Residual gas expulsion from young globular clusters

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ABSTRACT

The results of $N$-body simulations of the effects of the expulsion of residual gas (that gas not used in star formation) from very young globular clusters is presented. Globular clusters of a variety of initial masses, Galactocentric radii, concentration and initial mass function slope with star formation efficiencies of $\lesssim 50\%$ were simulated. The residual gas was expelled by the action of massive stars in one of three idealised ways: gradually by their UV flux and stellar winds; gradually by the input of energy by supernovae; and in a 'supershell' expanding from the cluster centre. The clusters were compared shortly after the gas expulsion with the results of Chernoff & Shapiro (1987) to estimate whether they would survive for a Hubble time. It is found that the expulsion of $\gtrsim 50\%$ of a globular clusters mass in a short period of time considerably affects the structure of the cluster. However, many clusters are estimated to be able to survive with reasonable initial conditions, even if their star formation efficiencies are possibly as low as 20%. It is found that the central density required within a proto-globular cluster at star formation in order for it to survive at a given Galactocentric radius is independent of the mass of stars in the cluster. For globular clusters in the inner few kpc of the Galaxy this value is found to be around $10^3 M_\odot \text{ pc}^{-3}$, falling as Galactocentric radius increases. This value is similar to the central densities found in giant molecular clouds in the Galaxy today. It is suggested that a globular cluster could reasonably form with that central density with a star formation efficiency of $\approx 40\%$ and and initial mass function slope $\alpha \approx 3$.

Key words: Globular clusters: general

1 INTRODUCTION

This paper reports the results of an $N$-body simulation of the early stages of the evolution of globular clusters. These simulations differ from previous models in that they include the residual gas (that gas not used in star formation) left in the cluster immediately after their formation. The simulations model the dynamical effects upon the cluster of the expulsion of this gas and the implications for the cluster’s survival. The aim of these simulations is to constrain in a parameter space containing star formation efficiency, initial mass function, Galactocentric radius and the initial spatial and energy distributions of the stars the conditions that will result in a long-lived globular cluster.

The observed properties of globular clusters and candidate globular clusters provide a number of strong constraints upon the initial conditions in those clusters, which may then be applied to theoretical models. Possibly the most important observational result from which it is possible to constrain these initial conditions is that relating to the chemical composition of globular clusters. Galactic globular clusters, with the important exceptions of $\omega$ Cen and M22, are extremely chemically homogeneous with differences in their internal metallicities of only about one tenth of a dex (Fahlman, Richer and VandenBerg 1985 and Kraft et al. 1992). This implies that the stars in globular clusters formed from a well-mixed gas cloud in the proto-Galaxy in a single burst of star formation. This star formation may have been internally induced (Fall and Rees 1985, Harris & Pudritz 1994 and Brown, Burkert & Truran 1991, 1995) or caused by external effects such as cloud-cloud collisions (Lin & Murray 1991, Murray & Lin 1992, Kumai, Basu & Fujimoto 1993 and Lee, Schramm & Mathews 1995) or by the interaction of shock waves with clouds (Shapiro, Clocchiatti & Kang 1992). Notwithstanding the inducing mechanism, the star formation must have been rapid and efficient. Star formation must be completed before the massive stars could ionise the residual gas and impede further star formation, as the ionisation of the gas will prevent the condensation of further stars from the gas in the proto-cluster. Further to this condition, the remaining gas in the cluster must have been expelled from the cluster. This expulsion must have occurred in order to prevent another generation of stars forming which would have had a higher metallicity due to the enrichment of the residual gas by the supernovae
of the massive stars within the cloud. Hence the $10^6$ to $10^7$ yr lifetime of the massive stars gives an upper limit to the star formation timescale of the entire cluster (Lin & Murray 1991). The loss of the gaseous component of the cluster will reduce the binding energy of the cluster making it more susceptible to disruption. The expulsion of the residual gas is presumed to be driven by the rapid evolution of the most massive stars. The expulsion is assumed to be caused either by the energy released into the residual gas by supernovae or the cumulative effect of ionisation and stellar winds from these stars.

Presuming that a globular cluster survives the loss of its residual gas, two further processes act upon the cluster which may disrupt it. Firstly, the cumulative mass loss caused by the stellar evolution of individual stars within the cluster will lower the mass of the cluster over a long timescale. In addition, the evaporation of stars from the cluster will reduce the total cluster mass. Evaporation may be caused by the action of the galactic tidal field or via relaxation where stars involved in energetic two-body encounters will be pushed into the high-velocity tail of the Maxwellian velocity distribution where they reach escape velocity and leave the cluster. There have been many previous simulations of the evolution of globular clusters, although none of these simulations have included any residual gas loss. The early work using $N$-body codes, the Fokker-Planck equation and hydrodynamic codes is summarised in Spitzer (1987) and Elson, Hut & Inagaki (1987). More recent work has been primarily using the Fokker-Planck equation due to the large number of particles that can be included in these simulations (Weinberg & Chernoff 1988 and Chernoff & Weinberg 1990). Additionally, Chernoff & Shapiro (1987) used a King model based simplified evolutionary calculation following the evolution of basic King model parameters. It is to these calculations that the $N$-body results in this paper will be compared.

The reasoning for using an $N$-body code in preference to a Fokker-Planck simulation is two-fold. Firstly, the $N$-body method makes no a priori assumptions about the dynamical evolution of the system. Secondly, an $N$-body approach allows the effects of an variable external potential to be very easily modelled. The code is based upon Aarseth’s nbody2 code (Aarseth 1985) with an initial mass function for particles. The code included stellar evolution and a variable external potential to simulate the residual gas loss. $N$-body simulations of open clusters including some gas loss have concentrated upon open star clusters (Lada, Margulis & Dearborn 1984, Terlevich 1987) due to the particle number limitations of $N$-body codes in comparison to the Fokker-Planck approximation. Recently $N$-body codes have been applied to the evolution of globular clusters in a statistical approach as in Fukushige & Heggie (1995) where the results of $N$-body and Fokker-Planck simulations were found to be qualitatively similar (this approach been analysed by Giersz & Heggie 1994a, 1994b). The $N$-body approach has been useful in discovering a number of properties of the evolution of large $N$ systems such as core collapse (Aarseth & Lecar 1975). It has also been used to test weak-scattering Fokker-Planck models (Aarseth, Hénon & Wielen 1974) and the tidal evolution of globular clusters (Oh, Lin & Aarseth 1992).

The simulations were run to cover $50 < T < 100$ Myr, that is the timescale for the loss of the residual gas (approximately the life time of the most massive stars) and a ‘settling down’ period afterwards. Following the evolution for such a relatively short period of time reduces the computing time and the problems of an $N$-body calculation.

In order to estimate the final fate of the cluster (whether it will survive or disrupt) the condition of the cluster after the simulation is compared to the results of Chernoff & Shapiro (1987), hereafter CS. The simulations of CS, based upon King models, give the final fate of a cluster for a wide range of initial Galactocentric radii, initial mass functions and concentrations. The range of these initial conditions is far wider than those of any other simulation. They are found to have broad agreement with Fokker-Planck (eg. Chernoff & Heggie 1990) and $N$-body (Fukushige & Heggie 1995) simulations of globular cluster evolution, although it should be noted that Fukushige & Heggie (1995) do conclude that the types of Fokker-Planck simulations used to model globular clusters may be ‘quantitatively very unreliable’.

# 2 INITIAL CONDITIONS

The initial conditions are set to be fairly similar to clusters that CS estimate will survive a Hubble time.

## 2.1 Tidal Cutoff Radius

The outer edge of the cluster is taken to be the King (1962) tidal radius $r_t$ (the point at which the density is assumed to vanish), given by

$$r_t = R_G \left( \frac{M_{cl}}{M_G(R_G)} \right)^{1/3}$$  \hspace{1cm} (1)

where $M_{cl}$ is the mass of the cluster, $R_G$ is the Galactocentric distance of the cluster and $M_G(R_G)$ is the mass of the Galaxy interior to $R_G$. $M_G(R_G)$ is derived from the rotation curve of the galaxy using $M_G(R_G) = R_G v_c^2 / G$ where $v_c$ is the circular rotation velocity ($\approx 220$ km s$^{-1}$) and $G$ is the gravitational constant. This approximation is used for $R_G > 2$ kpc. Such a model is used instead of more complex Galactic potentials as globular clusters are very old objects which formed during the early stages of Galactic evolution when the Galactic potential was probably very different from that today. If the Galaxy was still in the process of accreting mass when globular clusters were formed this would produce a correspondingly lower tidal radius at the time of formation. However, as $r_t \propto M_G(R_G)^{-1/3}$ this effect may not be too important.

