and axial vector coupling constants, and the various r.m.s. radii of the nucleon are evaluated to order α_s . The results are compared with experimental data and previous calculations.

KEYWORDS

Perturbative cavity QCD, M.I.T. bag model, nucleon observables

INTRODUCTION

Over the past two decades, cavity QCD (Chodos *et al.*, 1974; DeGrand *et al.*, 1975) has been much used to gain a quantitative understanding of the properties of the hadrons. Similar to cavity QED which is the Abelian counterpart of cavity QCD, the boundary conditions that the fields must obey on a static sphere are consistent with the gauge symmetry and the field equations. Cavity QCD, as much as cavity QED, is therefore presumably a well-defined and renormalizable relativistic quantum field theory on its own, which could in principle be calculated perturbatively to arbitrary order. Apart from its non-Abelian character, the main feature that distinguishes cavity QCD from cavity QED is that the latter can be realized and observed in nature while it is still not clear whether perturbative cavity QCD provides a consistent framework for the description of the hadronic world.

One of the reasons for this deficiency is that the boundary conditions on the fields of the colour carrying particles are imposed in a somewhat arbitrary and *ad hoc* fashion, in order to mimic confinement, while confinement is generally believed to be a consequence of the dynamics of the non-Abelian gauge theory. Secondly, by confining particles to a static sphere, one breaks both, translational and Lorentz invariance of the underlying gauge theory. It is difficult to restore these symmetries consistently without explicitly introducing the dynamics of the boundary, which necessarily amounts to solving the centre-of-mass problem exactly. Chiral symmetry is also broken, and this may be restored by introducing the coupling to an elementary pion field, as is done for instance in the chiral and cloudy bag models (Brown and Rho, 1979; Brown *et al.*, 1979; Thomas, 1983). The price one has to pay for this remedy though, is that these chiral field theories are in

general not renormalizable, which makes perturbative calculations dependent on a large number of renormalization parameters, with substantial loss of predictive power. Moreover, there remains the problem of double counting, as the pion can be treated both, as elementary and quark-antiquark degree of freedom. Thirdly, in the framework of cavity field theory, it is technically rather difficult to calculate higher-order Feynman diagrams (Stoddart and Viollier, 1988), especially the diverging ones (Goldhaber *et al.*, 1983, 1986; Stoddart and Viollier, 1990, 1991, 1992; Page *et al.*, 1993; Chin *et al.*, 1981; Stoddart, 1990), as the singularities have to be extracted numerically. While these higher-order corrections may be important, they have been largely ignored in the past, with a few exceptions (Maxwell and Vento, 1983; Goldhaber *et al.*, 1983, 1986). Fourthly, it is by no means clear that perturbative cavity QCD will converge any better than ordinary free-space QCD. However, the fact that the field operators are expanded in cavity modes instead of plane waves helps at least to get rid of the infrared divergences plaguing free-space QCD. Indeed, a confined quark cannot radiate off a gluon as easily as a free quark, as the emitted gluon has a non-zero minimum mode energy.

Despite all of these shortcomings, it is important to investigate the higher-order predictions of cavity QCD in more detail, in order to establish whether perturbative cavity QCD makes sense as a possible description of the hadronic world. Thus, the purpose of this paper is to calculate all the $O(\alpha_S)$ diagrams, including loop diagrams, which contribute to the magnetic moments μ_p and μ_n , the vector and axial coupling constants g_V and g_A , and the root-mean-square radii $\langle r^2 \rangle_p^{1/2}$, $\langle r^2 \rangle_n^{1/2}$ and $\langle r^2 \rangle_A^{1/2}$. Assuming that the nucleon contains only up and down quarks, we restrict our attention to massless quarks. We do not attempt to restore chiral symmetry, translational and Lorentz invariance, as this cannot be done in a model-independent way.

THE CAVITY REGULARIZATION TECHNIQUE

Recently, a reliable method for regularizing cavity loop-diagrams has been developed and applied to the self-energies of confined quarks (Stoddart and Viollier, 1990, 1991, 1992; Cuthbert and Viollier, 1992) and gluons (Schreiber and Viollier, 1992), and the axial and vector coupling constant of the nucleon (Page *et al.*, 1993). In this method, divergences due to the unreflected part of the propagators are extracted from loop integrals by using techniques similar to those of dimensional regularization in free space.

