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Fixing Two Chiral Perturbation Theory Parameters by Fitting pion-pion Phase Shifts

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Abstract

In this paper, we fix the parameters $\bar{\ell}_1$ and $\bar{\ell}_2$ of the one loop corrected chiral perturbation theory by fitting the low energy pion-pion S- and P- experimental phase-shifts. The corresponding values of the scattering lengths are calculated. Related approaches for data fitting, in particular the unitarization program of current algebra, are also discussed.

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1 Introduction

Many results have been derived in the early sixties from the assumptions of a local chiral $SU(2) \times SU(2)$ algebra of vector and axial vector currents densities together with the partial conservation hypothesis relating the derivative of the axial vector current to the pion field. In 1979, Weinberg [1] suggested that it is possible to summarize these previous results in a phenomenological Lagrangian that incorporates all the constraints coming from chiral symmetry of the underlying theory. In a set of very important and fundamental papers, Gasser and Leutwyler [2] have developed Chiral Perturbation Theory (ChPT), that allows one to compute many different Green functions involving low energy pions as functions of lowest powers of their momenta, their masses and a few undetermined parameters. These parameters are not yet capable of being predicted from the fundamental theory and instead are obtained phenomenologically. This is an unfortunate since the parameters contain a great deal of information on the structure of the theory.

Two of these parameters, namely $\bar{\ell}_1$ and $\bar{\ell}_2$, were first determined [2] from pion-pion D -wave scattering length. There are in the literature other attempts to fix these parameters by fitting experimental data. These include the use of Padé expansion [3] for the amplitudes, inverse amplitude method [4] and explicit introduction of fundamental fields describing resonances[5]. In the present paper we will be concerned to fix $\bar{\ell}_1$ and $\bar{\ell}_2$ by fitting the experimental pion-pion S - and P - wave phase shifts. The present method was also used early [6] in the unitarization program of current algebra where, working with an amplitude with same dependence in s , t and u than one loop ChPT one, we succeeded to fix model parameters, equivalent to $\bar{\ell}_1$ and $\bar{\ell}_2$, by fitting suitable defined experimental phase shifts data. Following the unitarization program of current algebra philosophy, we show in this work how to fix the combination $\bar{\ell}_2 - \bar{\ell}_1$ by P -wave phase shift fitting and $\bar{\ell}_1$ by isospin zero S -wave phase shifts fitting.

We present in section 2 the ChPT $\pi\pi$ total amplitude [2] to be used through this letter and the corresponding isospin defined S - and P -wave used to fit experimental data. A quick discussion on the elimination of some parameters are also presented in this section. Section III is devoted to a discussion about phase shift and scattering length definition. The successful fits are shown, and the corresponding low energy parameters determined. We also show the comparison between Weinberg soft-pion amplitude[7] and each component of $O(p^4)$ ChPT corrected amplitude. Section IV is a brief summary of the main results of the present work and perspectives.

2 ChPT partial wave phase shifts

The one-loop amplitude for elastic pion scattering obtained from ChPT Lagrangian, first computed is [2]:

$$\begin{aligned} A(s, t, u) &= (s - m^2)/F_\pi^2 + B(s, t, u) + C(s, t, u) + O(E^6) \\ B(s, t, u) &= \{3(s^2 - m^4)\bar{J}(s) + [(t(t - u) - 2m^2t + 4m^2u - 2m^4)\bar{J}(t) + (t \leftrightarrow u)]\} / (6F_\pi^4) \\ C(s, t, u) &= [2(\bar{\ell}_1 - 4/3)(s - 2m^2)^2 + (\bar{\ell}_2 - 5/6)(s^2 + (t - u)^2)] / (96\pi^2 F_\pi^4), \end{aligned} \quad (1)$$

where: $\bar{J}(s) = \frac{1}{16\pi^2} \left[\sigma \ln \frac{\sigma-1}{\sigma+1} + 2 \right]$, and $\sigma = \left(\frac{s-4m^2}{s} \right)^{1/2}$.

In this expression we have omitted the parameters $\bar{\ell}_3$ and $\bar{\ell}_4$ because they produce a numerically small shift between F and M parameters of the chiral invariant Lagrangian and the corresponding physical values of the pion decay constant F_π and the pion mass m . In any case, they would not play a major role in the present data fitting.

