Local Multiplicity Fluctuations in Z decay

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Abstract

Local multiplicity fluctuations of hadrons produced in the decay of $Z^0$ were studied on the basis of L3 data. In addition to the normalized-factorial-moment method, the fluctuations were studied for the first time by the use of bunching parameters. A strong multifractal structure was observed inside jets. JETSET 7.4 PS describes the fluctuations in the azimuthal angle defined with respect to the beam axis reasonably well. For the fluctuations in rapidity, defined with respect to the thrust axis and in the four-momentum-difference variable, JETSET 7.4 PS overestimates the fluctuations.

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1 Introduction

The quest for local multiplicity fluctuations is the quest for short-range correlations in a multiparticle system in which particles have a tendency to form so-called “spikes” according to underlying stages of the multiparticle process. In high-energy physics, spikes are seen as dynamical peaks in the phase-space distribution of individual events. The dynamical occurrence of spikes leads to the intermittency phenomenon [1–3], defined as a power-law behavior of the normalized factorial moments (NFMs)

\[ F_q(\delta) = \frac{\langle n[q] \rangle}{\langle n \rangle^q} \propto \delta^{-\phi_q}, \quad n[q] = n(n-1) \ldots (n-q+1), \]  

where \( n \) is the number of particles in a restricted phase-space interval of size \( \delta \), \( \langle \ldots \rangle \) is the average over all events in the sample, and \( \phi_q > 0 \) is the intermittency index.

Of course, this method of local-fluctuation analysis is not unique. The fluctuations can be investigated by any quantity characterizing the multiplicity distribution in \( \delta \), if we know \textit{a priori} its behavior in the case of statistical fluctuations, i.e., when the occurrence of spikes is caused by purely statistical reasons. In the simplest approach, one can restrict oneself to those quantities which have \( \delta \)-independent behavior for purely statistical phase-space fluctuations, as is the case for NFMs.

From a theoretical point of view, the NFMs have the additional advantage that they filter out Poissonian noise [1]. This property is of vital importance for the comparison of theoretical models involving an infinite number of particles in an event to the experimental data.

Bunching parameters (BPs) have similar properties for the local-fluctuation study [4–7]. The definition of the BPs is

\[ \eta_q(\delta) = \frac{q}{q-1} \frac{P_q(\delta)P_{q-2}(\delta)}{P_{q-1}^2(\delta)}, \]  

where \( P_n(\delta) \) is the probability of having \( n \) particles inside a restricted phase-space interval of size \( \delta \). The main advantage of these quantities over the NFMs is that they are more sensitive to the structure of local fluctuations [7]. Another property is that for multifractal local fluctuations, \( \eta_q(\delta) \) is a \( \delta \)-dependent function for all \( q > 2 \), while for monofractal behavior, one has \( \eta_q(\delta) = \text{const} \) for \( q > 2 \) [4]. This property simplifies the multifractal analysis significantly: any observation of the power-law \( \delta \)-dependence of \( \eta_q(\delta) \) for \( q = 3, 4, \ldots \) means that the anomalous fractal dimension \( d_q = \phi_q/(q-1) \) has a tendency to increase with increasing \( q \), so that the sample exhibits a multifractal property.

In addition, from an experimental point of view, BPs have the following two important properties [7]: 1) They are less severely affected by the bias from finite statistics than are the NFMs, since the \( q \)-th order BP resolves only the behavior of the multiplicity distribution near multiplicity \( n = q - 1 \); 2) For the calculation of the BP of order \( q \), one needs to know only the \( q \)-particle resolution of the detector, not any higher order resolution.
In this paper, we present an experimental investigation of local fluctuations in the final-state hadron system produced in $Z^0$ decays at $\sqrt{s} = 91.2$ GeV by using both NFMs and BPs. The final-state hadrons have been recorded with the L3 detector during the 1994 LEP running period. The calculations are based on approximately 810k selected hadronic events. Charged hadrons are selected by the standard L3 selection procedure, based on energy deposition in the electromagnetic and hadronic calorimeters and momentum measurement in the Central Tracking Detector including the L3 Silicon Microvertex Detector.