We now briefly describe this method in the case of the three $O(\alpha_S)$ loop diagrams contributing to the anomalous magnetic moment of the nucleon. Starting with the Gell-Mann and Low theorem (Fetter and Walecka, 1971) in its symmetric formulation (Sucher, 1957), the order g^2 correction to the magnetic moment is given by

$$\mu^{(2)} = \lim_{\epsilon \rightarrow 0} \frac{3i\epsilon}{2} \frac{(-i)^3}{3!} \int d^4x_1 d^4x_2 d^4x_3 \left\langle \hat{N} \left| T \left[\hat{\mathcal{H}}_{int}^\epsilon(x_1) \hat{\mu}^\epsilon(x_2) \hat{\mathcal{H}}_{int}^\epsilon(x_3) \right] \right| \hat{N} \right\rangle_\epsilon, \quad (1)$$

where $|\hat{N}\rangle$ is the nucleon state with spin up, $\hat{\mu}^\epsilon(x) = \frac{1}{2}(\vec{x} \times \vec{\gamma})_z Q e^{-\epsilon|t|}$ is the z -component of the magnetic moment density operator and Q is the quark charge operator. The relevant term in the interaction Hamiltonian density $\hat{\mathcal{H}}_{int}^\epsilon(x)$ is given by

$$\hat{\mathcal{H}}_{int}^\epsilon(x) = -g\hat{\psi}(x) \frac{\lambda_a}{2} \hat{A}_a(x) \hat{\psi}(x) e^{-\epsilon|t|}. \quad (2)$$

Here $\hat{A}_a^\mu$ denotes the gluon field, the λ_a 's are the Gell-Mann matrices and $e^{-\epsilon|t|}$ is an adiabatic damping factor with $\epsilon \downarrow 0$. The time-ordered product in eq. (1) may be expanded using Wick's theorem, leading to the five Feynman diagrams, shown in fig. 1, which contribute to the nucleon

Figure 1: Feynman Feynman diagrams contributing to magnetic moments of the nucleon. The quantum numbers of the intermediate quarks are labelled by p and q , and the gluon by m, Σ .

magnetic moment. The index c in eq. (1) stands for connected Feynman diagrams. The three diagrams involving loops are divergent, while the one-gluon exchange diagrams are finite.

The amplitude for the three loop-diagrams may easily be calculated in free-space QCD (Ryder, 1985), where the sum of these diagrams is found to be gauge-independent. Moreover, for massless quarks, the singularities from the vertex diagrams containing self-energy inserts, fig. 1(b)-(c), exactly cancel that from the vertex correction, fig. 1(a). For massive quarks, the self-energy diagrams develop an additional divergence which can be absorbed by introducing a mass counter-term. However, here we shall restrict our attention to massless up and down quarks, i.e. $m_u = m_d = 0$.

As the singularities are a high-momentum or short-distance phenomenon, one expects to encounter in the cavity the same singularities as in free-space field theory. New singularities could only arise from the boundary conditions imposed at the surface of the cavity. However, in the case of the quark self-energy, it has been shown (Hansson and Jaffe, 1983) that the MIT boundary conditions do not introduce any new singularities, and since the vertex correction and self-energy diagrams are intimately related through the Ward-Takahashi identities, the former is also free from boundary-induced divergences. Hence, each cavity diagram should diverge in exactly the same way as its free-space counterpart, making it possible to extract the singularity using the analytic free-space field theory result. Furthermore, we expect the sum of these cavity loop-diagrams to be finite and gauge-independent as well.