To compare the given amplitude with low energy data on $\pi\pi$ scattering one expands the combinations with definite isospin in the s -channel:

$$T^0(s, t) = 3A(s, t, u) + A(t, s, u) + A(u, t, s),$$

$$T^1(s, t) = A(t, s, u) - A(u, t, s), \quad \text{and}$$

$$T^2(s, t) = A(t, s, u) + A(u, t, s)$$

into partial waves:

$$T^I(s, t) = 32\pi \sum_{\ell=0}^{\infty} (2\ell + 1) P_{\ell}(\cos \theta) t_{\ell}^I(s).$$

Using equation (1), the resulting P -wave amplitude is:

$$\begin{aligned} t_1^1(s) = & \frac{1}{3F_{\pi}^2}(s - 4m^2) + \frac{1}{18F_{\pi}^4}(s - 4m^2)^2 \bar{J}(s) - \\ & \left\{ \frac{m^4}{8} \left(\frac{s^2}{2} - \frac{13}{16} sm^2 - m^4 \right) \frac{L^2(s)}{(s-4m^2)^2} + \left(\frac{s^3}{288} - \frac{1}{18} s^2 m^2 + \frac{1}{4} sm^4 - \frac{m^6}{8} \right) \frac{L(s)}{\sigma(s-4m^2)} \right. \\ & - \frac{1}{864(s-4m^2)} (s^3 + 37s^2 m^2 - 149sm^4 + 120m^6) - \\ & \left. \frac{s-4m^2}{288} [s - 12m^2 + 2s(\bar{\ell}_2 - \bar{\ell}_1)] \right\} \frac{1}{F_{\pi}^4 \pi^2}, \end{aligned}$$

the isospin $I = 0$, S -wave is:

$$\begin{aligned} t_0^0(s) = & \frac{1}{F_{\pi}^2}(2s - m^2) + \frac{1}{2F_{\pi}^4}(2s - m^2)^2 \bar{J}(s) + \\ & \frac{1}{F_{\pi}^4 \pi^2} \left\{ \frac{m^4}{8} \left(-s + \frac{25}{6} m^2 \right) \frac{L^2(s)}{(s-4m^2)} + \left(\frac{7}{144} s^2 - \frac{5}{18} s m^2 + \frac{25}{48} m^4 \right) \frac{L(s)}{\sigma} - \right. \\ & \left. \frac{1}{(s-4m^2)} \left(\frac{11}{864} s^3 - \frac{29}{144} s^2 m^2 + \frac{19}{16} sm^4 - \frac{253}{108} m^6 \right) + \right. \\ & \left. \left(\frac{11}{144} \bar{\ell}_1 + \frac{7}{72} \bar{\ell}_2 - \frac{79}{432} \right) s^2 - \left(\frac{5}{18} (\bar{\ell}_1 + \bar{\ell}_2) - \frac{19}{54} \right) sm^2 + \frac{m^4}{18} \left(\frac{11}{2} \bar{\ell}_1 + 7\bar{\ell}_2 - \frac{389}{48} \right) \right\} \end{aligned}$$

and the isospin $I = 2$, S -wave is:

$$\begin{aligned} t_0^2 = & \frac{1}{F_{\pi}^2}(2m^2 - s) + \frac{1}{2F_{\pi}^4}(2m^2 - s)^2 \bar{J}(s) + \\ & \frac{1}{F_{\pi}^4 \pi^2} \left\{ \frac{m^4}{16} \left(s + \frac{m^2}{3} \right) \frac{L^2(s)}{(s-4m^2)} + \left(\frac{11}{288} s^2 - \frac{1}{9} sm^2 + \frac{m^4}{48} \right) \frac{L(s)}{\sigma} - \right. \\ & \left. \frac{1}{(s-4m^2)} \left(\frac{25}{1728} s^3 - \frac{7}{288} s^2 m^2 - \frac{5}{16} sm^4 + \frac{77}{108} m^6 \right) + \right. \\ & \left. \left[\left(\frac{1}{72} \bar{\ell}_1 + \frac{1}{18} \bar{\ell}_2 - \frac{7}{108} \right) s^2 - \left(\frac{1}{36} \bar{\ell}_1 + \frac{7}{36} \bar{\ell}_2 - \frac{35}{108} \right) sm^2 + \left(\frac{1}{18} \bar{\ell}_1 + \frac{2}{9} \bar{\ell}_2 - \frac{193}{432} \right) m^4 \right] \right\}, \end{aligned}$$