2 Experimental definitions

In order to increase the statistics and to reduce the statistical error in observed local quantities when analyzing experimental data, we use the bin-averaged NFMs and BPs [7]:

1) **Horizontal NFMs**:

$$F_q(\delta) = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n^q_m \rangle}{\langle \bar{n} \rangle_q},$$

where $n_m$ is the number of particles in bin $m$, $\langle \bar{n} \rangle_q = \bar{N}/M$, $\bar{N}$ is the average multiplicity for full phase space, $M = \Delta/\delta$ is the total number of bins, and $\Delta$ represents the full phase-space volume.

2) **Horizontal BPs**:

$$\eta_q(\delta) = \frac{q}{q-1} \frac{\bar{N}_q(\delta)\bar{N}_{q-2}(\delta)}{\bar{N}^2_{q-1}(\delta)}, \quad \bar{N}_q(\delta) = \frac{1}{M} \sum_{m=1}^{M} N_q(m, \delta),$$

where $N_q(m, \delta)$ is the number of events having $q$ particles in bin $m$, $M = \Delta/\delta$.

Both quantities (3) and (4) are equal to unity for a purely independent particle production following a Poissonian multiplicity distribution in restricted bins.

Note that the definitions presented above can be used in practice for a flat single-particle density distribution. To be able to study non-flat distributions, we carry out a transformation from the original phase-space variable to that in which the underlying distribution is approximately uniform [8, 9].

3) **Generalized integral BPs**:

Recently, a new set of bunching-parameter measurements has been proposed that make use of the interparticle-distance measure technique [7]. To study fluctuations of spikes, we will consider the generalized integral BPs using the pairwise squared four-momentum difference $Q_{12}^2 = -(p_1 - p_2)^2$. The definition of the BPs is given by

$$\chi_q(Q_{12}^2) = \frac{q}{q-1} \frac{\Pi_q(Q_{12}^2)\Pi_{q-2}(Q_{12}^2)}{\Pi^2_{q-1}(Q_{12}^2)},$$

(5)
where $\Pi_i(Q_{12}^2)$ represents the number of events having $i$ spikes of size $Q_{12}^2$, irrespective of how many particles are inside each spike. To define the spike size, we used the so-called Grassberger-Hentschel-Procaccia (GHP) counting topology [10,11], for which a $g$-particle hyper-tube is assigned a size $\epsilon = Q_{12}^2$ that corresponds to the maximum of all pairwise distances. For purely independent particle production with the spike multiplicity distribution characterized by a Poissonian law, the BPs (5) are equal to unity for all $q$.

3 Analysis

Two samples of multihadronic events are generated with JETSET 7.4 PS. The first sample contains all charged final-state particles with a lifetime greater than $10^{-9}$s (generator-level sample). This sample is generated with initial state photon radiation. The second, detector-level sample, includes distortions due to detector effects, limited acceptance, finite resolution and the event selection. Both the generator-level and detector-level samples have the same statistics (810k hadronic events).

A corrected NFM or BP is found by means of the following correction procedure

$$D_{q}^{\text{cor}} = C_q D_{q}^{\text{raw}}, \quad C_q = \frac{M_{q}^{\text{gen}}}{M_{q}^{\text{det}}}.$$  \hspace{1cm} (6)

Here, $M_{q}^{\text{gen}}$ and $M_{q}^{\text{det}}$ symbolize an NFM or BP of order $q$ calculated from the generator-level and detector-level Monte-Carlo samples, respectively. $D_{q}^{\text{raw}}$ represents an NFM or BP calculated directly from the raw data. The same correction procedure has been used in [12,13].