In free-space field theory, the singular part of the vertex correction diagram, fig. 1(a), may be found using the method of dimensional regularization. Working in the Feynman gauge, the vertex function in $D = 4 - 2\epsilon$ dimensions is given by (Ryder, 1985)

$$\Lambda^\mu(p, p') = \frac{g^2 C}{16\pi^2} \gamma^\mu \Gamma(\epsilon) + \text{finite terms}, \quad (3)$$

where $C = 4/3$ is a colour factor. Transforming this Feynman amplitude into configuration space, and using the integral representation of the gamma function, $\Gamma(\epsilon) = \int_0^\infty e^{-z} z^{-1+\epsilon} dz$, the singular

part of this diagram, in the limit $\epsilon \rightarrow 0$, is given by

$$\mu_S = \int_0^\infty dz \mu_S(z) = \frac{g^2 C}{16\pi^2} M_{nn} \int_0^\infty dz \frac{e^{-z}}{z}, \quad (4)$$

where the matrix element of the magnetic moment operator is defined as

$$M_{nn'} = \frac{1}{2} \int d^3 r \bar{u}_n(\vec{r}) (\vec{r} \times \vec{\gamma})_z u_{n'}(\vec{r}). \quad (5)$$

Let us now calculate the analogous quantity in cavity field theory, starting with the quark and gluon propagators in an arbitrary covariant gauge

$$iS(x, x') = i\delta_{cc'} \delta_{ff'} \sum_n u_n(\vec{x}) \bar{u}_n(\vec{x}') \int \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}}{\omega - \epsilon_n \pm i0} \quad (6)$$

and

$$iD_{ab}^{\mu\nu}(x, x') = i\delta_{ab} \sum_{m\Sigma\Sigma'} a_{m\Sigma}^\mu(\vec{x}) a_{m\Sigma'}^\nu(\vec{x}') \int \frac{d\omega}{2\pi} \left[\frac{-g^{\Sigma\Sigma'}}{q^2 + i0} - \frac{1 - \lambda}{\lambda} \frac{q^\Sigma q^{\Sigma'}}{q^4 + i0} \right] e^{i\omega(t'-t)}, \quad (7)$$

respectively. Here, $u_n(\vec{x})$ and $a_{m\Sigma}^\mu(\vec{x})$ are the quark and gluon cavity modes, respectively (Viollier *et al.*, 1983; Buser *et al.*, 1988), with energy eigenvalues ϵ_n and Ω_m^Σ . The subscripts n and m denote the radial and angular quantum numbers of the quarks and gluons, respectively. $\Sigma \equiv S, L, M, E$ labels the scalar, longitudinal, magnetic and electric polarizations of the gluon, respectively, and λ is the gauge parameter. The diagonal metric $g^{\Sigma\Sigma}$ is defined as $g^{SS} = -g^{LL} = -g^{MM} = -g^{EE} = 1$, and the polarization vector by $q^\Sigma \equiv (q^S, q^L, q^M, q^E) = (\omega, \Omega_m^\Sigma, 0, 0)$.

Inserting the Feynman gauge ($\lambda = 1$) propagators in eq. (1), the contribution to the magnetic moment from the vertex correction $\mu_a^{(2)}$ is found to be

$$\mu_a^{(2)} = ig^2 C \sum_{pqm\Sigma} g^{\Sigma\Sigma} \bar{Q}_{np}^{m\Sigma} M_{pq} Q_{qn}^{m\Sigma} I_{pq}^{m\Sigma}, \quad (8)$$

where the vertex form factors describing the emission or absorption of gluons by quarks has been introduced

$$Q_{nn'}^{m\Sigma} = i \int d^3 r \bar{u}_n(\vec{r}) \gamma_\mu u_{n'}(\vec{r}) a_{m\Sigma}^\mu(\vec{r}) = -\bar{Q}_{n'n}^{m\Sigma}. \quad (9)$$

The energy denominators $I_{pq}^{m\Sigma}$ are defined as

$$I_{pq}^{m\Sigma} = i \int \frac{d\omega}{2\pi} \frac{1}{(\epsilon_n + \omega - \epsilon_p)(\epsilon_n + \omega - \epsilon_q)(\omega^2 - \Omega^2)}, \quad (10)$$

where for simplicity $\Omega = \Omega_m^\Sigma$.