This form of the tidal radius is an idealised case which is spherically symmetric. In real clusters the shape of the equipotential surfaces will be elongated in the radial direction pointing towards the Galactic centre (see Spitzer 1987 and Heggie & Ramamani 1995). For globular clusters with large Galactocentric distances ($\geq 6$ kpc) the extent of this asymmetry is small and the distance of the tidal boundary from the cluster centre is large in comparison to the half-mass radius. The simplification to a spherically symmetric boundary will not greatly effect the rate of stellar evaporation in the short periods of time covered by these simulations where the effects of a more complex treatment of tidal
forces, such as those used in Terlevich (1987) and Fukushima & Heggie (1995), should not be important.

For present day globular clusters this tidal radius is normally in the range $50 < r_t < 100$ pc (using Galactocentric radii and masses from Chernoff & Djorgovski 1989). If a star passes beyond this distance (ie. its energy is above the escape energy of $-GM^2/r_t$) it is assumed to be outside of the gravitational influence of the cluster and to leave the cluster forever. This is not necesserally the case, but is usually accepted as a first approximation in models of globular clusters (Spitzer 1987). This mass loss due to evaporation of stars from the cluster will, of course, reduce the tidal radius allowing further evaporation to occur more easily.

### 2.2 Initial Mass Function and Stellar Evolution

The initial mass function (IMF) is taken to be a power law of the form

$$N(M) \propto M^{-\alpha} \quad (2)$$

where this corresponds to the Salpeter (1955) IMF for the solar neighbourhood when $\alpha = 2.35$. Such forms for the IMF are used in virtually all simulations of globular clusters (for example CS, Chernoff & Weinberg 1990 and Fukushima & Heggie 1995). Three values of $\alpha$ are used in the models of 2.35, 3.5 and 4.5 which cover the most acceptable probable initial values of the IMF of a globular cluster and to enable simple comparisons of these simulations to CS, Chernoff & Weinberg (1990) have shown that no cluster with an IMF slope of $\alpha = 1.5$, or lower, is able to survive the large amount of mass loss due to stellar evolution in the first 5Gyr of its life so such low IMF slopes were not considered. The IMF is implemented in the code by assigning each particle a different mass according to the slope of the IMF. Each of these particles represents a collection of similar mass stars close together in phase space. Each particle contains from a few tens to a few thousands of stars, and the average mass of particles in the simulations can range from tens of solar masses to tens of thousands of solar masses depending upon the initial total mass of the cluster being simulated and the number of particles used to represent the cluster. The range of initial masses studied in this paper ranges from $10^5 M_\odot$ to $10^6 M_\odot$, normally using 1000 particles. This is obviously a gross approximation but allows the code to simulate the overall evolution of the cluster in a simple way.

The actual upper mass limit of the stars in the cluster is taken to be $12 M_\odot$. As stated above each particle in the simulation represents far more than one star, as stars of above $12 M_\odot$ are very rare, it would be inappropriate to assign to them more than one individual particle.

The lower mass limit of the IMF is taken to be $0.15 M_\odot$ corresponding to the observed downturn of the mass function in globular clusters (Paresce, De Marchi & Romaniello 1995).

One consequence of this treatment of the IMF is that the particle representing the highest mass stars is far larger than other particles in the simulation. In order to see if this could effect the results a number of simulations were run where this one large particle was split into 10, or more, different particles which evolved, in total, like the larger particle. The results of simulations with one large particle and several smaller particles were virtually identical. The use of such a large particle is justified as it does not survive for long enough before evolving to have a significant effect upon the dynamics of the other particles through two-body encounters.

Mass loss due to stellar evolution plays a very important part in the evolution of a globular cluster, especially in the early stages of its life when the rapid evolution of massive stars will substantially alter the mass of the cluster and correspondingly lower the tidal radius (Chernoff & Weinberg 1990). The effects of stellar mass loss have been included by the fitting of a simple straight line to the end times of stellar evolution calculated by Maeder & Meynet (1988) for various masses of Solar metallicity stars

$$\log_{10} \left( \frac{M}{M_\odot} \right) = 1.524 - 0.370 \log_{10} \left( \frac{T}{\text{Myr}} \right) \quad (3)$$

The majority of globular clusters are of lower than solar metallicity and these stellar evolutionary times are calculated for solar metallicity. However, as the only evolutionary times of interest are those of high mass stars these evolutionary times are expected to be close to those for low metallicity stars.

Once a star has reached its final age it evolves to the appropriate end state for a star of that mass. Stars are divided into three mass categories each of which leave a different stellar remnant at the end of their evolutionary times. Stars of $M_r > 8 M_\odot$ become type II supernovae, leaving behind a $1.4 M_\odot$ neutron star. For stars of intermediate mass, $4 < M_r/M_\odot < 8$ two end states are possible. The first is that the star becomes a type $\frac{1}{2}$ supernova which catastrophically disrupts leaving no remnant (Iben & Renzini 1983). Alternatively the star may evolve into a white dwarf of mass $1 M_\odot$. It is not known how important each of these processes are relative to each other, so both end states are used in the code. The loss of mass from the cluster is obviously greater in the first case but due to the steep slopes of the IMF used (hence the relatively small number of intermediate mass stars) this difference is quite low. Low mass stars of $1 < M_r/M_\odot < 4$ all evolve into white dwarfs of mass $0.58 + 0.22(m - 1)$, where $m$ is the initial mass of the star, all in solar masses (Iben & Renzini 1983). Because of the short timescale followed by the code (the evolutionary timescale is only followed to a maximum of 100 Myr) only high mass stars and maybe a very few intermediate mass stars have the time to evolve.

The evolution of each particle is not instantaneous. Each particle is assumed to contain stars of a similar mass, but not all exactly the same mass. The range of masses contained in each particle fills half of the gaps between it and the masses of particles on either side in the mass spectrum. This removes the wide spacing in masses at the upper end of the mass spectrum, especially when the IMF slope is high. Thus the total change in mass of the particle from $N_\star$ stars to $N_r$ lower mass remnants will not occur simultaneously. This is due to the dependence of lifetime upon stellar mass. This difference is small for high mass stars, but can become very pronounced for stars of a low mass (see equation (3)). The code spreads the change in mass of a particle over a few Myr, this has the advantage of not allowing drastic instantaneous changes which are obviously not realistic (especially...
when a particle whose average stellar mass is $12M_\odot$ becomes a particle of neutron stars of mass $1.4M_\odot$).

This multi-mass model of the IMF tends to clump similar mass stars in phase space. To see if there were significant differences between this method and one in which particles have a uniform mass and contain the full range of masses comparison simulations were made. The two methods are qualitatively similar over the timescales of these simulations ($\approx 50$ Myr). Discrepancies occur in the numbers of particles that escape beyond the tidal radius. The total mass of escaping particles is similar, but more particles are lost in the multi-mass case, the majority from the low-mass end of the IMF. Single-mass simulation clusters also appear to have a slightly higher survivability. The difference is small and probably unimportant in view of the qualitative nature of the results presented.

The mass lost from stars due to stellar evolution is treated in two ways depending upon whether the residual gas is still present in the cluster. Whilst the residual gas is still in the cluster, the mass lost by stars is negligible in comparison to the total mass of gas in the cluster and so can be ignored (even though the energy provided by this mass loss is the driving force behind much of the gas expulsion in the cluster). After residual gas has left the cluster further loss is the driving force behind much of the gas expulsion in the cluster. After residual gas has left the cluster further mass lost through stellar evolution is assumed to leave the cluster immediately. Even if the gas does not leave the cluster instantaneously, the timescales followed in the simulation would not allow significant amounts of gas to collect.