3.1 In the detector frame

To update the results already presented in [14], we carried out measurements of horizontal NFMs (3) as a function of the number $M = 2\pi/\delta\varphi$ of partitions of the full angular interval $2\pi$, where $\delta\varphi$ denotes the bin size in azimuthal angle $\varphi$ defined with respect to the beam axis. Since the event averaged distribution in $\varphi$ is uniform, the Ochs-Bialas-Gazdzicki transformation [8,9] was not performed here. In Fig. 1, the corrected data are shown by full symbols and the generator-level of JETSET 7.4 PS tuned by the L3 Collaboration by open symbols. Here and below, the smallest bin size is estimated from the Monte-Carlo study of the charged-track resolution of the L3 detector in the particular variable.

The statistical errors on the data shown in Fig. 1 are derived from the covariance matrix of the horizontally averaged factorial moments. They include the statistical error on the correction factor $C_q$ in (6). To combine the statistical error on the correction factor, we assume that the statistical errors for the generator-level and detector-level Monte Carlo's are independent. This (strong) assumption leads to an upper limit of the error derived for $C_q(\delta)$.  

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Figure 1: NFM as a function of the number $M$ of bins in azimuthal angle defined with respect to the beam axis.

Figure 2: BPs as a function of the number $M$ of bins in azimuthal angle defined with respect to the beam axis.

The error bars on the Monte-Carlo predictions include both statistical and systematical errors. The systematical errors have been estimated by varying, by one standard deviation, the LUND fragmentation parameter PARJ(42), the width of the Gaussian $p_x$ and $p_y$ hadronic transverse momentum distribution (PARJ(21)), and the $\Lambda$ value used for $\alpha_s$ in parton showers (PARJ(81))\(^3\). For the given statistics, the errors on Monte Carlo are dominated by the systematical errors, so that the open symbols represent the values of NFM with the L3 default and the error bars indicate the maximum and minimum values obtained after the parameter variations.

Fig. 1 shows that the Monte-Carlo predictions slightly oscillate around the corrected data, but reasonably reproduce the experimental results.

The same hadronic sample is used to calculate the horizontal type of the BPs (4). The behavior of $\ln \eta_q(M)$ as a function of $\ln M$ is presented in Fig. 2. Being more sensitive to the structure of fluctuations, the BPs show that JETSET 7.4 PS slightly overestimates the increase of the second-order BP and oscillates around the third-order BP calculated from the data. The Monte-Carlo predictions reproduce the higher-order BPs reasonably well. The observed decrease of the high-order BPs with increasing $M$\(^3\)

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\(^3\)The value of these parameters have been tuned by the L3 Collaboration to reproduce the single-particle spectra and global-shape distributions.
reflects a particle antibunching of the order $q > 2$ due to jet structure, i.e., particles are bunched together inside each jet, but there are phase-space intervals between the bunches due to energy-momentum conservation.

### 3.2 In the event frame

With the above observation in mind, it is obvious that the NFM$s and BPs calculated so far are strongly influenced by the jet structure of events. In order to study genuine fluctuations inside jets, we used the rapidity $y$ defined with respect to the thrust axis. The NFM$s as a function of the number of bins in $y$ are shown in Fig. 3. The predictions of the JETSET 7.4 PS tuned by the L3 Collaboration are presented by open circles. The behavior of the NFM$s shows the same trend as that in the azimuthal angle $\varphi$ defined with respect to the beam axis. The signal observed, however, is much smaller for the present calculations. As we see, JETSET 7.4 PS overestimates the intermittency effect for large $M$. This discrepancy is enhanced with rising moment order.

