ABSTRACT

The null hypothesis that the three dimensional power spectrum measured from the APM Survey is consistent with the one dimensional power spectrum measured from the pencil beams surveys of Broadhurst et al, and of Szalay et al, is tested. The external estimates of the mean power that we make are sensitive to details of the model for the survey geometry and to the assumed level of the distortion of the pattern of galaxy clustering caused by peculiar motions. We find that the measured 3D clustering of galaxies can account for the presence of peaks in the one dimensional power spectrum, but is less successful in recovering the detailed appearance of the observations. We find no strong evidence for any additional large scale structure in the deep pencil beams beyond that recovered from the APM Survey. We conclude therefore that it is unlikely that large scale structure can be responsible for the steep local number counts of bright galaxies.

Key words: surveys-galaxies: clustering -dark matter - large-scale structure of Universe

1 INTRODUCTION

The claim by Broadhurst et al, (1990, hereafter BEKS) that the universe contains significant structure on scales larger than ∼ 100h⁻¹Mpc has provoked much controversy. Based upon the analysis of several deep, narrow angle or pencil beam redshift surveys of galaxies in the directions of the Galactic Poles, BEKS noted a striking periodicity in the pair counts of the galaxies. This periodicity is revealed as a spike in the one dimensional power spectrum of the radial galaxy distribution at k ∼ 0.05 h Mpc⁻¹, corresponding to a wavelength of λ ∼ 128h⁻¹Mpc. Using estimates of the mean power from the observed power spectrum (internal) and using a model for galaxy clustering combined with a window function for the survey (external), BEKS attached a high significance to the peak. The adopted null hypothesis, namely that the two point correlation function of galaxies on small scales has a power law form ξ(r) = (r/r₀)γ (Davis & Peebles 1983), with a truncation at r = 30h⁻¹Mpc and random correlations on larger scales, was ruled out.

In a comprehensive paper, Kaiser & Peacock (1991 - KP) argued that the mean power had been underestimated, which would reduce the significance of the power spectrum peak. Using a simple Poisson clumps model, KP demonstrated that the power spectrum at higher wavenumbers could be suppressed by peculiar velocities and redshift errors, leading to a lower internal estimate of the mean power. Furthermore, with more detailed modelling of the BEKS survey geometry and radial selection, KP showed that the external estimate of the power could be a factor of two larger than the figure obtained by BEKS. Note that KP employ a different null hypothesis, in that the observed small scale clustering of galaxies is extended to all scales using the same power law.

Szalay et al. (1993) remark that assessments of the significance of the peak in the BEKS power spectrum based upon external estimates of the mean power are unreliable. Such estimates are sensitive to assumptions made about the clustering of galaxies on large scales and about the survey geometry. This point is reinforced by Luo & Vishniac (1993), who using the same model for galaxy clustering but a slightly different window geometry and selection function to KP, obtain a value for the mean power that is intermediate between that of Szalay et al. (1993) and KP.

The original BEKS data has now been supplemented by additional pencil beams giving a wider angular coverage (Szalay et al. 1993, Broadhurst et al. 1995). These authors claim that the aliasing of small scale power to large scales is reduced for the extended survey geometry, which has a narrower window function in k-space. In addition, there are new claims for excess power on scales of λ ∼ 100h⁻¹Mpc, compared with standard models of structure formation such as the Cold Dark Matter scenario and its variants, from power spectrum analysis of the slices of the Las Campanas Redshift Survey (Landy et al. 1996), though the longest baseline of this survey is shorter than that of the BEKS data.

Measurements of the three dimensional power spectrum on scales λ > 100h⁻¹Mpc are now available from large redshift surveys (e.g. Feldman, Kaiser & Peacock 1994, Tadros & Efstathiou 1996) and from the deprojection of the clustering information in angular catalogues such as the APM Survey (Peacock 1991, Baugh & Efstathiou 1993, 1994).
In this paper we shall ask the question is the three dimensional power spectrum measured in real space from the APM Survey (Baugh & Efstathiou 1993, 1994) consistent with the one dimensional power spectrum of the BEKS survey and the new data reported by Szalay et al. (1993) and Broadhurst et al. (1995). The models adopted for the survey geometry of the observations are outlined in Section 2, with the effects of including redshift space distortions discussed in Section 3. The significance of the observed peaks are assessed by comparison with our external estimates of the mean power in Section 4.