### 2.3 Initial Distribution of Stars and Gas

Immediately after star formation a globular cluster will be composed of roughly equal proportions (to within an order of magnitude) of stars and gas. The initial ratio of stellar mass to total mass determines the star formation efficiency (SFE) of the proto-cluster cloud. The initial distributions of both the stars and gas in the cloud are represented by a Plummer (1911) potential given by

$$\phi(r) = -\frac{GM}{r^2 + R_\text{S}^2}$$

where $M$ is the total mass of the residual gas or stars, $R_\text{S}$ is the scale length of the potential and $G$ the gravitational constant. The SFE is then given by

$$\eta = \frac{M_\text{stars}}{(M_\text{stars} + M_\text{gas})}$$

The initial positions and velocities have been determined using the technique described in detail in Aarseth, Hénon & Wielen (1974). The stars are first distributed randomly in phase space in such a manner that they produce a Plummer distribution. The positions and velocities can be scaled to produce a distribution of the desired scale length and virial ratio (see below). The gas is represented by an external potential acting upon the star particles. The scale lengths for the stars and gas are equal for any model. This represents an assumed dependence of star formation upon gas density. A Plummer potential was chosen for the initial conditions in preference to a King model due to the simple analytic form of the potential. The gas is assumed to always be a spherically symmetric distribution and the stars are assumed to have no effect upon the gas, ie. the viscosity is negligible and the gas is expelled from the system before any gravitational effect from the stars can make itself manifest upon the distribution of the gas. Any change in the gas potential is idealised (see section 2.4) so that it remains in a Plummer model to simplify the potential and force calculations. Selecting the value of $R_\text{S}$ correctly the Plummer model can be made very similar to a King model.

Particules are initially placed in the inner region of the cluster in a sphere of $\approx 10$ to $30$ pc radius, depending upon the initial concentration of the cluster and its Galactocentric radius. This positioning is also independent of the mass of the particle. Stars are not assumed to be formed over an extended region of space, certainly not up to the tidal radius.

The initial velocity distribution of the stars is scaled according to the initial virial ratio, $Q$, of potential energy, $\Omega$, to kinetic energy, $T$ where $Q = T/|\Omega| = 0.5$ corresponds to a system in virial equilibrium. The virial ratio of the system is given by the equation summing over the number of particles in the simulation

$$Q = \frac{1}{\sum_i m_i} \sum_{i \neq j} \frac{G m_i m_j}{(r_{ij3}^2 + \epsilon^2)^{3/2}} + \sum_i \frac{G m_i}{r_{iS}^{3+\epsilon}(\text{gas})}$$

where $m_i$ is the mass of particle $i$, $M_\text{gas}$ is the total mass of gas in the system, the radius of particle $i$ is $r_i$, the inter-particle distance between two particles $i$ and $j$ is given by $r_{ij} = |r_i - r_j|$, $\epsilon$ is the softening parameter (see below) and $R_{S(i\text{gas})}$ is the scale length of the gas potential.

It is important to note that even clusters initially in virial equilibrium will not be in complete dynamical equilibrium. It seems highly unlikely that after the star formation episode that the stellar component of the cluster would be in dynamical equilibrium, so some settling of stars into an equilibrium distribution after star formation would be expected, even if there were no gas or no gas expulsion.

After the mass loss episode in the cluster, the evolution will proceed along normal lines for a globular cluster with evaporation and stellar evolution competing against core collapse to decide the final fate of the cluster.

### 2.4 Residual Gas and Mechanisms for gas loss

As argued in the introduction, a globular cluster must expel all of its residual gas before another generation of stars is able to form in order to retain its extreme chemical homogeneity. This mass loss will have an important effect upon the dynamics of the cluster possibly leading to its destruction. The simplest application of the virial theorem to star clusters implies that for a SFE of less than 50% a cluster initially in virial equilibrium cannot lose all of its residual gas and still remain bound. More sophisticated simulations of open cluster dynamics show that a system may retain a bound core of stars with a SFE as low as 30% (Lada et al. 1984).

This expulsion of gas is assumed to be driven by the high mass ($M > 8M_\odot$) stars in the cluster through their photoionisation of the surrounding medium by intense UV radiation (Tenorio-Tagle et al. 1986), strong stellar winds and their final supernovae explosions (Dopita & Smith 1986 and Morgan & Lake 1989). These three different mechanisms are simulated to represent the alternative routes by which the gas may be lost.
The first mechanism of gas expulsion modelled is that due to photoionisation and strong stellar winds (referred to throughout as gradual early gas expulsion). Tenorio-Tagle et al. (1986) used a hydrodynamic code to model the expulsion of gas from a cluster. They showed that for relatively low masses of gas ($10^3$ to $10^5 M_☉$), the photoionisation caused by 100 O5 stars, each producing some $4 \times 10^{32}$ ergs of UV radiation in their lifetime (Chiosi & Maeder 1986), may expel all of the gas within a cluster, while for higher gas masses only a small, inner region is ionised and no gas is lost. The timescale for this gas loss is only a few Myr starting around 4Myr after the formation of the massive stars. These gas masses are far lower than the masses of residual gas left in young globular clusters. However, a proto globular cluster will normally contain more than enough stars to expel the residual gas, at least by the action of supernovae (for a caveat see section 3.5). The number of massive stars of above some mass $M_{SN}$ in a cluster with an IMF of slope $\alpha$ is given by

$$N_{SN} = M_{cl} \frac{(\alpha - 2) (M_{SN}^{-\alpha -1} - M_{up}^{-\alpha -1})}{(\alpha - 1) (M_{low}^{-\alpha -2} - M_{up}^{-\alpha -2})}$$

where $M_{low}$ and $M_{up}$ are the upper and lower mass limits of the IMF respectively and $M_{cl}$ is the total mass of the cluster. Using $M_{SN} = 8 M_☉$, $M_{up} = 15 M_☉$ (note that this is the highest mass of star assumed present in the cluster, rather than the mass of star represented by the largest particle) and $M_{low} = 0.15 M_☉$ then $\alpha = 3.5$ will give $N_{SN} \approx 150$ for a cluster where $M_{cl} = 10^5 M_☉$. An additional source of energy not included in Tenorio-Tagle et al. is that each O5 star will also add around $10^{49}$ ergs into the interstellar medium through strong stellar winds at typical globular cluster metallicities (Kudritzki, Pauldrach & Puls 1987). It may well, then, be reasonable to assume that in some cases mass loss can be driven solely by these two processes. This mechanism for mass loss will begin only a few Myr, at most, after the end of star formation and expel the majority of the residual gas before the first supernovae explode.

This type of mass loss is modelled in the code by reducing the mass of gas $M_{gas}$ in the Plummer potential gradually with time. The loss of the gas begins after $\approx 4$ Myr and continues at a constant mass loss rate (based upon those found in Tenorio-Tagle et al. 1986) until no gas is left within the cluster (cf. Lada et al. 1984). During the gas expulsion episode the scale length of the gas remains constant (and equal to that of the stars).

The second and third mechanisms of gas loss are both via the supernovae explosions of massive stars.

The second mechanism is similar to the loss of gas via photoionisation and strong stellar winds above in that the mass of gas in the gas potential is gradually reduced. However, this reduction begins at a later time as it is caused by the supernovae of massive stars expelling the gas (refered to throughout as gradual late residual gas expulsion). It is partly based upon the calculations of the effects of supernovae on gas clouds (Dopita & Smith 1986 and Morgan & Lake 1989) as to the number of supernovae required to disrupt a certain mass of gas. The main difference between this mechanism and the first is that the onset of mass loss is delayed until the most massive stars ($\approx 15 M_☉$) and below reach the end of their lives and go supernova ($\approx 10$ Myr or more).

Both gradual mechanisms are assumed to have timescales of gas expulsion that are independent of the mass or Galactocentric radius of the cluster. There seems little reason to suspect that Galactocentric radius would effect the expulsion timescale other than that the gas would require longer to escape beyond the tidal radius of high $R_G$ clusters. This is assumed to be unimportant as most stars are initially well within the tidal radius. There would probably be a dependence of expulsion rate with cluster mass. However, the mechanics of gas expulsion are very poorly understood and so an independence is assumed. This assumption is used as a first approximation as higher mass clusters will contain more high mass stars which expel the gas.

