Evolution Kernels of Twist-3 Light-Ray Operators in Polarized Deep Inelastic Scattering

B. Geyer, D. Müller, D. Robaschik
Fakultät für Physik und Geowissenschaft der Universität Leipzig
Augustusplatz, D-04109 Leipzig, Germany

Abstract

The twist three contributions to the $Q^2$-evolution of the spin-dependent structure function $g_2(x)$ are considered in the non-local operator product approach. Starting from the perturbative expansion of the T-product of two electromagnetic currents, we introduce the nonlocal light-cone expansion proved by Anikin and Zavialov and determine the physical relevant set of light-ray operators of twist three. Using the equations of motion we show the equivalence of these operators to the Shuryak-Vainshtein operators plus the mass operator, and we determine their evolution kernels using the light-cone gauge with the Leibbrandt-Mandelstam prescription. The result of Balitsky and Braun for the twist three evolution kernel (nonsinglet case) is confirmed.

1 Introduction

According to the present status of quantum chromodynamics nonperturbative inputs, being not calculable from first principles, are necessary for the interpretation and prediction of experimental results. Nevertheless, these inputs, for instance parton distribution and fragmentation functions, satisfy evolution equations whose integral kernels are calculable perturbatively. For the twist two parton distribution functions all evolution kernels are now known up to next-to-leading order.

However, also higher twist effects are experimentally accessible. Recently, in deep inelastic scattering the first moments of the polarized structure function $g_3(x)$ are measured [1]. In leading order of the momentum transfer $Q^2$ this structure function is determined by twist two as well as twist three contributions. In comparison with the twist two case the treatment of twist three is technically more subtle due to the appearance of a set of operators mixing under renormalization and constrained by relations between themselves. Up to now there exist already several papers which determine the local anomalous dimensions [2, 3, 4] as well as nonlocal operators and the evolution kernels for the distribution functions [2, 5, 6].
The two most complete calculations of the evolution kernels including the flavor singlet case, \[2\] and \[6\], are based on complicated techniques; moreover, the comparison between both approaches is by no means trivial and both papers contain also some misprints. Therefore, in a first step, our aim is to confirm the known results using an independent approach and compare it with the local anomalous dimensions. In a next step we will study the solution of the evolution equations including the singlet case. Here we present the first results for the nonsinglet case. Technically we apply the nonlocal operator product expansion \[7\], \[8\] proved by Anikin and Zavialov in renormalized quantum field theory. To avoid the complications with ghost and gauge-variant operators which are present in covariant gauge we apply the light-cone gauge using the Leibbrand-Mandelstam prescription \[9\], \[10\].

2 Perturbative Analysis

The part of the hadronic tensor being relevant for polarized deep inelastic scattering is determined by the antisymmetric part (with respect to Lorentz indices) of the absorptive part of the virtual forward Compton amplitude. In the Bjorken region the leading terms in \(Q^2\) of the Compton amplitude correspond to the leading light-cone singularities in the coordinate space.

Our analysis starts with a perturbative investigation of the time ordered product of two electromagnetic currents. For simplicity, we will neglect flavor and color indices in the following. In leading order the antisymmetric part is determined by both twist two and twist three contributions. A perturbative expansion up to the first order in the coupling constant \(g\) provides additionally twist four contributions:

\[
\{ T\bar{\psi}(x)\gamma^\mu\psi(x)\bar{\psi}(y)\gamma^\nu\psi(y) \}^{as} = \epsilon^{\mu\nu\rho\sigma} \{ i\partial^\rho D^c(x,y) \bar{\psi}(x)\gamma^\tau\gamma^\nu U(x,y)\bar{\psi}(y) \\
+ \frac{g}{2} D^c(x,y) \bar{\psi}(x)\gamma^\tau\gamma^\nu \int_0^1 dt (2t - 1) U(x,z)F_{\rho\lambda}(z)(x - y)^\lambda U(z,y)\bar{\psi}(y) \\
- \frac{ig}{2} D^c(x,y) \bar{\psi}(x)\gamma^\tau \int_0^1 dt U(x,z)\tilde{F}_{\rho\lambda}(z)(x - y)^\lambda U(z,y)\bar{\psi}(y) \} + (x,\mu \leftrightarrow y,\nu),
\]

where \(z = z(t)\) with \(z(1) = x\) and \(z(0) = y\); \(\tilde{F}_{\alpha\beta} = 1/2\epsilon_{\alpha\beta\mu\nu} F^{\mu\nu}\) is the dual field strength tensor and the path ordered phase factor

