Comparisons of Cluster Mass Determinations by X-ray Observations and Gravitational Lensing

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ABSTRACT

Gravitational lensing by clusters of galaxies has been detected on scales ranging from $\sim 10^{-1}$ Mpc to $\sim 10$ Mpc, namely, arcs/arclets, weak lensing and quasar-cluster associations. This allows us to derive an overall radius matter distribution of clusters of galaxies. While the dynamical analysis of the X-ray observations has yielded a great number of data for the virial cluster masses, it becomes possible to statistically compare the cluster mass determinations by these two independent methods. In this letter we show that as compared with gravitational lensing, the dynamical analysis under the assumption of isothermal and hydrostatic equilibrium has systematically underestimated the cluster masses inside the Abell radius by a factor of $\sim 2$ with scatter between 0.7 and 5. Because the same correction factor should be applicable to the gas baryon fraction of clusters of galaxies obtained from the X-ray data, it is probably too premature to claim a baryon crisis in today’s cosmology.

Subject headings: cosmology: theory — galaxies: clusters: general — gravitational lensing
1. Introduction

A combination of the primordial nucleon abundances predicted by the standard Big Bang Nucleosynthesis (BBN) and those inferred from astronomical observations has set a tight constraint on the baryonic matter component of the universe (Walker et al. 1991): $0.04 < \Omega_b h_{50}^2 < 0.06$. This indicates that the baryon fraction, $f_b \equiv \Omega_b / \Omega$ with $\Omega$ being the average mass density of the universe in units of critical density, is smaller than $\sim 0.06 \, h_{50}^{-2}$ in the prevailing cosmological model of $\Omega = 1$, and a significant fraction of the mass in the universe should be invisible (non-baryonic matter). However, such a standard scenario has been challenged in recent years by the X-ray observations which detect a considerably large amount of the hot X-ray emitting gas in clusters/groups of galaxies. The resulting gas baryon fraction is a few time greater than the prediction of BBN, provided that the gas is in the state of hydrostatic equilibrium with the gravitational potentials of clusters/groups of galaxies. The baryon crisis thus arises if the matter in clusters/groups of galaxies is representative of the universe. In particular, this discrepancy probably implies that at least one of the basic hypotheses in our current theories of cosmological study needs to be modified or even abandoned (White et al. 1993).

Yet, the above claim should be taken very cautiously without carefully examining the reliability of the X-ray cluster mass determinations. Indeed, the existence of substructures and the recent detection of the complex two-dimensional temperature patterns in clusters of galaxies (e.g. Henry & Briel 1995; Markevitch 1996; Henriksen & Markevitch 1996; Henriksen & White 1996) strongly suggest that clusters of galaxies may not be the well-virialized dynamical systems as were believed before and the uncertainty in cluster mass determinations assuming hydrostatic equilibrium for the X-ray gas may be quite large (Balland & Blanchard 1996). Therefore, it is desirable that another independent cluster mass estimate is made to test the accuracy of the X-ray cluster mass determinations and
furthermore, to re-examine whether there is a baryon overdensity in clusters of galaxies.

It has been realized that gravitational lensing associated with clusters of galaxies can fulfill the task, which gives rise to cluster masses regardless of the cluster matter components and states. In several clusters of galaxies where both X-ray data and image distortions of background galaxies are available, comparisons of the virial cluster masses derived from X-ray observations and the gravitational cluster masses inferred from the distorted images of distant galaxies have been made (e.g. Wu 1994; Fahlman et al. 1994; Miralda-Escudé & Babul 1995; Squires et al. 1995). Today, gravitational lensing by clusters of galaxies has been detected on scales ranging from the inner core to the outer radius of ten arcminutes, including giant arcs/arclets, weakly distorted images of background galaxies and quasar-cluster associations. These lensing observations alone may allow us to derive an overall radius matter distribution of clusters. Therefore, it would become possible to statistically compare the cluster matter distributions given by dynamical analysis of the X-ray observations with those by gravitational lensing. This procedure is essentially different from the previous work which focused on individual clusters with both X-ray and gravitational lensing observations. We now select the two sets of data separately from literature. This letter presents the result of the comparisons and discusses its significance for cosmological study. Throughout this letter we adopt a matter-dominated flat cosmological model of $\Omega = 1$ and a Hubble constant of $H_0 = 50 h_{50}$ km s$^{-1}$ Mpc.