where: $L(s) = \ell n \left(\frac{1-\sigma}{1+\sigma} \right)^2$

These functions do fit experimental data. The first term of each partial wave corresponds to Weinberg amplitude[7]; it is followed by the s -channel loop contribution term containing the function $\bar{J}(s)$; the last term includes the parameters and is related to tree graphs contribution; all the remaining are crossed channel loop terms, introducing crossing symmetric correct left hand side cuts for the amplitudes. We take this occasion to comment on the phase shifts definition, in order to establish the difference between the present method to fix $\bar{\ell}_1$ and $\bar{\ell}_2$ and that used in other approaches.

It is well known that unitarity implies that in the elastic region $4m^2 < s < 16m^2$ the partial wave

amplitudes t_ℓ^I are constrained by:

$$I_m t_\ell^I(s) = \frac{\sigma}{16\pi} |t_\ell^I(s)|^2.$$

This relation, allows one to define real phase shifts δ_ℓ^I , such that:

$$t_\ell^I(s) = \frac{16\pi}{\sigma} \exp(i \delta_\ell^I(s)) \sin \delta_\ell^I(s). \quad (2)$$

Therefore one loop ChPT S - and P -wave amplitudes do not satisfy exactly elastic unitarity and (2) cannot be used. For this reason some authors[8][9][10] preferred using

$$\delta_\ell^I(s) = \frac{\sigma}{16\pi} R_\epsilon t_\ell^I(s),$$

which is valid for small values of δ 's.

We have adopted another definition for δ 's in the applications of our unitarization program of current algebra. Let us remember the main points of the referred program.

The program, proposed by one of us and applied to pion-pion[6] and to kaon-pion[11] scattering, consists in estimating the behavior of form-factors and propagators at low energies. As an example, we have estimated that the correction f_0 to scalar pion form factor F_0 magnitude is, near threshold, smaller than current algebra isospin zero $\pi\pi$ Weinberg[7] amplitude. To construct unitarized amplitudes, we look for the implications of elastic unitarity for form-factors and propagators in a peculiar way. For instance, the first order correction to F_0 is obtained from

$$I_m f_0^{(1)}(x) = \frac{1}{16\pi} \sigma(x) t_0^{ca}(x),$$

considered valid for $x \simeq m^2$, where $t_0^{ca}(x) = (2x - m^2) / F_\pi^2$ is the $I = 0$ Weinberg amplitude.

Functions denoted by a superscript (1) are of the order m^2/X^2 , X being of the order of magnitude of the vector meson mass in a vector dominance approximation and so, at low energies, they can be considered as corrections to the soft meson limit.

Considering known the imaginary part of each function composing the amplitude, the method consists in obtaining their real parts by the dispersion relation technique and introducing back in the crossing symmetric total amplitude the corrected form factor expressions. The partial wave projection of these corrections are of order $O(E^4)$ and results to be smaller than $O(E^2)$ contribution near threshold.

In order to exploit the S - and P - partial wave imaginary parts this procedure implies, we have adopted, in the unitarization program application, the following phase shift definition:

$$\delta_\ell^I(s) = \tan^{-1} \frac{I_m t_\ell^I(s)}{R_\epsilon t_\ell^I(s)}.$$

This definition will also be adopted in the present analysis.

3 Phase-shifts and low energy parameters

Using our definition of phase shifts, we realize that only S - and P - wave can be analyzed, for all other waves are described by real functions in the physical region. We have decided to first fix the combination $\bar{\ell}_2 - \bar{\ell}_1$ by fitting the isospin $I = 1$ P - wave (the ρ resonance) and then determine the remaining parameter $\bar{\ell}_1$ by adjusting the isospin $I = 0$ S -wave phase shifts. There is no free parameter in the resulting $I = 2$ S - wave.

The fitting parameters in figures 1 and 2 are: $\bar{\ell}_2 - \bar{\ell}_1 = -5.75$ and $\bar{\ell}_1 = 3.03$.

FIG. 1. P-wave phase shifts corresponding to $\bar{\ell}_2 - \bar{\ell}_1 = -5.75$ and experimental [16] data.