\[
U(x,y) = P \exp \left\{ -ig \int_y^x dw A_\mu(w) \right\}
\]

ensures gauge invariance. Instead of the free field propagator \(S^c_0(x)\) we used here the spinor propagator in an external gluon field (which is a straightforward generalization of the abelian case \[11\]). One gets up to the first order in \(g\):

\[
S^c(x,0) = S^c_0(x) U_s(x,0) + \frac{g}{4} D^c(x) \int_0^1 dt \{ (4t - 2)\gamma^\mu x^\nu - \gamma^\mu \not{x}\gamma^\nu \} U_s(x,tx)F_{\mu\nu}(tx)U_s(tx,0) + \cdots,
\]

with

\[
U_s(x,y) = P \exp \left\{ -ig \int_0^1 d\lambda A_\mu(w(\lambda))(x^\mu - y^\mu) \right\}, \quad w(\lambda) = x\lambda + y(1 - \lambda).
\]
The nonlocal light-cone expansion is obtained by approximating the vector $x$ by the light-like vector $\tilde{x}$ defined as $x = \tilde{x} + a(x, \eta) \eta$, where $\eta$ denotes a fixed auxiliary vector (e.g. normalized by $\eta^2 = 1$) and $a$ the corresponding coefficient. In leading order we substitute $p x \to \tilde{p} \tilde{x}$, where $p$ denotes a momentum, but the $x^2$-singularities remain unchanged, $x^2 \to \tilde{x}^2$. In general, the coefficient function will depend on two auxiliary variables $\kappa_i$, whose range (according to the $\alpha$-representation of the contributing Feynman diagrams) is restricted by $0 \leq \kappa_i \leq 1$. Here we introduce these variables quite trivially through integration over two $\delta$-functions. In this more or less intuitive way we get the following light-cone expansion:

$$\{T\bar{j}^\mu(x)j^\nu(0)\}^{\text{as}} = \epsilon^{\mu\nu\rho\sigma} \int_0^1 d\kappa_1 \int_0^1 d\kappa_2 \delta(\kappa_1 - 1)\delta(\kappa_2)$$

$$\{i\partial_\rho^\rho D^\rho(x) \tilde{\psi}(\kappa_1 \tilde{x}) \gamma_\sigma \gamma_5 U_s(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) \psi(\kappa_2 \tilde{x}) - \frac{g}{2} D^\rho(x) \tilde{\psi}(\kappa_1 \tilde{x}) \gamma_\sigma \int_0^1 d\tau U_s(\kappa_1 \tilde{x}, z) [\gamma_5 (2\tau - 1) F_{\rho\lambda}(z) - i \tilde{F}_{\rho\lambda}(z)] \tilde{x}^\lambda U_s(z, \kappa_2 \tilde{x}) \psi(\kappa_2 \tilde{x})\} + (\kappa_1 \leftrightarrow \kappa_2),$$

with $z = [(\kappa_1 - \kappa_2)\tau + \kappa_2] \tilde{x}$. In renormalized quantum field theory the proof of this representation is more complicated (for a rigorous derivation see [7]). In the following we use the light-cone gauge $\tilde{x} A = 0$, so that $U_s(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) \equiv 1$. Obviously, expression (2.3) contains two light-ray operators from which we start our consideration:

$$O_\rho(\kappa_1, \kappa_2) = \tilde{\psi}(\kappa_1 \tilde{x}) \gamma_\rho \gamma_5 \psi(\kappa_2 \tilde{x}),$$

$$O_{[\rho\sigma]}(\kappa_1, \kappa_2) = \frac{g}{4} \tilde{\psi}(\kappa_1 \tilde{x}) \gamma_\sigma \int_0^1 d\tau [\gamma_5 (2\tau - 1) F_{\rho\lambda}(z) - i \tilde{F}_{\rho\lambda}(z)] \tilde{x}^\lambda \psi(\kappa_2 \tilde{x}) - (\rho \leftrightarrow \sigma).$$

The operator $O_\rho(\kappa_1, \kappa_2)$ contains twist two as well as twist three contributions, whereas $O_{[\rho\sigma]}(\kappa_1, \kappa_2)$ contains contributions of twist 3 and higher (because of their coefficient functions the last operator is power suppressed).