2. X-ray cluster mass determination and gas baryon fraction

Under the assumption of the standard isothermal $\beta$-model for the X-ray surface brightness of cluster of galaxies, the total mass in gas within radius $r$ of cluster center is (Cowie, Henriksen, & Mushotzky 1987) $M_{\text{gas}}(r) = 4\pi n_0 r^2 \mu m_p \int_0^{r/c} x^2 (1 + x^2)^{-3\beta/2} dx$, while the equations of hydrostatic and dynamical equilibrium give the total virial mass
\[ M_{\text{vir}}(r) = 3\beta (kT/\mu m_p G) r^3/(r^2 + r_c^2), \]

where \( n_0 \) and \( r_c \) denote, respectively, the central number density and core radius of the gas profile, \( T \) is the gas temperature, \( k \), the Boltzmann’s constant and \( \mu m_p \), the mean particle mass. The ratio of \( M_{\text{gas}} \) to \( M_{\text{vir}} \) provides a conservative estimate of the cluster baryon fraction \( f_b \) since the galaxy contribution is not included. Fig.1 shows the measured gas baryon fractions of clusters of galaxies in literatures without any corrections, in which we have only utilized the virial masses obtained in the case of isothermality. Note that for most of the clusters the gas baryon fractions at radii of larger than \( \sim 1 \) Mpc are computed from the spatially-unresolved measurements of the gas temperature. This leads to an underestimate of gas baryon fraction if temperature decreases with radius as it is naturally expected (e.g. Henriksen & Mamon 1994). The relatively low gas baryon fraction at the largest radius \( r \approx 7.1 - 10 \) Mpc for A2142 in Fig.1 probably arises from such an oversimple assumption (Henriksen & White 1996). It appears that based on the current data, we have not detected any apparent variations of the gas baryon faction of clusters of galaxies with radius. The mean gas baryon fractions are \( \overline{f_b} \approx 14\% \) and \( \overline{f_b} \approx 18\% \) with and without those data at \( r = 0.5 \) Mpc given by Edge & Stewart (1991). It is thus concluded that the mean baryon fraction in clusters of galaxies is about 2 – 4 times larger than the prediction of BBN, if the hot X-ray gas is in hydrostatic, isothermal equilibrium with the binding cluster gravitational potentials.

**EDITOR: PLACE FIGURE 1 HERE.**

### 3. Cluster masses from gravitational lensing

Arcs/arclets are the strongly/moderately distorted images of distant galaxies by foreground clusters of galaxies. The projected cluster mass within the position \( (r_{\text{arc}}) \) of arc/arclet can be easily obtained if one assumes a spherical matter distribution for the
lensing cluster and approximates the alignment parameter of the background galaxy to zero:

\[ m_{\text{lens}}(r_{\text{arc}}) = \pi r_{\text{arc}}^2 \Sigma_{\text{crit}}, \]

where \( \Sigma_{\text{crit}} \equiv \left( c^2 / 4\pi G \right) \left( D_s / D_d D_{ds} \right) \) is the critical mass density with \( D_d, D_s \) and \( D_{ds} \) being the angular diameter distances to the cluster, to the galaxy and from the cluster to the galaxy, respectively. For the complex arc/arclet configurations, cluster mass can be estimated by constructing an asymmetrical lens model [see Fort & Mellier (1994) for a recent review]. We illustrate in Fig.2 the cluster masses given by modeling of arcs/arclets. One major uncertainty comes from the unknown redshifts for some arclike images, for which we have assumed \( z_{\text{arc}} = 0.8 \). For a typical arc-cluster at redshift of \( \sim 0.3 \), this leads to an overestimate of cluster mass by a factor of 1.4 if the background galaxy is actually located at \( z_{\text{arc}} = 2 \).