FIG. 2. S-wave phase shifts corresponding to $\bar{\ell}_1 = 3.03$ and experimental[16, 17, 18] data.

In order to study model dependence on parameters variation we present, for a 10% changing of the relevant parameters the corresponding phase shifts. We observe that both are quite dependent on parameter variation, as can be seen in figures (3) and (4).

FIG. 3. P-wave phase shifts dependence for $\pm 5\%$ and $\pm 10\%$ variation of the combination $\bar{\ell}_2 - \bar{\ell}_1$. The central curve corresponds to no variation of the parameter.

FIG. 4. S-wave phase shifts dependence for $\pm 5\%$ and $\pm 10\%$ variation of the parameter $\bar{\ell}_1$. The central curve corresponds to no variation of the parameter.

To analyze what makes P - wave fit the ρ resonance, and what makes S -wave to fit experimental data, we exhibit in figures (5) and (6) the splitted contributions for amplitude real parts. We note that at threshold $O(E^2)$ is bigger than each of other contributions; we can even verify the points where the sum of the contributions is zero.

$I = 0$	$\epsilon_0^0 = -2.45 (\%)$	$\epsilon_0^{\prime 0} = -16.2 (\%)$	$A_0^0 = -.158 (m_\pi^{-1})$
$I = 2$	$\epsilon_0^2 = 2.17 (\%)$	$\epsilon_0^{\prime 2} = 28.4 (\%)$	$A_0^2 = 0.043 (m_\pi^{-1})$

Table 1:

FIG. 5. The contributions for P-wave real parts. (a) is current algebra Weinberg amplitude; (b) is the contribution of the function having right-hand cut ; (c) corresponds to left hand side contribution and (d) is tree graphs polinomial part contribution.

FIG. 6. The contributions for S-wave real parts. (a) is current algebra Weinberg amplitude; (b) is the contribution of the function having left-hand cut ; (c) corresponds to right-hand side contribution and (d) is tree graphs polinomial part contribution.

Finally we determine the corrected S-wave scattering length and effective range. These parameters are related with near threshold partial waves behavior, and its current definition (not the standard one) [2][10] uses the two first coefficient of the amplitude real parts expansion:

$$R_e t_0^I(s) = a_0^I + (s - 4m^2) b_0^I + O((s - 4m^2)^2).$$

In the table we present the corrections ϵ_0^I and $\epsilon_0^{\prime I}$, defined by:

$$a_0^I = a_0^{\prime ca} (1 + \epsilon_0^I), \quad \text{and} \\ b_0^I = b_0^{\prime ca} (1 + \epsilon_0^{\prime I}).$$

In order to exploit the imaginary part of the one loop amplitude, in the physical region, we calculate the S-wave scattering length A_0 by the standard definition:

$$\frac{\sqrt{s - 4m^2}}{2} \cot \delta_0^I(s) = -\frac{1}{A_0^I} + \frac{s - 4m^2}{8} r_0^I + \dots,$$

and we present the results in the same table I.

4 Conclusion

We have fixed ChPT $\bar{\ell}_1$ and $\bar{\ell}_2$ parameters by fitting low energy $\pi\pi$ suitable defined phase shifts. In contrast with other method [10] in our approach the ρ data fixes just a combination of these parameters. On the other hand, the inverse amplitude method [4] can equally well fit the experimental phase shifts; however, in contrast of our method, this method does not preserve crossing symmetry. The successful fits are presented for the numerical values of $\bar{\ell}_1 - \bar{\ell}_2$ and $\bar{\ell}_1$ equal to 3.03 and -2.72 respectively. The corresponding corrections, (ϵ and ϵ'), to the behavior of the real part of the amplitudes at threshold, in respect to current algebra predictions are presented in Table I. In this table, we also present the model prediction for S -wave scattering length.

Finally, as $O(p^6)$ ChPT calculations are being performed[12], we remark that next order contributions have to be small up to resonance region; the new available parameters do fit higher ℓ - partial waves, letting $\bar{\ell}_1$ and $\bar{\ell}_2$ almost unchanged. On the other hand we would like to mention that, one of us[13] gave the tools for constructing second order quasi-unitarized $\pi\pi$ scattering amplitude to be compared with two-loop ChPT calculation. The comparison between the unitarization program and one-loop ChPT has already been done [14][15].

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