### 3 Choice of Light-Ray Operators

As mentioned before, the operator (2.4) has mixed twist, so we look for a decomposition into its parts of definite twist. For the local light-cone operators there exists a well-known procedure: The tensor structure of operators with definite dimension decomposes into irreducible representations of the (formal) symmetry group $O(4)$. For light-ray operators, however, one has to take into account towers of such irreducible representations. Contraction with the light-cone vector $\tilde{x}$ projects onto the leading twist two piece,

$$O^{tw2}(\kappa_1, \kappa_2) = \tilde{x}^\rho O_\rho(\kappa_1, \kappa_2) = \tilde{\psi}(\kappa_1 \tilde{x}) \tilde{x} \gamma_5 \psi(\kappa_2 \tilde{x}).$$

The usual local twist two operators with spin $n$ follow according to $(\partial/\partial \kappa \equiv \tilde{x} \partial)$

$$O^{tw2}_n = (i \tilde{x} \partial)^{n-1} O^{tw2}(0, \kappa)|_{\kappa = 0} = \tilde{x}^{\mu_1} \ldots \tilde{x}^{\mu_n} O^{tw2}_{\mu_1 \ldots \mu_n}. $$

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where $O_{\mu_1...\mu_n}^{tw2}$ is symmetrised and traceless.

In order to construct the twist three part we represent the operator (2.4) in terms of local operators, extract the twist three part of them, and express the result again in terms of nonlocal operators. It turns out that

$$O_{\rho}^{tw3}(\kappa_1, \kappa_2) = -i(\kappa_2 - \kappa_1) \int_0^1 du \tilde{O}_\rho(\kappa_1, \kappa_1 \bar{u} + \kappa_2 u),$$

(3.3)

$$\tilde{O}_\rho(\kappa_1, \kappa_2) = \int_0^1 du \tilde{\psi}(\kappa_1 \bar{x}) \gamma_{[\rho} \tilde{x}_{\sigma]} D^\sigma(\kappa_1 \bar{x}, \kappa_2 \bar{x}) \gamma^5 \psi([\kappa_1 + \kappa_2 u] \bar{x}),$$

(3.4)

where we introduced the notation $D^\sigma(\kappa_1, \bar{x}, \kappa_2 \bar{x}) = \partial_{\kappa_2 \bar{z}} + igA^\rho([\kappa_1 \bar{u} + \kappa_2 u] \bar{x})$ and $\bar{u} = 1 - u$. Analogous to the twist two case our result can easily be checked by comparing it with the local operators of definite twist.

All operators with the same or lower twist can be mixed under renormalization. Therefore, we also have to take into account the following twist 3 operators:

$$\pm S_\rho(\kappa_1, \tau, \kappa_2) = ig\tilde{\psi}(\kappa_1 \bar{x}) \tilde{x} [i \tilde{F}_{\alpha \rho}(\tau \bar{x}) \pm \gamma^5 F_{\alpha \rho}(\tau \bar{x})] \tilde{x}^\alpha \psi(\kappa_2 \bar{x}).$$

(3.5)

These operators are the building blocks of the operator (2.5), which is contained in the light-cone expansion. For the local case, the importance of the operators (3.5) has been first observed by Shuryak and Vainshtein in [12]. They result from an application of the equations of motion to the local twist three operators.

The same can be shown directly and more easily also for the nonlocal operators. Applying the equations of motion to the operator $\tilde{O}_\rho$ and using the relation $A_\nu(\bar{u} \bar{x}) - A_\nu(v \bar{x}) = \int_v^u d\tau \bar{x}^\mu F_{\mu \nu}(\tau \bar{x})$ which is valid in the light-cone gauge we get

$$\tilde{O}_\rho(\kappa_1, \kappa_2) = \frac{i}{2}(\kappa_2 - \kappa_1) \int_0^1 du \left( \Omega_{EOM}^\rho(u) + \Omega_{REM}^\rho(u) - 2M_\rho(\kappa_1, \kappa_1 \bar{u} + \kappa_2 u) \right.

+ u \pm S_\rho(\kappa_1, \kappa_1 \bar{u} + \kappa_2 u, \kappa_2) + \bar{u} \mp S_\rho(\kappa_1, \kappa_1 \bar{u} + \kappa_2 u, \kappa_2),$$

(3.6)

where $\Omega_{EOM}$ is an equation of motion operator, $\Omega_{REM}$ contains residual trace terms (being proportional to $\tilde{x}_\rho$) and operators containing an overall derivative, and $M_\rho$ is the mass dependent operator defined by

$$M_\rho(\kappa_1, \kappa_2) = m \tilde{\psi}(\kappa_1) \sigma_{\alpha \rho} \tilde{x}^\alpha \gamma^5(\bar{x}D)(\kappa_2 \bar{x}) \psi(\kappa_2 \bar{x}), \quad \sigma_{\alpha \beta} = \frac{i}{2}[\gamma_\alpha, \gamma_\beta].$$

(3.7)

Corresponding to Eq. (3.3) also the operator $O_{\rho}^{tw3}$ is build up of the Shuryak-Vainshtein operators $\pm S_\rho$ and the mass operator $M_\rho$. Because of the following properties the additional operators $\Omega_{EOM}$ and $\Omega_{REM}$ can be neglected: If the operator $\Omega_{EOM}$ is sandwiched between physical states their matrix element vanishes. The trace terms in the remaining operators $\Omega_{REM}$ can be safely neglected and the obtained overall derivative vanishes in the forward case.