2 PENCIL BEAM WINDOW FUNCTIONS

The projected one dimensional power spectrum is a convolution of the three dimensional power spectrum with the Fourier transform of the window function of the survey:

\[ P_{1D}(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dy P_{3D}(y) I(k, y), \]

where \( I(k, y) \) depends upon the particular model adopted for the radial and angular selection of the survey. We shall discuss three survey geometries:

A: Uniform cylinder: a disk of fixed comoving radius \( R \), neglecting the radial selection function, with cylinder length \( L \) given by the highest redshift galaxies observed. Following BEKS, we use a cylinder with radius \( R = 3h^{-1}\text{Mpc} \) and length \( L = 2034h^{-1}\text{Mpc} \).

B: A conical window, with fixed opening angle and a model for the survey selection function. In order to reproduce the deep NGP and SGP pencil beams of BEKS, we use a conical beam with opening angle 10 arcminutes and a magnitude limit of \( b_J \leq 22.5 \).

C: Multiple conical windows distributed at random within some larger solid angle. We choose 21 beams with opening angle 15 arcminutes distributed at random within a cone of opening angle 5°, with galaxies in the magnitude range \( 17 \leq b_J \leq 20.5 \), as described by Szalay et al. (1993).

For a uniform cylinder, to second order in \( k \) the window function takes the form (following BEKS, KP equation (3.10))

\[ I(k, y) = \frac{1}{L} \exp(-\left[\sqrt{(y^2 - k^2)}R\right]^2 / 4). \]

To calculate the window function for a conical survey including the effects of a radial selection function, we follow the derivation given in Appendix A1 of KP, substituting the redshift distribution of galaxies per steradian \( dN/dz \) used by Baugh & Efstathiou (1993):

\[ dN/dz = \frac{3N}{2\pi^2} \exp(-z/z_c)^{1.5}, \]

where the median redshift is given by

\[ z_m(b_J) = 1.412 z_c \]
\[ = 0.016 (b_J - 17)^{1.5} + 0.046 \]

This parametric form for the redshift distribution was chosen to fit both the Stromlo-APM survey (Loveday et al. 1992) and the fainter surveys of Broadhurst et al. (1988) and Colless et al. (1990, 1993) (cf Figure 1 of Baugh & Efstathiou 1993).

With this model for the selection function, the window function is given by:

\[ I(k, y) = \frac{\int_0^\infty dx (dN/dx)^2 W^2(\theta x \sqrt{(y^2 - k^2)})}{\int_0^\infty dx (dN/dx)^2}, \]

where the angular window function is normalised to unity. For a single beam, the Fourier transform of the angular selection function is given by

\[ W(k\theta) = \exp(-k\theta)^2 / 8, \]

whilst for \( N \) beams each of opening angle \( \theta_1 \) placed at random within a solid angle of opening angle \( \theta_2 \), the angular selection is given by (equation 4.5 KP):

\[ W(k) = W(k\theta_1)W(k\theta_2) + 1/N, \]

with \( W(k\theta) \) given by equation 7.

The form of the window function Fourier transforms given above are plotted in Figure 1. This Figure shows the large effect that the adopted survey geometry can have upon the deduced mean power; increasing the number of pencil beams damps the aliasing of power from small scales.

3 REDSHIFT SPACE DISTORTIONS

The pattern of galaxy clustering measured in redshift space is distorted by the peculiar motions of galaxies. Coherent bulk flows lead to a boost in the measured power in three dimensions on large scales (Kaiser 1987), whilst the virialised motions of galaxies in groups and clusters results in
where small and large scales is given by:

\[ P(k) = P_s(k) + P_l(k) \]

where \( P_s(k) \) is the power spectrum on large scales and \( P_l(k) \) is the power spectrum on small scales. The effects of redshift space distortions on the small scale velocities follow an exponential distribution as indicated by N-body simulations. We have assumed that the small scale velocities follow an exponential distribution.