The third mechanism is a highly idealised simulation of the effects on a star cluster of mass ejection via a 'supershell' such as those observed in OB associations in the Galactic disc. These supershells are formed by the merging of many supernova shock fronts into one large and powerful shock (McCray & Kafatos 1987, McCray & Mac Low 1988). A moderate number of supernovae in the central regions of a proto-cluster cloud will be able to form a supershell to force the gas out of the gravitational influence of the cluster (Brown et al. 1995). A supershell will have a radius $r_{shell}$ dependent upon the rate of supernova events $N$. $N$ is calculated from the total number of supernovae events in the cluster, given by equation (7), assuming that the events are spread evenly throughout the whole time that supernovae are occuring. The radius is given by equation 9 in Brown et al. (1995) as

$$r_{shell} = \left( \frac{3 E N t}{10 \pi P_{ext}} \right)^{1/3}$$

where $E$ is the energy of each supernova (taken to be $10^{51}$ ergs), $P_{ext}$ is the external pressure of the gas in the cloud and $t$ is the time since the first supernova. The supershell is approximated in the code by assuming that the shell is spherically symmetric and that all matter interior to $r_{shell}$ has been swept into the shell. The finite size of the shell is also neglected. Stars with $r_s < r_{shell}$ feel no external potential due to the gas while stars with $r_s > r_{shell}$ feel the force due to the gas as if there were no supershell (applying Newton’s First Theorem). Once $r_{shell} > r_t$ then all of the gas is assumed to be lost into the intra-cluster medium. Again, the onset of this mechanism is delayed until the most massive stars go supernova.

While $r_{shell} < r_t$, the assumption of a central source of energy from supernovae is not good. The dynamics of the shell and the consequent effect upon the stellar dynamics of the cluster are not well modelled. This mechanism only has the effects of a very rapid depletion of the residual gas in the inner regions. However, this is only true for a short period (normally less than a crossing time) and so are not expected to be of great importance. This model is more concerned with the effects of a supershell upon the dynamics of outlyung particles which may wait a far longer before the shell passes their positions.

These expulsion timescales are often in excess of those used by Lada et al. (1984) whose maximum time for the expulsion of gas was set to be four crossing times.
These three expulsion mechanisms are treated in isolation in this paper. In reality the situation will be far more complex and some aspect of all three mechanisms, and mechanisms not considered within this paper, will conspire to expel the residual gas. The relative importance of the mechanisms, however, is unknown and it may be that any one (or none) is by far the most dominant.

2.5 Computational Aspects

Standard $N$-body units were used from Heggie & Mathieu (1986) to scale such that $M_0 = G = 1$ and $E_0 = -1/4$, where $M_0$ is the initial mass of the particles (ie. the initial stellar mass of the cluster) and $E_0$ is the initial energy of the particles. Conversion to units of time (in Myrs) for the treatment of stellar evolution was made using the relationship

$$T_c = M_0^{1/2} \left( 2 |E_0| \right)^{3/2} = 2 \sqrt{2} U_t$$

where $U_t$ is the unit of time within the code.

The softening, $\epsilon$, in the code was set to be of order the inter-particle distance in order to reduce the effects of two-body relaxation. For the duration (in ‘real’ time $\approx 50$ Myr) of the simulations the effects of two-body relaxation (see Spitzer 1987) would be neglegable. This was a primary reason for the choice of such short durations as it minimises one of the major problems associated with an $N$-body simulation, especially with the relatively small number of particles (1000) used in these runs.

The code was run on a Sun 10 Workstation at the Astronomy Centre, University of Sussex. A typical run of 1000 particles over 100 crossing times took approximately 2 hours.

2.6 Comparison with Chernoff & Shapiro

Given an initial mass, IMF, Galactocentric distance and the central potential parameter of the King model (for details of King models see King 1966) the results of CS enable the end state of the cluster to be estimated. The end states possible in CS are disruption, steady state King model, collapsing or core collapsed. In the context of this paper these possible states will be divided into two: disruption and survival. The initial conditions that lead to these two possible states are illustrated in fig. 1.

After the gas loss episode and a short settling period (simulations are normally run for 50 Myr) in the $N$-body simulations the continuing evolution of the cluster is assumed to be as CS. The state of the $N$-body simulation after this period is then compared to the lowest King model that CS say will survive. If the structure of the simulation is such that it is more concentrated and more bound that the lowest King model then it is assumed that the cluster would be able to survive. A comparison is made of the mass distributions and velocity dispersions of the simulation and King model to assess the survivability.

The comparison of young globular clusters to King models appears justified as observations show that young LMC clusters have a luminosity profile remarkably close to that of a King model (Elson 1991, Elson, Fall & Freeman 1987).

It should be noted that within the context of the CS paper the survival of clusters was maximised by the associated approximations. CS also took no account of other destructive mechanisms, such as bulge and disc shocking, which will effect the evolution of globular clusters, Aguilar, Hut & Ostriker (1988) discuss these, and other, mechanisms and their possible effects upon the globular cluster population of the Galaxy. For these reasons, many more clusters than
predicted in both CS and this paper may be disrupted over the course of a Hubble time.

3 RESULTS

In this section the results of the simulations are presented. In section 3.1 the general effects of residual gas loss upon a cluster and their dependence upon the environment of the cluster are investigated. Section 3.2 explores the three gas expulsion mechanisms themselves in more detail. Section 3.3 deals with the actual survivability according to CS of a wide variety of 50% SFE clusters, while section 3.4 examines the effect of the initial virial ratio upon this survivability. Finally, section 3.5 looks at the survivability of clusters that have an SFE of less than 50%.

Throughout this section particular attention is paid to clusters with an initial stellar mass of \(10^5M_\odot\). This is due to the overwhelming number of actual globular clusters that have approximately this mass.

3.1 The effects of residual gas expulsion

The effect of residual gas expulsion on a globular cluster are initially independent of the chosen IMF of that cluster. This result is as would be expected because of the relatively short timescale of the gas expulsion compared to other dynamical processes affecting the evolution of the cluster which would be dependent upon the mass spectrum of the stars (the amount of stellar mass lost is smaller with higher slopes to the IMF as the mass of high mass stars is obviously greater). In the few Myr covered by these simulations no dynamical processes such as relaxation, equipartition or mass loss due to the stellar evolution of anything other than very high mass stars should have any significant effect.

The number of massive stars may well effect the gas loss rate, but in these simulations it has been assumed that there are enough stars to expel the gas and that this expulsion will occur on an arbitrarily chosen timescale. Only in the supershell mechanism is any account taken of the numbers of massive stars present in the cluster. In this case the early stages of gas loss are relatively insensitive to changes in the number of massive stars present and so the effect upon the majority of stars (which populate the inner few pc) is effectively independent of the IMF. In simulations run without stellar evolution at the three different IMFs the absence of the additional mass loss from the stellar evolution is not found to alter the bulk effects of the gas expulsion mechanism. However, the absence of stellar evolutionary mass loss will effect the survivability of the cluster as clusters with low values of \(\alpha\) lose more mass and will have an extra disruptive influence upon them than clusters with higher \(\alpha\).

The initial mass of the cluster is found to have a significant effect upon the effects of residual gas loss as shown in fig. 2. Note that the number of particles has been chosen instead of the cluster’s stellar mass as it does not include the loss of mass due to stellar evolution, just the escape of particles beyond the tidal radius. For otherwise identical clusters in IMF, Galactocentric radius, SFE, initial stellar and gas distribution and mass loss mechanism and timescale, higher mass clusters are found to be far more stable to gas loss than those of a lower mass. Both the reduction in the binding energy of the stars and the expansion of the radial mass distribution are far less extreme in higher mass clusters. More massive clusters also lose fewer stars. The reason for this dependence appears to lie in a very strong correlation between the number of crossing times over which gas loss occurs and the disruptive effect of that gas loss. As the mass loss timescales for each mechanism are the same for all clusters, regardless of mass, then the gas loss occurs over more crossing times in higher mass clusters (see section 2.5). Any dependence of expulsion timescale with cluster mass would be expected to increase the timescale in larger clusters (as \(N_{SN} \propto M\) but \(\Omega \propto M^2\)). Longer expulsion timescales are less disruptive as the potential changes more slowly which would further increase the relative survivability of high mass clusters compared to lower mass clusters (as the number of crossing times over which gas loss would occur would be further increased).

The sudden increase in the rate of particle loss from the \(10^5M_\odot\) cluster at \(\approx 18\) Myr is due to a large overflow of particles from the tidal radius. This is caused as the tidal radius is smaller than the edge of the new equilibrium distribution of the cluster after mass loss. The cluster expands to reach an equilibrium state and in doing so causes over 30% of its particles to escape immediately. The loss rate slows somewhat after 10 Myr, as all of the very energetic particles are lost, but still continues at a far higher rate than for the two more massive clusters until the destruction of the cluster. This effect is self perpetuating in that the loss of particles (mass) causes the tidal radius to shrink even further and enhances the loss rate. The two larger clusters (with their correspondingly larger tidal radii) do not have this effect as fewer of their particles are able to reach this distance from the cluster core. This effect results in the very rapid destruction of the \(10^4M_\odot\) cluster over \(\approx 100\) Myr.