4 Evolution Kernel of the Light-Ray Operators

Before we present our result for the forward evolution kernel of the nonsinglet twist three light-ray operators we will discuss shortly the mixing properties of these operators. If we take into account all possible mixing partners up to trace terms, then
we get the following set of operators

$$\{ \tilde{O}_\rho, \; \pm S_\rho, \; M_\rho, \; \Omega^\text{EOM}_\rho \}. \quad (4.1)$$

Also the equation of motion operator $\Omega^\text{EOM}_\rho$ possesses an anomalous dimension and will be mixed with the other operators. However, from the general renormalization properties of gauge invariant operators it is to be expected that the counter term of $\Omega^\text{EOM}_\rho$ is only given by the operator itself [14]. In fact, this was explicitly shown in [2] and recently in [4]. Thus, the anomalous dimension matrix is triangular. Forming physical matrix elements the equation of motion operators as well as their anomalous dimensions drop out completely from the renormalization group equation. Furthermore, because of the constraint (3.6) we can eliminate the operator $\tilde{O}_\rho$ and are left with the operators $\pm S_\rho$ and $M_\rho$ only.

There exists an additional property of the operators $\pm S_\rho$ which guarantees that $\pm S_\rho$ and $\mp S_\rho$ do not mix under renormalization. The contraction with $\epsilon_{\alpha\beta\rho} \tilde{\alpha}_\sigma \tilde{x}_\beta^* \tilde{\sigma}$, where $\tilde{x}_\beta^*$ is a second light-cone vector with $\tilde{x} \tilde{x}^* = 1$, transforms the operators $\pm S_\rho$ into $\pm \tilde{S}_\rho = \pm \pm S_\rho \gamma^5$ (here the vertices of $\tilde{S}_\rho$ and $S_\rho$ differ by a $\gamma^5$ matrix). Since the anomalous dimensions of $\pm \tilde{S}_\rho$ and $\pm S_\rho$ coincide the operators $\pm S_\rho$ are effectively eigenstates of the duality transformation and thus do not mix under renormalization.

Using FeynCalc and some own subroutines for doing the momentum integration in light-cone gauge we calculated the pole part of the contributing one-loop diagrams of the operators $\pm S_\rho$ and $M_\rho$ in space time dimension $n = 4 - 2\epsilon$ (the same set of diagrams as given in [6]). Applying $\mu^2 \frac{d}{d\mu^2} = -\epsilon \frac{\partial}{\partial y} + \cdots$ we get the result:

$$\mu^2 \frac{d}{d\mu^2} + S^\sigma(\kappa_1, \kappa_2) = \frac{\alpha_s}{4\pi} \int_0^1 dy \int_0^{1-y} dz \left\{ (2C_F - C_A) \left[ y \delta(z) + S^\sigma(-\kappa_1 y, \kappa_2 - \kappa_1 y) - 2z S^\sigma(\kappa_1 - \kappa_2(1 - z), -\kappa_2 y) \right] \\
+ K(y, z) + S^\sigma(\kappa_1(1 - y) + \kappa_2 y, \kappa_2(1 - z) + \kappa_1 z) \right\} + C_A \left[ (2(1 - z) + L(y, z)) S^\sigma(\kappa_1 - \kappa_2 z, \kappa_2 y) + L(y, z) S^\sigma(\kappa_1 y, \kappa_2 - \kappa_1 z) \right] + 2C_F (1 - y)^2 \delta(1 - y) M^\rho(\kappa_2 z - \kappa_1 y) \right\}. \quad (4.2)$$