A simple approach is to assume that galaxy velocities are given by linear theory on large scales plus an uncorrelated, random velocity dispersion which dominates on small scales (e.g. Peacock & Dodds 1994; Cole, Fisher & Weinberg 1995). The redshift space power spectrum \( P_S(k, \mu) \) is then given in terms of the real space power spectrum \( P_R(k) \) by

\[ P_S(k, \mu) = P_R(k) + \beta \mu^2 \]

where \( \beta = \Omega/\Omega_b \), \( \Omega \) is the density of the universe in units of the critical density, \( \Omega_b \) is the bias between fluctuations in the galaxy distribution and the underlying mass distribution, \( \mu \) is the 1D velocity dispersion and \( \mu = k \cdot \hat{z}/|k| \) is the cosine of the angle between the line of sight and the wavevector. We have assumed that the small scale velocities follow an exponential distribution as indicated by N-body simulations (Park et al. 1994).

Extending the results of Section 4.3 of KP the projected power including the effects of redshift space distortions on small and large scales is given by:

\[ P_{1D}(k) = \frac{1}{2\pi} \int_0^\infty dy y P_{1D}(y) I(k, y) R(k, y), \]

where

\[ R(k, y) = (1 + \beta (k/y)^2)^2/(1 + (\sigma_v/\sigma_p)^2)^2 \]

The effects of including redshift space distortions on the shape of the window function in Case C, multiple pencil beams, is shown in Figure 2. Estimates of the parameter \( \beta \) show a large spread in values. Determinations using the Stromlo/APM and APM Surveys by Loveday et al. (1995) and Baugh (1996) find \( \beta \sim 0.5 \); a careful analysis of the errors by Tadros & Efstathiou (1996) shows however that values as large as \( \beta \sim 1 \) cannot be ruled out by the present generation of redshift surveys. Tadros & Efstathiou find one dimensional velocity dispersions in the range \( \sigma_v = 300 - 700 \text{ km s}^{-1} \) in N-body simulations of the standard Cold Dark Matter model and its variants.

### 4 PEAK SIGNIFICANCE

We calculate the mean 1D power spectrum at \( k = 0.049 h \text{ Mpc}^{-1} \) for each survey geometry, using the real space three dimensional power spectrum recovered in each of four zones into which the APM Survey was split by Baugh & Efstathiou (1993), with the results given in Table 1. For geometry B, we compute the mean expected power for two apparent magnitude limits. In case C for multiple pencil beams, we use two estimates of the real space power spectrum in 3D. The parameter \( \alpha \) describes the evolution of clustering with redshift; for \( \alpha = 0 \) clustering is fixed in comoving co-ordinates, for \( \alpha = 2 \) clustering evolves according to linear perturbation theory.

In the case of a uniform cylinder, we note that the APM power spectrum gives a mean power that is lower than that obtained with a \( \xi(r) = (r/5)^{1.8} \) power law extended to all scales by 30% (KP find \( P = 0.019 \)). Furthermore, if we truncate the APM power spectrum at \( \lambda = 30 h^{-1} \text{ Mpc} \), the mean power for a cylinder falls by a factor of two. More realistic modelling of the survey window function boosts the mean power by a factor of 3–5, confirming the claim of KP that there is little evidence for periodicity in the deep NGP + SGP beams alone.

### Table 1. Mean power for survey geometries in real space

<table>
<thead>
<tr>
<th>Survey geometry</th>
<th>Mean power ( \pm 1\sigma )</th>
<th>( P_{peak}/ &lt;P&gt; ) ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>0.014 ( \pm 0.002 )</td>
<td>15.2</td>
</tr>
<tr>
<td>( b_J \leq 21.5 )</td>
<td>0.065 ( \pm 0.009 )</td>
<td>3.3</td>
</tr>
<tr>
<td>( b_J \leq 22.5 )</td>
<td>0.047 ( \pm 0.007 )</td>
<td>4.5</td>
</tr>
<tr>
<td>(C)</td>
<td>( \alpha = 0 ) 0.0085 ( \pm 0.0016 )</td>
<td>10.0</td>
</tr>
<tr>
<td>(C)</td>
<td>( \alpha = 2 ) 0.0106 ( \pm 0.0020 )</td>
<td>8.0</td>
</tr>
</tbody>
</table>