Another powerful tool of probing the matter distribution of cluster of galaxies is to study the weak gravitationally induced distortions in the images of faint galaxies behind cluster of galaxies. By analyzing the shear field \( (\gamma_T) \) around the cluster, the statistics

\[ \zeta(r) = \int_r^{r_{\text{max}}} \langle \gamma_T \rangle \left( 1 - r^2 / r_{\text{max}}^2 \right)^{-1} d\ln r \]

measures the mean surface mass density in units of \( \Sigma_{\text{crit}} \) interior to \( r \) minus that in the annulus from \( r \) to \( r_{\text{max}} \) (Fahlman et al. 1994). Therefore, a lower bound on the projected cluster mass within the radius \( r \) can be found through

\[ m_{\text{lens}}(r) = \pi r^2 \zeta(r) \Sigma_{\text{crit}}. \]

Cluster mass reconstructions have been made for several clusters of galaxies in which the statistically significant shear patterns are detected. The resulting cluster masses are plotted in Fig.2. Again, there has been so far no information available about the redshifts of background galaxies and a mean value of \( \langle z \rangle = 1 - 3 \) has been often assumed in the computations. This brings about an uncertainty of cluster mass by a factor of \( \sim 1.3 \) for a lensing cluster at redshift of \( \sim 0.3 \).

Gravitational magnification can also enhance the number density of background sources around a foreground cluster of galaxies, which has been recently confirmed by discovering
the so-called quasar-cluster associations on scale of up to $\sim 10$ arcminutes (Wu & Fang 1996 and references therein). One can figure out the mean cluster mass which is required to produce the reported quasar overdensity in terms of gravitational lensing. It turns out that clusters of galaxies should contain considerably large gravitational masses extending to a radius of $\sim 10$ Mpc in order to account for the quasar enhancements. We compute the projected cluster masses over the association areas simply by $m_{\text{lens}}(r) = \pi r^2 \Sigma$ and show the results in Fig. 2, where the mean cluster surface mass density $\Sigma$ have been given by Wu & Fang (1996) for the four measurements of quasar-cluster associations.

It is remarkable that the projected cluster masses revealed statistically by three different lensing methods over two decades in radius from $\sim 10^{-1}$ Mpc to $\sim 10$ Mpc can be well fitted by a power-law: $m_{\text{lens}}(r) = 10^{15.39\pm0.17}(r/\text{Mpc})^{1.51} M_\odot$, where (also hereafter) the error bar represents the scatter of the best-fit average value rather than the real uncertainty in the measurement which is difficult to estimate. Because the weak lensing method usually provides a low limit to the cluster mass, most of its results are smaller than the mean value. The fitting without weakening lensing data becomes slightly steeper: $m_{\text{lens}}(r) = 10^{15.56\pm0.11}(r/\text{Mpc})^{1.63} M_\odot$. While one might argue the reliability of the cluster masses up to the radius of $r \approx 10$ Mpc derived from the quasar-cluster associations, we give the fit by removing the four results of quasar-cluster associations from Fig. 2: $m_{\text{lens}}(r) = 10^{15.22\pm0.15}(r/\text{Mpc})^{1.32} M_\odot$.

4. **Comparisons**

We also display in Fig. 2 the X-ray cluster masses $M_{\text{vir}}(r)$ derived from the isothermal $\beta$-model under the assumption of hydrostatic equilibrium, including the results for five compact groups of galaxies. Recall that $M_{\text{vir}}(r)$ are the dynamical masses used in Fig. 1 for computations of the gas baryon fractions in clusters of galaxies. The discrepancy of $M_{\text{vir}}$
and $m_{\text{lens}}(r)$ is clearly seen at small radius, while the two sets of data seem to merge beyond $r \sim 1$ Mpc. Therefore, an intuitive speculation for such a variation is the projection effect. We have then tested the conventional $r^{-2}$ profile and the so-called universal density profile found by the standard CDM simulations (Navarro, Frenk & While 1995). Neither of these profiles can fit both the three-dimensional masses and the two-dimensional projected ones. In fact, the deprojection of $m_{\text{lens}}(r)$ recovers the corresponding three-dimensional masses $M_{\text{lens}}(r)$, if one assumes a spherical matter distribution as we have already adopted in the above sections. We list the resulting $M_{\text{lens}}(r)$ in Table 1, together with a least-square fit of a power-law to the X-ray cluster masses $M_{\text{vir}}(r)$ and the ratios of $M_{\text{lens}}(r)/M_{\text{vir}}$ at different cluster radii.