At this point, we note that since the cavity theory is identical to its free-space counterpart in the time co-ordinate, the integral over the ω variable is actually the same as the integral over the loop energy k^0 in the free-space diagram. Thus, if the ω integral is evaluated using the free-space method, the parametric form of $\mu_a^{(2)}$ will be equivalent to the corresponding free-space expression. We thus adopt the same procedure as in free-space field theory: rotate to Euclidean space with the shifts $\omega \rightarrow i\omega$ and $\epsilon_1 \rightarrow i\epsilon_1$; multiply the numerator and denominator by the complex conjugate of the latter; and elevate the denominators into exponential factors using $1/a = \int_0^\infty e^{-ax} dx$. After

some variable shifts, the ω integral becomes a standard Gaussian which is easily evaluated. The result can be written in Minkowski space as the spectral form

$$I_{pq}^{m\Sigma} = \int_0^\infty dz I_{pq}^{m\Sigma}(z) \quad (11)$$

with the spectral function

$$I_{pq}^{m\Sigma}(z) = \int_0^1 dx \int_0^{1-x} dy \left[(\epsilon_p + \epsilon_n(1-x-y)) (\epsilon_q + \epsilon_n(1-x-y)) z - \frac{1}{2} \right] \times \sqrt{\frac{z}{4\pi}} \exp \left\{ -z [\Omega^2(1-x-y) - \epsilon_n^2(x+y)(1-x-y) + x\epsilon_p^2 + y\epsilon_q^2] \right\}. \quad (12)$$

As the evaluation of the x - and y -integrals is rather lengthy, it will be presented elsewhere (O'Connor and Viollier, 1995). Making a convenient shift of variable, $z = y^2$, in order to avoid a trivial integrable singularity at the origin, the cavity vertex correction becomes

$$\mu_a^{(2)} = g^2 C \int_0^\infty dy \mu_a^{(2)}(y) = g^2 C \int_0^\infty 2y dy \sum_{pqm\Sigma} g^{\Sigma\Sigma} \bar{Q}_{np}^{m\Sigma} M_{pq} Q_{qn}^{m\Sigma} I_{pq}^{m\Sigma}(y). \quad (13)$$

The sum over the spins of the intermediate quarks and gluons can easily be calculated analytically, leaving three infinite sums over the radial modes and two infinite sums over partial waves. The remaining partial wave sum is constrained by conservation of angular momentum and is thus finite. As these infinite sums are to be evaluated numerically, they must be truncated at some point, e.g. by including only cavity modes whose energy is less than a certain cut-off value $E_{\max}$.

The sum over vertex integrals, without the z -dependent terms, can be checked using the sum rules (O'Connor and Viollier, 1995), and the complete expression for $\mu^{(2)}$, as a function of z , can be compared with the free-space divergent terms. As we noted previously, the divergent parts of the free-space and cavity results should be the same. It is thus convenient to shift the variable $z \rightarrow y^2$ in eq.(4) as well.

The contribution from the vertex correction diagram is shown in the upper frame of fig. 2, where the functions $\mu_a^{(2)}(y)$ and $\mu_S(y)$ are plotted together as a function of y . The initial and final quark are both in the $1s_{1/2}$ state, and the value of the energy cut-off used is $E_{\max} = 140$. The colour factor is not included. As expected, the numerical calculation produces a curve which, for small y , lies exactly on the free-space divergent term. The lower frame of fig. 2 shows the finite remainder, after subtracting the free-space divergence from the cavity expression. In principle, the finite contribution of the vertex correction diagram to the anomalous magnetic moment (up to a finite renormalization) could be obtained by integrating this remainder.

Note that the function $\mu_a^{(2)}(y)$ has not been calculated all the way to zero on the y -axis. This is because the error introduced by truncating the infinite sum over cavity modes appears in the low y , or high energy region, creating a sharp kink. By using different values for the cut-off $E_{\max}$, one can establish that this point is at $y_{\min} \approx \pi/E_{\max}$. Apart from bringing the kink closer to the origin, increasing $E_{\max}$ has little other effect on the result. The 'missing' piece of $\mu_a^{(2)}(y)$ in the error region can be approximated by extrapolation using Chebychev polynomials.