### Table 2. Mean power for geometry C in redshift space

<table>
<thead>
<tr>
<th>( \beta ) ( \text{(kms}^{-1}) )</th>
<th>Mean power ( \pm 1\sigma )</th>
<th>( P_{peak}/ &lt;P&gt; ) ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 ( \sigma_v = 500 )</td>
<td>0.0107 ( \pm 0.0021 )</td>
<td>7.9</td>
</tr>
<tr>
<td>0.5 ( \sigma_v = 1000 )</td>
<td>0.0094 ( \pm 0.0018 )</td>
<td>9.1</td>
</tr>
<tr>
<td>1.0 ( \sigma_v = 500 )</td>
<td>0.0127 ( \pm 0.0024 )</td>
<td>6.7</td>
</tr>
<tr>
<td>1.0 ( \sigma_v = 1000 )</td>
<td>0.0150 ( \pm 0.0028 )</td>
<td>5.7</td>
</tr>
</tbody>
</table>
Figure 3. Gaussian realisations of the 1D power spectrum predicted by the 3D power spectrum of the APM survey after convolution with the survey window function of Szalay et al. (1993). The vertical dotted line shows the wavenumber of the spike in the power spectrum reported by BEKS and Szalay et al. (1993). The lower dashed line shows the mean power level at $k = 0.049 h^{-1}$ Mpc. The upper dashed line shows the peak amplitude reported by Broadhurst et al. (1995). Two realisations are shown (a) and (c) that contain a sharp peak with no other peak higher than $P \sim 0.02$; such spectra occur much less often than the type shown in (b), where there are peaks several times higher than the mean power at other wavenumbers. Panel (d) shows an example of a spectrum without any high peaks.

The final column in Table 1 gives the ratio of the observed peak power to the external estimate of the mean power. For the BEKS data, (geometries A and B), the peak power is $P = 80.92$. KP point out that 380 galaxies have redshifts less than $z = 0.5$, which gives an amplitude in our units of $P = 80.92/380 = 0.213$, which is used to calculate the peak to mean power ratio for cases A and B. Figure 4 of Broadhurst et al. (1995) gives the height of the peak for many beams as $P = 0.085$, which is used in case C.

KP found that projected power spectra have an exponential distribution of amplitudes, as expected from the central limit theorem (Fan & Bardeen 1995): the probability of observing a peak in the power spectrum in excess of $P$ at some selected frequency given the mean power $\langle P \rangle$ is (Kendall & Stuart 1977) $Pr(>P) = \exp(-P/\langle P \rangle)$.

There is a factor of $\sim 2$ difference in the peak to mean power ratio for different survey geometries which illustrates the difficulty in making an external estimate of the mean power. When the effects of redshift space distortions are included, the peak to mean power ratio can fall as low as 5.7, which gives a field point probability that the peak arises from a Gaussian random field of $3.3 \times 10^{-3}$. This is much higher than the most pessimistic example in which there are no redshift space distortions and $\alpha = 0$, where the probability is $4.5 \times 10^{-5}$. However, in both cases the field point probability is much larger than the figure $4 \times 10^{-7}$ quoted by Broadhurst et al. (1995) for the null hypothesis of no large scale clustering of galaxies. Note that the amplitude of the APM power spectrum could be low by as much as 20%, due to merging corrections to APM images which we have not applied here (Maddox et al. 1990, 1996) or uncertainties in the form of the redshift distribution (Baugh & Efstathiou 1993, Gaztañaga 1995).

We have simulated 1D power spectra using the pro-
jected amplitude for \( P_{1D}(k) \) computed from the APM power spectrum after convolution with the multiple beam window function. Real and imaginary Fourier components are generated with a Gaussian distribution. The resolution of the 1D power spectrum is taken to be \( \delta k = 0.007 h^{-1} \text{Mpc}^{-1} \) as calculated by KP for the combined sample of BEKS, and which matches the full width at half maximum of the peak in Figure 4 of Broadhurst et al. (1995). Examples of these synthetic spectra are shown in Figure 3. We generate a large number of realisations of the Gaussian power spectrum and look for peaks in the wavenumber range 0.028hMpc^{-1} < k < 0.063hMpc^{-1}, corresponding to wavelengths of 100h^{-1} \text{Mpc} < \lambda < 224h^{-1} \text{Mpc}. We find peaks in excess of the height reported by Broadhurst et al. (1995) 3% of the time using the estimated real space mean power, rising to 6% of realisations when redshift space distortions with \( \beta = 1.0 \) and \( \sigma_v = 500 \text{km} \text{s}^{-1} \) are included. A typical power spectrum generated with a high peak in the range reported by Broadhurst et al. is shown in Figure 3(b). The power spectrum realisation contains several peaks that are a few times higher than the mean power, indicated by the dotted line, in contrast with the power spectrum shown by Figure 4 of Broadhurst et al. Roughly 1 in every 1500 realisations contains a peak in excess of \( P = 0.085 \) in the specified wavenumber range, without having another peak in the range \( 0.0 < k < 0.3 \) that is higher than \( P = 0.02 \); two examples are shown in Figure 3(a) and (c).