There is a significant discrepancy between the virial and lensing cluster masses inside the core radius of the X-ray gas profile, $r_c \sim 0.25$ Mpc, where arcs/arclets are observed. It seems that lensing method using arclike images can give rise to the cluster masses of $\sim 5$ times larger than the virial masses. This is essentially comparable to the previous similar studies for individual clusters (Wu 1994, Miralda-Escudé & Babul 1995). Around the radius of $r \sim 1$ Mpc, at which the lensing data are dominated by the weak lensing observations, the mean ratio of $M_{\text{lens}}/M_{\text{vir}}$ is about $2 - 3$. However, considering the large scatters, one cannot exclude the possibility that the cluster masses derived from the two methods are consistent. Yet, most of the data based on the weakening lensing method correspond to the low limits to cluster masses. The similar conclusion can be applicable to the Abell radius of 3 Mpc, where one may expect a mean factor of $\sim 2$ in $M_{\text{lens}}/M_{\text{vir}}$. The mass discrepancy

EDITOR: PLACE FIGURE 2 HERE.

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might vanish at the outer radii of clusters ($r > 5$ Mpc) if we take out the results given by the quasar-cluster associations which dominate the cluster mass determinations with gravitational lensing at large cluster radius.

5. Discussion and conclusions

It appears that as compared with the gravitational lensing method, the dynamical analysis based on the isothermal, hydrostatic equilibrium has systematically underestimated the cluster masses within the Abell radius (3 Mpc) by a factor of $\sim 2$ with scatter ranging from 0.7 to 5. The mass discrepancy is rather remarkable inside the cluster core of $r_c \sim 0.25$ Mpc but diminishes along the outgoing radius. As an immediate result of this discrepancy, the baryon fractions in clusters of galaxies provided by the X-ray cluster masses should be correspondingly reduced by the same factor, which thus opens a possibility to solve or partially remove the recently claimed baryon crisis in clusters of galaxies. Meanwhile, our finding indicates that clusters of galaxies may not be regarded as the well-relaxed virialized systems.

The above conclusions are strongly supported by the spatially resolved spectra for some clusters of galaxies obtained with *ASCA*, *GINGA* and *ROSAT*, which show the complex temperature patterns over the cluster faces (see Henry & Briel 1995; Markevitch 1996; Henriksen & Markevitch 1996; Henriksen & White 1996). These significant temperature variations cannot be described by a simple analytic profile like a $\beta$-model, and non-isothermality in the hot X-ray emitting gas of clusters of galaxies is apparently required. So, cluster mass determinations using the isothermal hypothesis for the X-ray gas may lead to large errors.

Gravitational lensing is a robust estimate of gravitational mass in a celestial body
or system. We have found the consistency between the cluster masses derived from three different lensing phenomena over scale $0.1 \text{ Mpc} < r < 10 \text{ Mpc}$. The total masses inferred from lensing can be well represented by a single power-law of $\sim r^{1.5\pm0.2}$, indicative of a density profile of $\sim r^{-1.5\pm0.2}$. This is steeper than the singular isothermal matter distribution $\sim r^{-2}$ and lies between the $\sim r^{-1}$ and $\sim r^{-3}$ universal profile predicted by the standard CDM simulations (Navarro et al. 1995).

Nonetheless, the mass discrepancy between dynamical analysis and gravitational lensing at the central regions of clusters of galaxies could arise from the cooling flow and/or the contribution of the nonthermal pressure such as magnetic field (Loeb & Mao 1994; Ensslin et al. 1996). Alternatively, gravitational lensing may overestimate cluster masses if the lensing cluster is prolate with the major axis along the line-of-sight (Miralda-Escudé & Babul 1995). Furthermore, it is somewhat hard to understand the cluster mass extension to a radius of $\sim 10 \text{ Mpc}$ where the quasar-cluster associations are detected.