This effect is, to a certain extent, a result of the N-body simulation. In all masses of cluster 1000 particles have been used to simulate the cluster. In the nbody2 code, the only place where the total actual mass of the particles (as opposed to their relative masses) is in the conversion from
after about 200 Myr where a lower number of particles tends to lead to disruption far faster than a larger number. However, it is only the first 50 Myr, or so, that these simulations cover, in which time the loss rates are comparable.

Figure 4 also provides an idea of the noise present in any one individual run in the mix of the curves and the lack of an obvious trend with particle number at these early times. To test the validity of the results generally, most runs were repeated at least once (usually several times) with slightly varying initial conditions. A few of the border-line cases were run 10 or 20 times, this process has the advantage of testing the statistical robustness of the results. The results from these varied runs were not found to differ significantly, hopefully showing that the changes observed for different sets of initial conditions were not the results of statistical fluctuations in the computations (see Heggie 1995).

The effect of the particular gas loss mechanism is found to be relatively consistent, independent of the mass or Galactocentric radius of the cluster, subject to the caveats mentioned in the previous section. The three different mechanisms of gas expulsion are found to have quite different effects upon the stability of the cluster.

In this subsection all the clusters described are $10^5 M_\odot$ at a Galactocentric radius of 10 kpc with an IMF slope of $\alpha = 3.5$ and Plummer model scale lengths initially $R_G = 4.4$ pc (corresponding to a mean harmonic particle radius of 7.5 pc), with central stellar densities of $\approx 280 M_\odot$ pc$^{-3}$. This set of parameters is chosen as an example to illustrate the general effects of the gas expulsion mechanisms. The clusters were all initially in virial equilibrium.

Figure 5 shows the change in the virial ratio $Q = T/|\Omega|$ for three otherwise identical clusters each with a different mechanism of gas expulsion.

Figure 6 shows the loss of particles with time for each of the different residual gas expulsion mechanisms illustrated in fig. 5 as well as an identical cluster but with no gas present to be expelled. As can be seen from fig. 6 no loss of the particles even before the onset of mass loss (at 5 Myr) as the higher Galactocentric radii. The cluster at 2 kpc starts losing the code time units (in units of the crossing time) to physical time (in Myrs) see equation (9). The system can be easily set-up in such a way that, in all but this conversion, the system is identical. With this situation the higher stability of higher mass clusters is obvious after 50 Myr of physical time. It appears that this effect may be representative of a real physical effect in actual clusters.

Further hydrodynamic simulations of gas expulsion from such systems, similar to those of Tenorio-Tagle et al. (1986), would be required to assess the true validity of the assumption that expulsion timescales in this cluster mass range are, indeed, as independent of initial cluster mass and tidal radius (and, even, the cluster IMF slope).

A dependence of the survivability of clusters upon Galactocentric radius is also found. Due to the dependence of tidal radius upon Galactocentric radius the amount of mass loss by overflow beyond the tidal radius stimulated by the residual gas loss and the expansion of the cluster to a new equilibrium position is increased with decreasing Galactocentric radius, due to the linear dependency of $r_1$ upon $R_G$ (equation (1)).

In fig. 3 the escape rate of particles from clusters at four different Galactocentric radii is illustrated. The severity and time of onset of mass loss is significantly decreased with higher Galactocentric radii. The cluster at 2 kpc starts losing particles even before the onset of mass loss (at 5 Myr) as the cluster moves towards a dynamical equilibrium.

The scaling and properties of any N-body simulation vary with the number of particles used. The selection of computational parameters is such as to try and reduce these effects and provide as good an approximation to reality as possible. Figure 4 shows the change in the number of particles of a typical run as the number of particles used were varied from 500 to 4000. This particular set of parameters was chosen as they result in a disrupted cluster when compared with the constraints of CS (see section 3.3). As can be seen within this range the results of the change in particle numbers remain fairly consistent. The results diverge more after about 200 Myr where a lower number of particles tends to lead to disruption far faster than a larger number. However, it is only the first 50 Myr, or so, that these simulations cover, in which time the loss rates are comparable.

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were initially identical with supershell starting at 12 Myr (dash-dot line). The three clusters lasting for 5 Myr (dashed line). A supershell will sweep the inner few pc of a cluster of its gas in only a few tens, possibly hundreds, of thousands of years. This corresponding change in the potential in these central regions then occurs on a timescale less than a crossing time. With such a short timescale, the stars in the cluster are not able to adjust their orbits to the new potential. This sudden change pushes the cluster far out of virial equilibrium with \( Q_{\text{max}} \approx 0.8 \) (fig. 5). As can be seen, however, the cluster is able to very rapidly readjust to the new potential and bring itself back into virial equilibrium very rapidly (within \( \approx 10 \) crossing times).

It should be noted that while the supershell mechanism induces far more mass loss than either of the other two mechanisms it may not, necessarily, be too much more disruptive. At higher Galactocentric radii and masses a larger and larger bound central mass remains which appears as if it could well survive. A relatively small decrease in the scale length of the Plummer model can vastly reduce the level of mass loss from a supershell and prevent its disruption (see section 3.3).

In fig. 6 the particle loss can be seen to be delayed by 10 to 15 Myr after the initiation of residual gas expulsion. This delay is due to the travel time of particles with escape velocity from the cluster centre to the tidal radius at which point they are removed.

During the gas loss episode the globular cluster expands from its initial central distribution into a new equilibrium distribution. In the settling process after star formation (see section 2.3) some stars will escape beyond the tidal radius in a slow process that will continue for the life of the cluster. This process is, however, enhanced and encouraged by a gas loss episode. The more rapid the gas loss, the more pronounced is the stellar mass loss associated with that gas loss.

### 3.3 Survivability and gas loss from 50% SFE globular clusters

In this sub-section the effects of residual gas expulsion from clusters with an SFE of 50% is investigated. The state of the clusters after the gas expulsion episode is then compared to the survivability estimates of CS (fig. 1).

The effect of the residual gas expulsion described in the previous two sub-sections is to weaken the binding of a cluster. In some cases this almost immediately (within a few hundred million years at most) results in the disruption of a cluster that, according to the estimates of CS, would have survived for a Hubble time without the gas expulsion. In
Figure 7. The change in the mass distribution with radius during the time of gas expulsion. The two dashed lines show the mass distribution of the simulation at 0 and 50 Myr. The dotted line is the mass distribution expected of a King model with $W_0 = 3.5$ (the lowest surviving King model with these initial conditions from CS).

other cases the effect of the gas expulsion is far slower, but will still eventually result in the disruption of the cluster.

Figure 7 shows the effect of residual gas loss upon the distribution of mass with radius inside a cluster. This cluster has a stellar mass of $10^5 M_\odot$ initially, an IMF slope of $\alpha = 3.5$ at a Galactocentric radius of 5 kpc. The cluster has a initial Plummer length scale $R_S = 4.4$ pc, which gives a central stellar density initially of $\approx 280 M_\odot$ pc$^{-3}$. The gas expulsion is modelled as the least disruptive, gradual early loss via stellar winds and the UV flux. After 50 Myr (which includes the residual gas expulsion and the evolution of the most massive stars) its mass distribution and rms velocity distribution are compared with those of the lowest King model that can CS estimate can survive for a Hubble time with the same initial mass, Galactocentric radius and IMF as the model cluster. The use of a radial mass distribution is justified by the approximate sphericity of the clusters. The initial conditions produce spherically symmetric clusters without rotation that retain their sphericity at least for the duration of the simulations. Variations in the sphericity are seen of a few percent but these fluctuations are not found to be in any way systematic.

The King models used for comparison were numerically integrated from King (1966) and set to be similar to the initial condition King models used in the N-body calculations of Fukushige & Heggie (1995). They were given a $W_0$ equal to the lowest surviving value taken from fig. 1 and a total mass equal to the mass of the cluster at that time (which may significantly lower than the initial mass due to particle loss). However, at lower masses fig. 10 indicates that survival is actually easier, so this effect may underestimate survivability slightly.