We now turn to the two remaining loop-diagrams containing self-energy inserts, fig. 1(b)-(c). These are usually discarded in cavity QCD calculations, with the argument that the fields, masses, and charges appearing in the cavity QCD Lagrangian are renormalized quantities, and as such, already include self-energy effects (Maxwell and Vento, 1983). However, this argument is fallacious. In fact, one has to include these diagrams since the order g^2 corrections to the observables are not

Figure 2: The contribution from the vertex correction $\mu_a^{(2)}(y)$ and the free-space divergence $\mu_s(y)$.

gauge-independent without them. Furthermore, the finite parts of these diagrams do not vanish (in contrast to free-space field theory) and hence should have observable consequences. This is supported by the fact that, the first order correction to the vector coupling constant g_V only vanishes when the self-energy diagrams are included, as expected from CVC (Page *et al.*, 1993).

Since each of the self-energy diagrams has the same contribution, only one graph needs be considered. The contribution from fig. 1(b) is

$$\mu_b^{(2)} = g^2 C \sum_{pqm\Sigma} g^{\Sigma\Sigma} M_{nq} \tilde{Q}_{np}^{m\Sigma} Q_{qn}^{m\Sigma} K_{pq}^{m\Sigma}, \quad (14)$$

where the energy denominator is given by

$$K_{pq}^{m\Sigma} = \begin{cases} i \int \frac{d\omega}{2\pi} \frac{1}{(\epsilon_n - \epsilon_q)(\omega + \epsilon_n - \epsilon_p)(\omega^2 - \Omega^2)} & \text{if } \epsilon_q \neq \epsilon_n \\ -\frac{i}{2} \int \frac{d\omega}{2\pi} \frac{1}{(\omega + \epsilon_n - \epsilon_p)^2(\omega^2 - \Omega^2)} & \text{if } \epsilon_q = \epsilon_n \end{cases} \quad (15)$$

Figure 3: The finite self-energy term and the divergent wave function renormalization arising from the self-energy diagrams.

Some care is needed in deriving eq. (15); in particular, integrals and limits involving the adiabatic damping factor $e^{-\epsilon|t|}$ appearing eq. (1) must be evaluated rigorously in order to obtain an unambiguous result. Similar attention is required for the free-space analogue of this diagram (Feynman, 1949). Note that the case $\epsilon_q \neq \epsilon_n$ in eq.(15) is, for massless quarks, equivalent to the finite part of the self-energy in free space. For massive quarks, this term develops a divergence, equivalent to the mass renormalization constant, which can be absorbed by a mass counter-term. The remaining part of $K_{pq}^{m\Sigma}$ for $\epsilon_q = \epsilon_n$ is the divergent wave function renormalization.

The integral $K_{pq}^{m\Sigma}$ is evaluated according to the standard prescription given earlier, and the result is similar to that found for the cavity vertex correction (O'Connor and Viollier, 1995). A plot of twice $\mu_b^{(2)}(y)$ is shown in fig. 3, where the finite self-energy and divergent wave function renormalization terms are shown separately. Clearly, since the singular part of $\mu_b^{(2)}(y)$ also diverges like $\mu_s(y)$, the sum $\mu_a^{(2)} + \mu_b^{(2)} + \mu_c^{(2)}$ is finite.

To complete the discussion of the one-body loop diagrams, we need to show that their sum is gauge-independent. To this end, we shall use the second terms on the r.h.s. of the expression for the gluon propagator, eq. (7), which is the 'remaining' piece after subtracting the Feynman-gauge propagator. Inserting this gauge-dependent part of the propagator into eq. (1), the vertex correction turns out to be

$$\Delta\mu_a^\lambda = -g^2 C \frac{1-\lambda}{\lambda} \int_0^\infty dz \sum_{pqm} \tilde{Q}_{np}^{mS} M_{pq} Q_{qn}^{mS} \sqrt{\frac{z}{4\pi}} e^{-z\Omega^2}. \quad (16)$$

The gauge-dependent contributions from the two diagrams with self-energy inserts, Fig.1(b) and