![Figure 3: NFM$s as a function of the number $M$ of bins in rapidity $y$ defined with respect to the thrust axis.](image1)

![Figure 4: BPs as a function of the number $M$ of bins in rapidity $y$ defined with respect to the thrust axis.](image2)

Fig. 4 shows the results for the horizontally normalized BPs (4) in the rapidity $y$ defined with respect to the thrust axis. In contrast to Fig. 2, all high-order BPs...
show a power-law increase with increasing $M$, indicating that the fluctuations in this variable are of multifractal type. The multifractality observed, therefore, appears to be a consequence of the cascade nature of parton branching, hadronization, resonance decays and Bose-Einstein correlations. Note that the conclusion on the multifractal type of the fluctuations becomes possible without the necessity for the calculation of the intermittency indices. In contrast, to reveal multifractality with the help of the NFM-method one needs to carry out fits of the NFM by a power law.

A disagreement with the Monte-Carlo predictions is observed for $q = 2, 3$, while higher-order BPs are described well by the model.

As mentioned in [15], the influence of Bose-Einstein (BE) correlations on quantities measured depends strongly on the type of quantity and variable used. Obviously, the BE correlations are a typical candidate for the cause of local fluctuations in 3-momentum phase space, which should lead to a rise of fluctuations in one-dimensional phase space as well. To demonstrate this effect, Figs. 3 and 4 also show a comparison of the JETSET 7.4 PS model without BE interference (open triangle symbols) to the data. Indeed, the model expectations without the BE effect in Fig. 3 have a smaller rise of the NFM than those that include the BE effect.

It is quite remarkable how well the influence of BE correlations on the local fluctuations in JETSET can be seen in the second-order BP (see Fig. 4). This influence is due to the fact that the BE effect is implemented in JETSET on the level of two-particle correlations, which are strictly related to the second-order BP (NFM).

### 3.3 In the four-momentum difference

Fig. 5 shows the behavior of the generalized BPs (5) as a function of $Q_{12}^2$. The dashed lines represent the behavior of these BPs in the Poissonian case. To contrast, all BPs rise with decreasing $Q_{12}^2$, which corresponds to a strong bunching effect of all orders, leading to multifractal fluctuations. The saturation and downward bending of the second-order BP at small $Q_{12}^2$ are caused by the influence of resonances at intermediate $Q_{12}^2$ (see below).

To investigate this disagreement in more detail, we present in Fig. 6 the behavior of the second-order BP as a function of $Q_{12}^2$ for multiparticle hyper-tubes (spikes) made of like-charged and that of unlike-charged particles. A large difference is observed between these two samples due to different particle dynamics. For like-charged particle combinations (i.e., for spikes with a maximum charge), a strong bunching effect ($\chi_2(Q_{12}^2) > 1$) is seen at small $Q_{12}^2$. However, the bunching is much smaller and even disappears at small $Q_{12}^2$ for unlike-charged particle combinations. This effect can be explained by resonance decays at intermediate $Q_{12}^2$, when decay products of short-lived resonances tend to be separated in the phase space. The antibunching effect ($\chi_2(Q_{12}^2) < 1$) for large $Q_{12}^2$ is caused by the energy conservation constraint [7].

The resonance effect is much weaker for like-charged combinations. In addition, the BE correlations strongly affect the like-charged particle combinations. Note, however, that JETSET 7.4 PS leads to a strong rise of $\chi_q(Q_{12}^2)$ for like-charged particle
Figure 5: Generalized integral BPs as a function of the squared four-momentum difference $Q_{12}^2$ between two charged particles.

Figure 6: Generalized second-order BP as a function of the squared four-momentum difference $Q_{12}^2$ between two charged particles.

combinations, even without the modeling of the BE interference.

JETSET 7.4 PS overestimates the data for unlike-charged combinations at intermediate values of $Q_{12}^2$ and underestimates the data for like-charged combinations at small $Q_{12}^2$. The latter disagreement can be reduced, in part, by varying the BE parameters in JETSET 7.4 PS\textsuperscript{4}. However, we have verified that the disagreement for unlike-charged combinations cannot be reduced by only varying the BE parameters.