Figure 7 shows that a cluster initially well within the survivability range of fig. 1 will, in this border-line case, fall to become a cluster only just expected to survive. The cluster has lost nearly 10% of its mass through both stellar evolution (\approx 1%) and the escape of particles (\approx 8%). The 'jump' in the $T = 0$ Myr mass profile at $\approx 7$ pc is caused by the large particle that represents the most massive stars before they have evolved. As explained in section 2.2, the presence of a particle of this size for a short period does not significantly effect the dynamics of the simulation. The similarity of the state of the cluster after 50 Myr to the $W_0 = 3.5$ King model is further illustrated in fig. 8. The rms velocity dispersions of the simulation can be seen to be very similar to those calculated for the fitted King model. The dissimilarity at large radii is largely due to the low number of particles at these radii with which to construct the velocity dispersion. The similarity of clusters to King models in these simulations is striking, even shortly after a disturbing episode of gas expulsion.

It should be noted that the lowest survivability King models from CS are those before the most massive stars in the cluster evolve. In these comparisons the simulations have already lost the most massive ($M > 8 M_\odot$) stars. The change in the mass distributions of clusters with no residual gas (and hence no gas expulsion) is shown in fig. 9 for a cluster with the same initial conditions as that in fig. 7. The range in radius has been reduced in this figure to show more clearly the increase in the central density of this simulation when compared to those with residual gas expulsion. The settling of the cluster and the evolution of the most massive
stars can be seen to produce a slight lowering of the mass profile in the outer regions. The King model illustrated is, again, the lowest surviving King model from fig. 1 this time with a mass of $9.6 \times 10^3 M_\odot$ as the cluster has lost $4 \times 10^3 M_\odot$ due to the stellar evolution of the most massive stars but no particles have escaped in this case. The $T = 0$ Myr ‘jump’ in the mass profile at $\approx 18$ pc is, again, caused by the presence of the massive star particle.

This process of estimating the survival, or otherwise, of a cluster was followed for a wide range of initial conditions similar to those in CS. Comparisons were made after the 50 Myr evolutionary period to try and determine if the cluster would still be bound after a Hubble time.

It is found, unsurprisingly, that the minimum concentration required for the survival of a cluster is always increased. This increasing concentration in terms of a Plummer potential corresponds to an increasing $W_0$ of King models for that initial mass and Galactocentric distance.

The preferential survival of clusters at lower Galactocentric radii is more than compensated for by the increased disruptive effects of residual gas expulsion at those radii. The lowering of the required King model at low Galactocentric radii becomes an upturn, leading to a higher concentration for survival at low Galactocentric radii.

Figure 10 shows the effects of residual gas expulsion on the survivability of globular clusters. Figure 10 is laid-out in a similar fashion to fig. 1 for the three different initial stellar masses and IMF slopes. It should be noted that in fig. 10 the steepest slope to the IMF is the uppermost line and clusters that survive are below the lines (the opposite of fig. 1) this is due to the choice of scale length as the abscissa. It should be noted that all of these clusters are in virial equilibrium, the effects of non-equilibrium initial conditions are discussed below.

The results are given in terms of the length scale, $R_S$, of the Plummer model for the stars used as the initial conditions. The length scale can be converted into a central density via the formula

$$\rho_0 = \frac{3M_{\text{tot}}}{4\pi R_S^3} \quad (10)$$

An interesting feature of a conversion to central densities is that the central densities required for survival are dependent only upon the IMF slope and the Galactocentric radius, and are approximately independent of the initial mass of the cluster. An order of magnitude increase in the initial mass causes an increase in the scale length required for survival. When $\alpha = 3.5$ and $4.5$ this increase is approximately a doubling of the required $R_S$. For $\alpha = 2.35$, however, the relationship is not as clear and depends more upon Galactocentric radius. The increase required ranges from $\approx 30\%$ to 100% at higher radii. This relationship is not present in the results of CS and must therefore appear as a property of the residual gas expulsion. Although this result depends upon the slope of the IMF, it appears to hold for each of the three different values of $\alpha$.

The clusters represented in fig. 10 have all undergone the gradual early gas expulsion, the least disruptive of the three expulsion mechanisms. The quoted values for $R_S$ in fig. 10 should therefore only be thought of as upper limits for the gas expulsion mechanisms.

The change in the length scale required for survival produced by each of the other two gas expulsion mechanisms is not very sensitive to IMF slope or Galactocentric radius. Gradual late gas expulsion reduces the required $R_S$ from gradual early expulsion by around 4 or 5%, while a super-shell expulsion will reduce $R_S$ by about 8 to 10%. This has the effect of lowering the position of the lines on fig. 10 and hence survival requires a slightly higher central density. As the reductions are similar for all Galactocentric radii and IMF slopes, the independence of the required central density to mass is retained for all gas expulsion mechanisms.

Even if gradual early mass loss was not able to completely remove the residual gas from a cluster, its action may reduce the disruptive effects of one of the latter mechanisms. If the stellar winds and UV flux were able to remove some of the gas, or even just decrease the density of the

Figure 10. The minimum length scale of a Plummer model required for the survival of a cluster of the indicated mass. The IMF slope, $\alpha = 2.35, 3.5$ or $4.5$ of the simulation are indicated on each line. If a cluster is to survive it must lie below the line.
residual gas in the central few pc of the cluster, then the latter mechanisms would not be as disruptive.

The cut-off line on fig. 10 should be considered merely as a guide to the actual position of the cut-off between survival and disruption. Many of the simplifying assumptions made in this paper and in CS mean that it is only a first order estimate of the true value which will depend on many initial properties individual to any particular cluster.

The quoted value of the mass is the total initial stellar mass of the cluster. After gas loss the cluster will, of course have lost 50% of its initial mass after losing the residual gas as all of these clusters have an SFE of 50%. In addition, some clusters that survive can lose from 5% to 20% of their initial stellar mass by stellar evolution and the escape of particles. This loss should not effect the survivability of the cluster after residual gas expulsion as lower mass clusters are more stable after residual gas expulsion than higher mass clusters (as illustrated in fig. 1).

All clusters on the border-line of survivability lose mass during the gas expulsion episode and the restabilisation period afterwards. The extent of this mass loss appears to be relatively independent of the Galactocentric radius of the cluster. The increase in $r_t$ acts to cancel the increase in the border-line value of $R_S$. The mass loss is still highly dependent upon the initial mass of the cluster, however. $10^5 M_\odot$ clusters only lose a few percent of their mass, the vast majority of this mass loss being the mass lost in the stellar evolution of the most massive stars. $10^4 M_\odot$ clusters, however, may lose up to 25% (typically 10 to 15%) of their mass during this phase. The reason for this appears to be the greater extent of the gravitational influence of the larger clusters. The tidal radius of a $10^5 M_\odot$ cluster will be approximately 5 times larger than that of a $10^4 M_\odot$ cluster at the same Galactocentric radius. The more massive clusters are then more capable of retaining a weakly bound extended halo of stars than those of lower mass. These extended halos will presumably be stripped relatively quickly by the Galactic tidal field, but in these simulations as they are still within the tidal radius, they are still counted as being part of the cluster. Due to this effect (and the lack of particles at large distances), the comparisons to King models were concentrated on the inner regions (usually within the half-mass radius) of clusters where they are expected to be more accurate indicators. In the vast majority of cases, 50% SFE clusters were found to be very similar to King models after gas expulsion.

### 3.4 Non-virial equilibrium clusters.

The assumption that the stars within a cluster are in virial equilibrium at formation has underlay the previous discussion of the survivability of a globular cluster. The validity of this assumption, however, is not known. The gas from which the stars form is expected to be in virial equilibrium itself in globular cluster formation models with internally induced star formation. These stars may then be expected to also be in, or very close to virial equilibrium. If star formation is induced by external effects then there seems no a priori reason to expect the stars themselves to be in equilibrium. On the contrary, it may be expected that the stars would not be virialised.