	0th order	1st order corrections			0th + 1st order		experiment
		1-body	2-body	total	$\alpha_s = 0.6$	$\alpha_s = 2.2$	
μ_p	1.924	$-0.8034\alpha_s$	$0.0503\alpha_s$	$-0.7531\alpha_s$	1.472	0.267	2.793
μ_n	-1.282	$0.5356\alpha_s$	$0.0308\alpha_s$	$0.5664\alpha_s$	-0.942	-0.036	-1.913
$\langle r^2 \rangle_p$	0.5314	$-0.2598\alpha_s$	$-0.0147\alpha_s$	$-0.2745\alpha_s$	0.3667	-0.0725	0.69
$\langle r^2 \rangle_n$	0	0	$-0.0125\alpha_s$	$-0.0125\alpha_s$	-0.0075	-0.0275	-0.12
g_A	1.088	$0.2927\alpha_s$	$-0.0501\alpha_s$	$0.2426\alpha_s$	1.234	1.622	1.261
$\langle r^2 \rangle_A$	0.4979	$0.0104\alpha_s$	$-0.0441\alpha_s$	$-0.0337\alpha_s$	0.4777	0.4238	0.7

Table 1: Nucleon observables to order α_s for various values of α_s compared with experimental data (Particle Data group, 1992). The magnetic moments are in units of the nuclear magneton $\mu_N = e\hbar/2m_p$ and the radii in fm.

1(c), are equal and given by

$$\Delta\mu_b^\lambda = -g^2 C \frac{1-\lambda}{\lambda} \int_0^\infty dz \sum_{pqm} \tilde{Q}_{qp}^{mS} M_{nq} Q_{pn}^{mS} \left(\frac{\delta_{n,q}}{2} - \frac{\epsilon_q - \epsilon_p}{\epsilon_n - \epsilon_q} \right) \sqrt{\frac{z}{4\pi}} e^{-z\Omega^2}. \quad (17)$$

The sum of $\Delta\mu_a^\lambda$ and $2\Delta\mu_b^\lambda$ is zero, i.e. the sum of these diagrams is also gauge-independent in the cavity.

The remaining contributions to the nucleon magnetic moment are the diagrams involving one-gluon-exchange, fig. 1(d)-(e). The result for Fig.1(a) is

$$\mu_d^{(2)} = g^2 C' \sum_{\substack{m\Sigma \\ p \neq n_4}} g^{\Sigma\Sigma} \tilde{Q}_{n_1 n_2}^{m\Sigma} Q_{n_3 p}^{m\Sigma} M_{pn_4} \frac{\delta(\epsilon_1 + \epsilon_3, \epsilon_2 + \epsilon_4)}{(\epsilon_p - \epsilon_4)[(\epsilon_1 - \epsilon_2)^2 - \Omega^2]}, \quad (18)$$

where C' is a two-body colour and flavour matrix element (O'Connor and Viollier, 1995). In this case, it is not necessary to transform the ω integral into a spectral form since the diagram is finite; it can simply be evaluated as a contour integral to produce the usual energy denominator. We also include the large contribution from zero-energy scalar and longitudinal gluon modes, which is necessary if these diagrams are to remain gauge-independent.

RESULTS AND DISCUSSION

It is easy to generalize the method, outlined in the preceding section in the case of the magnetic moment, to other nucleon observables that are given by expectation values of one-body operators: e.g.

$$\begin{aligned} r^2 \gamma_0 Q & \text{ charge radius squared} \\ \gamma_0 Q & \text{ vector coupling constant} \\ \gamma_3 \gamma_5 \tau_3 & \text{ axial vector coupling constant} \\ r^2 \gamma_3 \gamma_5 \tau_3 & \text{ axial vector radius squared.} \end{aligned} \quad (19)$$

The results are summarized in table 1. As a test, it is interesting to note that zeroth order and one-body part of the first order correction still respect the $SU(6)_{FS}$ relation $\mu_p/\mu_n = -\frac{3}{2}$, while the two-body part does not, as expected.

	Maxwell and Vento	present paper
μ_p	-0.11	0.1107
μ_n	0.22	0.0678
$\langle r^2 \rangle_p$	0.032	-0.0323
$\langle r^2 \rangle_n$	-0.028	-0.0275
g_A	0.183	-0.1102
$\langle r^2 \rangle_A$	0.124	-0.0970

Table 2: Comparison of results for the one-gluon-exchange of Maxwell and Vento and the present paper