The most probable shortcomings leading to the discrepancies found are the simulation of hadronization\textsuperscript{5} and the BE effect. As an example, the residual distortion of the decay products of short-lived resonances by BE correlations not yet implemented in the JETSET 7.4 PS model may be a good candidate as an explanation for such a discrepancy. The importance of the latter effect was realized recently, when a significant mass shift of $\rho^0$ was observed by OPAL and DELPHI \cite{17,18}.

The production rate of $f_0(975)$ and $f_2(1270)$ measured by DELPHI \cite{18} is another challenge for the JETSET model. In this respect, it is not improbable that a much

\textsuperscript{4}Note that the study of the BE effect for charged particles has not been performed by the L3 Collaboration so far.

\textsuperscript{5}For $2.5 < -\ln Q_{12}^2 < 5.0$, our calculations show a large sensitivity of the results obtained to LUND fragmentation, since large systematic errors for this domain of $Q_{12}^2$ come mainly from varying the LUND fragmentation parameter PARJ(42) and PARJ(21) by one standard deviation.
larger fraction of the observed final-state hadrons results from resonance decays than is usually assumed. In this case, the negative correlations should be larger, and a better agreement with the data for the intermediate values of $Q_{12}^2$ would be achieved for unlike-charged particles. Indeed, we have found that a realistic small variation of the production of resonances ($\rho$, $\omega$, $\eta$, $\eta'$) responsible for the unlike-charged particle fluctuations in the JETSET 7.4 PS can lead to a better agreement. This is not likely to improve the discrepancy fully, however, since the JETSET 7.4 PS tuned by the L3 Collaboration shows a reasonable agreement with the production rates of the main resonances [19] and the variation of the parameters should not be large.

As an additional verification, the default version of JETSET 7.4 PS has been compared to the data. The same kind of the disagreement is found (not shown).

Of course, the disagreement for the unlike-charged particle combinations in $Q_{12}^2$ (and, hence, for the all-charged combinations shown in Fig. 5) can also lead to the disagreement between the JETSET 7.4 PS and the data in the case of the one-dimensional variables $\varphi$ and $y$ presented in subsections 3.1 and 3.2.

4 Discussion

For the first time, local multiplicity fluctuations have been studied by means of bunching parameters. Using this method, fluctuations in rapidity defined with respect to the thrust axis and in the four-momentum difference $Q_{12}^2$ are found to exhibit a strong multifractal behavior. The multiplicity distributions in these variables, therefore, cannot be described by conventional distributions (Poisson, geometric, logarithmic, positive-binomial, negative-binomial), which have $\delta$-independent high-order BPs [4]. Recently, more general multiplicity distributions with power-law high-order BPs have been considered [5]. Such types of distributions, therefore, appear to be more relevant to the situation observed. However, a phenomenological description of these distributions can, to only a slight extent, provide a physical explanation of the nature of multifractal behavior.

For an $e^+e^-$ interaction, one can be confident that, at least on the parton level of this reaction, perturbative QCD calculations can give a hint for understanding the problem. Analytical calculations based on the double-log approximation of perturbative QCD show that the multiplicity distribution of partons in ever smaller opening angles is inherently multifractal [20–22]. Of course, the choice of variable can affect the observed signal, and the final conclusion about agreement between QCD and the data can only be derived after the calculation of local quantities in angular variables that are defined with respect to the thrust (or sphericity) axis.

In part, the disagreement found between JETSET and the data is due to problems in tuning the JETSET 7.4 PS model by the L3 Collaboration, since the tuning of the model has been performed by means of global observables only. However, the problem of the discrepancy found can be more complicated, and an additional study of JETSET itself is necessary: it has been shown that JETSET 7.4 PS overestimates in the $Q_{12}^2$
variable fluctuations of spikes made of unlike-charged particles.

Thus, it appears that some important point in the simulation of hadronization and BE interference is missing in the present version of JETSET and further modifications of the model are needed. A similar conclusion has been derived in [23], where it was shown that JETSET fails to reproduce the multiplicity dependence of the intermittency index.

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