![Figure 11. The evolution of the virial ratio $Q$ for two $10^5 M_\odot$ clusters with $R_S = 4.4$ pc, $\alpha = 3.5$ at 5 kpc. The evolution shown by the dotted line is that of a cluster with an initial virial ratio of $Q = 0.8$, while the solid line is for a cluster with initial virial ratio $Q = 0.2$. The clusters both underwent the same form of gradual early mass loss beginning at 5 Myr, and lasting for 5 Myr.](image)

![Figure 12. The mass distribution of three otherwise identical clusters with different initial virial ratios, $Q = 0.2$, $Q = 0.5$ and $Q = 0.8$, after 50 Myr with gradual early mass loss. The $Q=0.5$, $R_S = 4.4$ pc cluster is at the limit of survivability for a $10^5 M_\odot$ cluster with $\alpha = 3.5$ at 5 kpc.](image)

Figure 11 shows the evolution over 50 Myr of two clusters with initial virial ratios of $Q = 0.2$ and $Q = 0.8$. The $Q = 0.2$ cluster can be seen to settle into virial equilibrium very quickly (within 5 Myr), the subsequent variations in the virial ratio being the expected random level of fluctuation. The cluster initially at $Q = 0.8$, however, is far more disturbed. The action of attempting to settling into a virial equilibrium distribution combined with residual gas expulsion and mass loss due to stellar evolution in the first 10 Myr of the cluster’s evolution prevent the cluster from reaching equilibrium in the 50 Myr shown. The cluster is in the process of attaining virial equilibrium, but the loss of energetic stars required to cause this has been extreme.

In a cluster where gas expulsion (by the gradual early mechanism) is included the $Q = 0.2$ cluster has lost exactly 10% of its stellar mass after 50 Myr (exactly the same mass...
loss as from an equivalent cluster starting with $Q = 0.5$). The $Q = 0.8$ cluster has, however, lost nearly 40% of its initial stellar mass in the same period. Figure 12 shows the effects that the different initial conditions have had upon the final mass distributions of the cluster. A cluster with the same initial conditions, but initially in virial equilibrium is a border-line case for surviving a Hubble time. As is illustrated, a different initial virial ratio will, in the case of $Q < 0.5$, improve the survivability of the cluster or, in the case of $Q > 0.5$, substantially decrease the chances of surviving for a Hubble time. The main cause of this effect appears to be the changes in concentration that occur within a cluster out of virial equilibrium in order to achieve virial equilibrium: $Q < 0.5$ clusters will collapse and $Q > 0.5$ clusters expand. These changes effectively increase or decrease the length scale, shifting the cluster down or up fig. 10.

### 3.5 Survivability and gas loss from less than 50% SFE globular clusters

The clusters discussed in the previous sections have all had an SFE of 50%. Considering the low SFEs observed in star forming regions in the Galaxy today, only of order a few percent (Larson 1986), it is of interest to examine the effects of the loss of residual gas from clusters that have only achieved an SFE of less than 50% and to estimate the survivability of those clusters.

Lada et al. (1984) followed a similar method in an N-body simulation of the effect upon star clusters of residual gas expulsion. They found that star clusters may remain bound with an SFE of 30%, however they will lose 10 to 80% of their original stars in this process. These simulations differ from the simulations presented within this paper in the size of the cluster that is simulated, the timescale of the gas expulsion and the number of particles used.

The effect of residual gas expulsion from a cluster with an SFE lower than 50% is, unsurprisingly, qualitatively no different from that from the clusters described above with an SFE of 50%. Quantitatively, an SFE of less than 50% decreases the scale length of the lowest surviving Plummer model as its effect upon the cluster is more disruptive. There would appear, theoretically, to be no lower limit to the SFE that will produce a bound cluster, the required central density would just continue to increase with decreasing SFE as long as the tidal radius is large enough to contain the expansion of the cluster after the residual gas expulsion. In practice, however, very low SFEs require a concentration and central density that would be highly implausible, if not impossible to achieve.

The effective limit of cluster SFEs occurs when a cluster does not contain enough massive stars to expel the residual gas. Following Morgan & Lake (1989) and using equation (7) it is possible to obtain a rough estimate of the minimum SFE for a particular mass and IMF slope. The following table shows the minimum values of the SFE allowed if the cluster is to be able to expel its residual gas via supernovae alone.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$10^4 M_\odot$</th>
<th>$10^5 M_\odot$</th>
<th>$10^6 M_\odot$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.35</td>
<td>1%</td>
<td>20%</td>
<td>56%</td>
</tr>
<tr>
<td>3.5</td>
<td>11%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td></td>
<td>75%</td>
<td>90%</td>
</tr>
</tbody>
</table>

Table 1. The lowest SFE required for a cluster to contain enough massive stars ($\gtrsim 8 M_\odot$) to expel its residual gas for each of the three slopes to the IMF. From the calculations of Morgan & Lake (1989).

For SFEs as low as 30% comparisons with CS to assess survivability are still thought to be fairly valid as after the gas expulsion episode the clusters retain a good resemblance to King models, especially within the half mass radius. On the other hand, the half mass radius the mass density and rms velocity dispersion often fall far more rapidly than a King model. For this reason survivability becomes far more difficult to assess by this method. There does appear to be a linear decline of the scale length (hence an increase in the central density) required for survival. For a 50% SFE $10^4 M_\odot$ cluster at $R_G = 5$ kpc the border-line occurs at $R_S \approx 4.4$ pc. With an SFE of 40% this falls to $R_S \approx 3.3$ pc and for an SFE of 30% to $R_S \approx 2.9$ pc.

In clusters with less than approximately a 30% SFE the resemblance of the clusters to King models after the gas expulsion episode is not good. For that reason, estimates of the survivability by the method of comparison to CS are not expected to be accurate in these cases. Proper estimates for the survivability of these clusters (by running the code for time periods comparable to a Hubble time) are beyond the scope of this present paper. Such a simulation would have to address a number of the problems associated with N-body simulations that this paper has attempted to minimise in the treatment of the clusters over a short time (with respect to any significant dynamical timescale).

The clusters tended to show a more concentrated core, well within the half-mass radius with a far larger number of stars escaping and forming a loosely bound or unbound halo around the cluster before being stripped by the Galactic tidal field. The existence of this concentrated core may indicate the possibility of forming a bound cluster of signif-
icantly lower mass than the original cluster (cf. Lada et al. 1984). The linear decline of the required scale length with SFE appears to continue below an SFE of 30%, due to the uncertainty in the validity of the application of the comparisons at these SFEs this result has very little weight. What does appear clear, however, is the very real possibility that these clusters could survive (albeit largely depleted) with such a low SFE.

4 DISCUSSION

This paper explored the effects of an episode of residual gas expulsion a few Myr after star formation in a globular cluster. This episode clears the cluster of all the gas not turned in to stars. An N-body code, based upon Aarseth’s nbody2 code, with a variable external potential representing gas acting upon the stellar particles was used to perform a large number of simulations over a wide variety of initial conditions (IMF slope, Plummer scale length, SFE, virial ratio and Galactocentric radius). The results of these simulations after 50 Myr were compared to the results of Chernoff & Shapiro (1987) to provide an estimate of the survivability of the globular cluster. The short timescale of the N-body simulations helps to reduce the errors inherent in N-body calculations.

The residual gas loss from globular clusters was modelled using three idealised cases. All three mechanisms of gas expulsion rely upon the input of energy in to the cluster gas by massive stars to provide the impetus for the expulsion. The first mechanism assumes that the UV flux and stellar winds from the most massive stars will gradually force the residual gas out of the cluster and beyond the tidal radius where it will be lost into the general Galactic environment. The second mechanism follows the same path of gradual expulsion but is caused by the supernovae of these massive stars. Both of these mechanisms involve the reduction of the mass of gas in the external potential over a timescale of a few Myr. The third mechanism, however, is based upon the expulsion of the gas in a ‘supershell’ formed by the merging of the shock fronts of many supernovae in the central regions of the cluster. This mechanism sweeps the cluster clean of gas on a timescale dependent upon the number of supernovae and the mass and distribution of the gas.

The imposition upon a globular cluster of the condition that it must expel its residual gas soon after star formation can be seen to place strong constraints upon the set of initial conditions that will result in a cluster being able to survive for a Hubble time. As illustrated in fig. 10 these constraints relax in an approximately linear way with increasing Galactocentric radius.

There are two schools of thought as to the extent of the original globular cluster population. The first suggests that the present population of globular clusters is a fairly complete sample of the original population (some of these arguments are summarised in Laird et al. 1988). If this is the case then all proto-globular clusters must have formed in such a way as to survive residual gas loss, dynamical evolution and destructive processes. In such a case the initial conditions implied by these simulations would represent a lower limit upon those of actual globular clusters. Aguilar, Hut & Ostriker, however, conclude from their study of the destructive mechanisms that operate upon globular clusters that the current population may only be ‘...but a shadow of its former self.’ Even in the case where a large number of globular clusters are destroyed by the Galaxy, there is a lower limit of ≈ 2% of the initial population surviving, as globular clusters still comprise 2% of the visible halo mass. Whatever the case may be, then, a significant number of original globular clusters are expected to survive their residual gas expulsion phase.