$$\mu^2 \frac{d}{d\mu^2} - S^\sigma(\kappa_1, \kappa_2) = \frac{\alpha_s}{4\pi} \int_0^1 dy \int_0^{1-y} dz \left\{ (2C_F - C_A) \left[ y \delta(z) - S^\sigma(\kappa_1 - \kappa_2 y, -\kappa_2 y) - 2z S^\sigma(-\kappa_1 y, \kappa_2 - \kappa_1(1 - z)) \right] \\
+ K(y, z) - S^\sigma(\kappa_1(1 - y) + \kappa_2 y, \kappa_2(1 - z) + \kappa_1 z) \right\} + C_A \left[ (2(1 - z) + L(y, z)) - S^\sigma(\kappa_1 y, \kappa_2 - \kappa_1 z) + L(y, z) - S^\sigma(\kappa_1 - \kappa_2 z, \kappa_2 y) \right] + 2C_F (1 - z)^2 \delta(1 - y) M^\rho(\kappa_2 z - \kappa_1 y) \right\}, \quad (4.3)$$

where

$$K(y, z) = \left[ 1 + \delta(z) \frac{1 - y}{y} + \delta(y) \frac{1 - z}{z} \right].$$

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\[ L(y, z) = \left[ \delta(1-y-z) \frac{y^2}{1-y} + \delta(z) \frac{y}{1-y} \right]_+ - \frac{7}{4} \delta(1-y)\delta(z), \]

and

\[ \mu^2 \frac{d}{d\mu^2} M^\rho(\kappa) = -4 C_F \frac{\alpha_s}{4\pi} \int_0^1 dy \left( 1 + y - \left[ \frac{1}{1-y} \right]_+ \right) M^\rho(\kappa y). \quad (4.4) \]

To condense the notation we used \( \pm^S^\rho(\kappa_1, \kappa_2) = \pm^S^\rho(\kappa_1, 0, \kappa_2) \), \( M^\rho(\kappa) = M^\rho(0, \kappa) \), and the standard plus-prescription fulfilling \( \int dy[...]_+ = 0 \) and \( \int dydz[...]_+ = 0 \), respectively; \( C_F = 4/3 \) and \( C_A = 3 \) are the usual Casimir operators of SU(3).

Since charge conjugation transforms \( \pm^S^\rho(\kappa_1, \kappa_2) \) into \( -^S^\rho(\kappa_2, \kappa_1) \) also the evolution kernels in Eq. (4.2) and Eq. (4.3) are related to each other. It is easy to see that our kernels satisfy this symmetry condition. Up to a different definition of \( \pm^S^\rho \) (and one small misprint) our result coincides with that of Balitsky and Braun [6] restricted to the forward case and \( m = 0 \).

Finally, we want to add some technical remarks.

Our operators are not completely traceless. Therefore, terms proportional to \( \tilde{x}_\rho \) have been omitted. Furthermore, the auxiliary pole of the gluon propagator is regularized by the Leibbrandt-Mandelstam prescription [9]

\[ \frac{1}{k\tilde{x}} = \frac{k\tilde{x}^*}{(k\tilde{x})(k\tilde{x}^*) + i\epsilon} \quad (4.5) \]

with the light-like vector \( \tilde{x}^* \). There are different advantages of this prescription proved in one loop order, e.g., consistency of tensor integral relations and the validity of power counting [9, 13]. The investigation of local anomalous dimension shows that \( \tilde{x}^* \)-dependent operators as well as special nonlocal operators may appear [10]. However, their anomalous dimensions decouple from the anomalous dimensions of gauge invariant operators [14]. Also in our calculation a \( \tilde{x}^* \)-dependent operator appears and, as expected, does not contribute to the physical sector.

Let us point out shortly, that in our calculation different prescriptions of the auxiliary pole in the gluon propagator do only affect terms which are proportional to two \( \delta \)-functions. In the final result this effect is compensated by the wave function renormalization. As an example, we consider a triangular diagram of the operator \( \pm^S^\rho \) with two external fermion lines and one external gluon line

\[ \int d^{4-2\epsilon} k \frac{P(\gamma, k, p_1, p_2)}{((p_1 + k)^2 - m^2)((p_2 + k)^2 - m^2)k^2} \frac{e^{i((p_1+k)^2\tilde{x}_k_1-(p_2+k)^2\tilde{x}_k_2)}}{k\tilde{x}}. \quad (4.6) \]

The exponential can be rewritten as

\[ e^{i(p_1\tilde{x})\kappa_1-(p_2\tilde{x})\kappa_2} \left\{ \frac{e^{i(\kappa_1-\kappa_2)k\tilde{x}}}{k\tilde{x}} - \frac{1}{k\tilde{x}} + \frac{1}{k\tilde{x}} \right\}. \quad (4.7) \]

The first term in the curly bracket is analytic in \( k\tilde{x} \) so that the typical problem of light-cone gauge occur only in the second term which, however, contributes to a trivial \( \kappa \)-structure only.
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