In the recent past, calculations of the two-body corrections to nucleon observables have been presented in several papers (Tsushima *et al.*, 1988; Ushio, 1985; Krivoruchenko, 1984; Kobzarev *et al.*, 1986; Høgaasen and Myhrer, 1988; Donoghue *et al.*, 1977). Høgaasen and Myhrer clarified earlier calculations by Ushio, Krivoruchenko and Kobzarev *et al.* by evaluating the contribution to $\mu^{(2)}$ due to the transverse magnetic gluon. They obtained $\mu_p^{(2)} = 0$ for the proton, and $\mu_n^{(2)} = 0.133\mu_N$ for the neutron, using $\alpha_s = 2.2$ and $R = 5 \text{ GeV}^{-1}$. These numbers are confirmed in the present calculation when the contribution from the scalar gluon is omitted. (The transverse electric and longitudinal gluons do not contribute anyway.) With the same parameters, Ushio finds the contribution from the transverse magnetic gluon to g_A to be 0.036, which we again can confirm. In his calculation, Ushio adds particular terms arising from the self-energy interaction to the contribution from the one-gluon-exchange. This is done to ensure that his Coulomb propagator satisfies the correct boundary conditions (see Chodos *et al.*, 1974; Chodos *et al.*, 1974; DeGrand *et al.*, 1975 for a discussion). This procedure is not necessary here since the zero-energy scalar gluon mode takes care of that problem.

In calculating these corrections, Maxwell and Vento use a similar formalism to that employed here which includes a rigorous treatment of the divergent vertex correction diagram (Maxwell and Vento, 1983). However, as the authors have not calculated the vertex diagrams containing self-energy inserts, their results for the one-body corrections are gauge-dependent, making a comparison with our results for these terms difficult, as these are performed in a different gauge. On the other hand, the one-gluon-exchange corrections are automatically gauge-independent, and can thus be compared with our calculation, shown in Table 2. Here we have used $\alpha_s = 2.2$, which is equivalent to the $\alpha_c = 0.55$ used by Maxwell and Vento.

Our results for the one-gluon-exchange contribution agree with those from Maxwell and Vento only in the case of the neutron charge squared radius, $\langle r^2 \rangle_n$, with differing signs being obtained for μ_p and $\langle r^2 \rangle_p$. The agreement in the case of $\langle r^2 \rangle_n$ suggests that the discrepancy could lie in the calculation of the zero-energy mode, since this is the only case for which the contribution from the zero-energy mode is zero.

In most of the nucleon observables investigated here, the $O(\alpha_s)$ corrections have made the agreement with experiment worse rather than better. The culprit is easily identified—it is those diagrams with self-energy inserts on the external legs, whose contributions may be divided into two parts for convenience. Firstly, there is a divergent part, equivalent to the free-space charge renormalization, which cancels the divergence from the vertex correction diagram, leaving a small finite remainder. Secondly, there is a part which is finite for massless quarks (the free-space analogue is zero). For massive quarks, this term also diverges, and would need to be cancelled by a mass renormalization counter-term (Cuthbert and Viollier, 1992). It is found that the finite part is responsible for the large $O(\alpha_s)$ corrections, and it seems to have the 'wrong' sign. As mentioned earlier, the

self-energy type diagrams are discarded by other authors, but, as we have seen, there are compelling reasons for retaining these diagrams in the cavity. The results might be indicating that the boundary conditions confining the fields to the cavity are inadequate—a fact already hinted at by the disconcertingly large self-energies of a confined quark (Stoddart and Viollier, 1990; Stoddart and Viollier, 1992; Goldhaber *et al.*, 1986) and gluon (Schreiber and Viollier, 1992). One should perhaps determine the boundary conditions in some self-consistent way such that the nucleon observables and self-energies have reasonable values, and this exercise may even lead to some insight into the mechanism of confinement. Alternatively, the poor fit of the calculated observables to the experimental values could be due in part to our omission of corrections such as the center-of-mass or effects of the pionic cloud at the cavity surface. It is also possible that higher-order diagrams should be included to obtain a better fit, and although that would be an extremely un-attractive task, it would be useful to know if the perturbative series really is converging.

ACKNOWLEDGEMENT

I would like to thank M.S. O'Connor, R.J. Lindebaum and P.R. Page with whom I have collaborated on this topic. Financial support by the Foundation of Research Development is gratefully acknowledged.

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