At the low Galactocentric radii (R_G < 6 kpc) that most globular clusters inhabit with an average mass of 10^5 M⊙ a central gas density in clusters just before the time of star formation of around 10^3 M⊙ pc^-3 is required (for an SFE of 50%) corresponding to mean densities within these clouds of ≈ 1 M⊙ pc^-3. As stated in section 2.6, the limits obtained in this paper should be regarded as lower limits only upon the initial conditions necessary for a cluster to survive. These densities are of the order of those observed in giant molecular clouds in the Galaxy today, although their cores, where star formation would be expected to occur, are far less massive than a globular cluster (Harris & Pudritz 1994 and references therein).

An interesting result to arise in this treatment of residual gas expulsion is that the central density requirement for survival is approximately independent of the initial mass. It depends only upon the IMF, Galactocentric radius and residual gas expulsion mechanism employed. If the formation mechanism of all globular clusters produces similar IMFs and the gas expulsion mechanism is the same then globular clusters of all initial stellar masses will be able to survive at all Galactocentric radii if the central density in the proto-cluster cloud is above some minimum value (≈ 10^4 M⊙ pc^-3 for a 50% SFE cluster). As noted above this is not an unreasonable figure to place upon the central densities of a proto-cluster cloud.

Unfortunately, any original independence of central density and mass will have been eliminated in the present day Galactic globular cluster system by dynamical evolution which can drive up central densities by gravothermal instabilities (Spitzer 1987) or possibly reduce it in clusters that are tending towards disruption. The best testing ground for this independence would be young globular clusters such as those around the LMC which have not had the time to significantly evolve dynamically. A detailed comparison of observations of young clusters to theoretical considerations of cluster survival is in preparation.

The higher values of the slope, α, of the IMF seem to lie fairly close together in fig. 10, while α = 2.35 provides far stronger constraints on the length scale. This effect is due mainly to the results of CS were the substantial dependence upon α is due to the mass loss from stellar evolution at later times than these simulations follow, rather than the gas loss mechanisms which are relatively insensitive to the IMF of the stars. Nevertheless, a cluster of α = 2.35 requires a central density at least an order of magnitude higher than α = 3.5 or 4.5 in order to survive. However, the calculations presented in table 1 appear to indicate that very high values of the IMF slope are not allowed as they do not provide enough high mass stars (M ≥ 8 M⊙) to expel gas in any cluster with even a ≥ 50% SFE.

Observed luminosity functions in clusters today appear to show that globular clusters have a low slope to their mass
functions for the low masses of stars present in clusters today, observed mass function slopes appear as low as \( \alpha \approx 2 \) (Fahlman 1993). The theoretical results presented here and elsewhere, however, show that a high slope to the IMF is advantageous for the survival of globular clusters. High slopes to the IMF will reduce the mass loss due to stellar evolution that occurs during the lifetime of the cluster (Chernoff & Weinberg 1990). A possible explanation of this apparent discrepancy in the observed and theoretically allowed values of the IMF slope is the action of dynamical evolution including mass segregation and the preferential loss of low mass stars which would act to flatten the IMF (Piotto 1993). Such a change in both the observed and actual mass function has been observed in the Fokker-Planck calculations of Chernoff & Weinberg (1990). It is not clear that this effect could cause the level of change required (reducing \( \alpha \) by 1 or 2) within a Hubble time, although Piotto (1993) concludes that the IMF of globular cluster stars will be ‘strongly modified’ by dynamical evolution. New observations of deep luminosity functions should help to clarify if there is indeed a discrepancy. It may also be instructive to run both Fokker-Planck and N-body simulations with different forms of the IMF to assess the effects that this might have upon the dynamical evolution of a system.

The gas expulsion mechanisms simulated in this paper are meant to provide a first approximation to the most plausible routes by which the residual gas may be expelled. They are, however, highly idealised and, as such, may not fully simulate the interaction of stars and gas during this episode, possibly leading to an over estimate of the disruptive effects of the gas expulsion. More realistic hydrodynamic simulations would need to be run to fully understand the effects of stellar winds, UV flux and supernovae on such a dense medium as that presumably found within very young globular clusters.

For low SFEs large numbers of massive stars are required to expel the residual gas which implies a shallow slope to the IMF. Apart from the problems this poses in later evolution, where low \( \alpha \) mitigate against survival, low SFEs may require the action of the more disruptive gas expulsion mechanisms to clear the cluster of residual gas. In these cases the central densities required of a cluster in order to survive will be extremely high \( (\gtrsim 10^4 M_\odot \text{pc}^{-3}) \). The obvious compromise would be to form clusters with moderate SFEs \((\approx 50\%)\) and mid-range IMF slopes \((\alpha \approx 3)\).

The suggestion has been made that globular clusters have any gas within them routinely removed by the ram pressure as they pass through the disc (Faulkner & Smith 1991). This may well be an adequate method of removing the relatively small amounts of gas that may collect within a cluster when it is lost by stars in the process of stellar evolution. However, it is not clear if this method could remove the large amount of residual gas presumed present and, if it could, that it would be any the less disruptive than the mechanisms described within this paper. It would seem that the removal of any large amount of residual gas by whatever mechanism will have a significant disruptive effect upon a globular cluster.

It has been shown in section 3.4 that clusters that have an initial virial ratio below that of virial equilibrium are considerably more stable than those clusters initially in or above virial equilibrium. The higher survivability of \( Q \approx 0.5 \) globular clusters may provide some evidence in favour of externally induced star formation models of globular cluster formation (such as those of Lin & Murray 1991, Murray & Lin 1992, Shapiro, Clochiatte & Kang 1992, Kumai, Basu & Fujimoto 1993, and Lee, Schramm & Mathews 1995) where stars may well form out of virial equilibrium. The star formation in such models will probably not occur in such simple distributions (and almost certainly not spherically) and so a proper investigation of such models is beyond the scope of this present paper. Although a limit may be placed on the types of star formation that is permitted by the consideration that they must appear as King models within only a few tens of Myr (the observational constraints from Elson, Fall & Freeman 1987).

The possible presence of dark matter with globular clusters could provide a stabilising influence. It has recently been suggested by Heggie & Hut (1995) that up to half of the mass present today in globular clusters may be unobserved. If a significant amount of dark matter were present at the time of the burst of star formation in a globular cluster this would provide additional mass to help reduce the disruptive effects of a gas expulsion episode. However, if, as Heggie & Hut suggest, this unseen mass is composed mainly of stellar remnants, the presence of dark matter may be evidence of a large number of high and intermediate mass stars in the cluster at formation. Such a presence of stars would only exacerbate the problems already posed by stellar mass loss during evolution by implying a low value of the IMF slope.

Of course, all of these problems would not be important if the star formation mechanism were able to produce star formation efficiencies greatly in excess of 50%, perhaps approaching 100%. Our knowledge of both star and globular cluster formation is sufficiently limited that such a possibility cannot be ruled out. Even if this were possible (and it is not clear how any system could achieve such high SFEs) the limitations from simulations not including residual gas expulsion would still hold.

The results presented in this paper can be briefly summarised as follows:

(i) The loss of a significant fraction of a clusters mass during a residual gas expulsion will effect the structure of the cluster significantly, disrupting many clusters that would otherwise be expected to survive. However, a globular cluster may still survive with SFEs as low as 20% if the initial concentration of the cluster is high enough.

(ii) The survivability appears only to depend upon the central density of a cluster at formation, not upon its mass (for a given IMF slope, Galacticentric radius and SFE).

(iii) For moderate SFEs \((\approx 50\%)\) the central density required for survival at low Galacticentric radii is around \(10^3 M_\odot \text{pc}^{-3}\), similar to the central densities seen in giant molecular clouds in the Galaxy today.

(iv) The survival of a globular cluster would appear to be a play-off between the SFE and IMF slope, \( \alpha \). The clusters that survive with attainable central densities (at least they are observed in the Galaxy today) with a moderate \( \alpha \approx 3 \) require SFEs \( \approx 40\% \). Such a value for the SFE is still well in excess of those values observed in...
star forming regions today, but is far more reasonable than previously assumed values approaching 100%.

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