After all, neither of the present X-ray cluster masses and gravitational lensing data forms a complete sample and therefore, it is impossible to evaluate the statistical significance of the result. What we would like to emphasize in this letter is that the comparisons of the updated cluster mass determinations by these two independent methods have revealed the possible existence of large mass uncertainties under the scenario of the hydrostatic equilibrium. So, it is probably too premature to claim a baryon crisis in today’s cosmology.

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Table 1: Comparisons of virial and lensing cluster masses.

<table>
<thead>
<tr>
<th>method</th>
<th>power-law fit</th>
<th>$M_{\text{tens}}/M_{\text{vir}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{\text{vir}}(r) = 10^{14.53 \pm 0.23}(r/\text{Mpc})^{1.91} \ M_{\odot}$</td>
<td>$r = 1$ Mpc $\quad r = 3$ Mpc $\quad r = 5$ Mpc</td>
</tr>
<tr>
<td>virial</td>
<td></td>
<td>$2.69_{-1.31}^{+2.56}$ $\quad 1.73_{-0.84}^{+1.65}$ $\quad 1.41_{-0.68}^{+1.35}$</td>
</tr>
<tr>
<td>lensing (1)</td>
<td>$M_{\text{tens}}(r) = 10^{14.96 \pm 0.06}(r/\text{Mpc})^{1.51} \ M_{\odot}$</td>
<td>$2.34_{-1.05}^{+1.93}$ $\quad 1.23_{-0.56}^{+1.00}$ $\quad 0.91_{-0.41}^{+0.74}$</td>
</tr>
<tr>
<td>lensing (2)</td>
<td>$M_{\text{tens}}(r) = 10^{14.90 \pm 0.03}(r/\text{Mpc})^{1.32} \ M_{\odot}$</td>
<td>$3.24_{-1.62}^{+3.22}$ $\quad 2.38_{-1.19}^{+2.37}$ $\quad 2.06_{-1.03}^{+2.05}$</td>
</tr>
<tr>
<td>lensing (3)</td>
<td>$M_{\text{tens}}(r) = 10^{15.04 \pm 0.07}(r/\text{Mpc})^{1.63} \ M_{\odot}$</td>
<td></td>
</tr>
</tbody>
</table>

(1)Arclike images, weak lensing and quasar-cluster associations; (2)Arclike images and weak lensing; (3)Arclike images and quasar-cluster associations
REFERENCES


Fig. 1.— Baryon fractions in clusters of galaxies derived from the X-ray observations under the assumption of isothermal and hydrostatic equilibrium. The dashed lines show the predictions of the standard nucleosynthesis for the cosmological models of $\Omega = 1$ and $\Omega = 0.3$. (see Fig.2 for references)

Fig. 2.— Comparisons of cluster masses inside radius $R$ derived from dynamical analysis and gravitational lensing. Open squares: the projected cluster masses from arcs/arclets (Wu 1994; Kneib & Soucail 1995); Fancy squares: the projected cluster masses from weak lensing technique (Fahlman et al. 1994; Tyson & Fischer 1995; Smail & Dickinson 1995; Luppino & Kaiser 1996; Seitz et al. 1996; Squires et al. 1996a,b); Diamonds: the projected cluster masses from quasar-cluster associations (Wu & Fang 1996); Octagons: the cluster masses from X-ray observations (Hughes et al. 1989; Edge & Stewart 1991; Briel, Henry, & Böhringer 1992; Miyaji et al. 1993; White et al. 1993; Briel & Henry 1994; White et al. 1994; Elbaz, Stewart, & Böhringer 1995; Dell’Antoio, Geller, & Fabricant 1995; White & Fabian 1995; Schindler 1995; Schindler & Wambsganss 1995; Ikebe et al. 1996; Böhringer et al. 1996; Schindler et al. 1996; Squires et al. 1996a; Henriksen & White 1996). The masses of compact groups of galaxies from X-ray observations are also illustrated (crosses) as a comparison (Henriksen & Mamon 1994; Pildis, Bregman, & Evrard 1995). The solid lines show the best fitting power-laws of the dynamical and lensing data, and the dashed line is the three-dimensional cluster masses obtained by the deprojection of the two-dimensional lensing masses.