5 DISCUSSION

We have examined the compatibility of the three dimensional power spectrum measured from the APM Galaxy Survey (Baugh & Efstathiou 1993, 1994) with the power spectrum recovered from deep pencil beam surveys by BEKS, Szalay et al. (1993) and Broadhurst et al. (1995). The probability of observing a peak in the 1D power spectrum, given our knowledge of the 3D power spectrum on these scales depends upon the ratio of the height of the observed peak compared with the external estimate of the mean power. External estimates of the mean power in the 1D datasets are extremely sensitive to how the survey geometry is modelled and to the magnitude of the redshift space distortions. For a survey consisting of many pencil beams, we find a factor of two difference in the amplitude of the mean power that we calculate, for various models of the distortions in the pattern of clustering due to the peculiar velocities of clusters.

In the case of our highest estimate of the mean power for a survey consisting of multiple pencil beams, we find that the field point probability of finding a peak of the observed height given the null hypothesis of the measured clustering in 3D, is roughly 1 in 300. We have also generated Gaussian realisations of the pencil beam power spectra, using the 1D power spectrum obtained by convolving the measured 3D power spectrum with the survey geometry. Using the same resolution as the observed 1D power spectrum, we find that peaks of the observed height or higher in the wavelength range \( 100 < \lambda < 200h^{-1} \text{Mpc} \) occur in \( 3 - 6\% \) of the realisations, depending upon the model assumed for the redshift space distortions.

The general appearance of these synthetic power spectra is quite different from that of the power spectrum of the observations in the majority of realisations. We find that a high peak in the power in the wavelength range specified above is generally accompanied by peaks that are a few times the amplitude of the mean power at other wavenumbers. In only a small number of cases, roughly 1 in 3000 do with find an isolated high peak with the other peaks in the spectrum less than twice the amplitude of the mean. KP discuss the effects that redshift errors or binning of the data in real space before taking the Fourier transform could have upon the appearance of the observed power spectrum. Also, we have neglected the consequences of the density distribution in 3D being skewed (e.g. Efstathiou et al. 1990, Saunders et al. 1991, Gaztaña 1994) which has been shown to further enhance the probability of observing a peak given a null hypothesis of the measured 3D clustering ( Amendola 1994).

The ability of the measured 3D clustering to account for the presence of high peaks in the observed 1D power spectrum makes it seem unlikely that large scale structure can be responsible for the steep observed slope of the galaxy counts at bright magnitudes, a conclusion already discussed by Maddox et al. (1990) and Loveday et al. (1992). The factor of two increase in the normalisation of the luminosity function desired to provide a better match to the faint counts (Shanks 1990) requires that the local universe out to a radius of around \( 130h^{-1} \text{Mpc} \), corresponding to the median redshift of galaxies brighter than \( b_j = 17 \), is massively underdense. The variance in the number of galaxies measured on these scales from the APM Galaxy Survey is on the order of 10% ( Baugh & Efstathiou 1993, Gaztaña 1994) many times smaller than would be necessary to result in a factor of two fluctuation in the number of galaxies found in a volume of such a large radius (see also the discussion in Glazebrook et al. 1994).

Acknowledgements

The author acknowledges George Efstathiou, Enrique Gaztaña and Shaun Cole for useful conversations and comments on an earlier version of the manuscript.

REFERENCES

Broadhurst, T.J., Ellis, R.S., Koo, D.C., Szalay, A.S., 1990, Nature, 343, 726 (BEKS)
C.M. Baugh