Scaling behavior at the tricritical point in the fermion-gauge-scalar model

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We investigate a strongly coupled U(1) gauge theory with fermions and scalars on the lattice and analyze whether the continuum limit might be a renormalizable theory with dynamical mass generation. Most attention is paid to the phase with broken chiral symmetry in the vicinity of the tricritical point found in the model. There we investigate the scaling of the masses of the composite fermion and of some bosonic bound states. As a by-product we confirm the mean-field exponents at the endpoint in the U(1)-Higgs model, by analyzing the scaling of the Fisher zeros.

1. Introduction

The fermion-gauge-scalar-model (\(\chi U\phi\) model) was suggested as a model for dynamical mass generation in \([1]\). It has a confining phase with dynamical chiral symmetry breaking (Nambu phase). The physical fermion is a bound-state of the fundamental fermion and the scalar: \(F = \phi \dagger \chi\). It is neutral under the (strong) gauge interaction and thus escapes the confinement. Nevertheless, it acquires a mass, which scales to zero at a 2\textsuperscript{nd} order phase transition (PT) \([2,3]\). These features suggest the existence of a new mass generation mechanism, different from the standard Higgs-Yukawa sector.

2. The model

The model is defined by the lattice action

\[
S_{\chi U\phi} = S_\chi + S_U + S_\phi ,
\]

where

\[

S_\chi = \frac{1}{2} \sum_{x} \sum_{\mu=1}^{4} \eta_{\mu x} \bar{\chi}_x \left[ U_{x,\mu} \chi_{x+\mu} - U_{x-\mu,\mu} \chi_{x-\mu} \right] + a m_0 \sum_x \bar{\chi}_x \chi_x ,
\]

Here \(U_P\) is the plaquette product of link variables \(U_{x,\mu}\) and \(\eta_{\mu x} = (-1)^{x_1 + \cdots + x_{\mu-1}}\). The gauge field link variables \(U_{x,\mu}\) are elements of the compact gauge group U(1). The complex scalar field \(\phi\) of unit charge satisfies the constraint \(|\phi| = 1\). The staggered fermion field \(\chi\) of charge one leads to the global U(1) chiral symmetry of the model in the chiral limit, i.e. when the bare fermion mass \(a m_0\) vanishes.

A detailed discussion of the phase diagram and its investigation can be found in \([2,3]\).

3. Scaling in the quenched approximation

A major tool in the examination of universality classes is the determination of critical exponents by finite size scaling analysis. To get experience with this method we investigate the model in the quenched approximation, which corresponds to \(a m_0 = \infty\). This model is known by itself under the names ‘scalar QED’ or ‘U(1)-Higgs model’. In this model the 1\textsuperscript{st} order Higgs-PT ends in a critical point (\(E_\infty\)). The scaling at this point was investigated in \([4]\) along the 1\textsuperscript{st} order line, indicating mean-field exponents. We investigate the scaling along different lines passing through \(E_\infty\).
First of all it is convenient to introduce two reduced couplings (fig. 1):

\( t \) : parallel to the 1st order PT line and

\( h \) : perpendicular to the PT line.

The letter \( t \) and \( h \) have been chosen in correspondence to temperature and external field of a magnetic system.

We now introduce critical exponents for both directions. The correlation length critical exponents \( \nu \) and \( \tilde{\nu} \) are defined by

\[
\xi \propto |t|^{-\nu} |h=0| \quad \text{and} \\
\xi \propto |h|^{-\tilde{\nu}} |t=0|
\]

(5)

(6)

with \( \xi \) being the correlation length. The corresponding exponents connected with the heat capacity are called \( \alpha \) and \( \tilde{\alpha} \).

To understand the connection between \( \nu \) and \( \tilde{\nu} \) we regard the scaling relation

\[
\xi = |t|^{-\nu} F \left( \frac{|h|}{|t|^{\Delta}} \right)
\]

(7)

with the scaling function \( F \) and \( \Delta = \beta + \gamma \) [5].

This transforms with \( \tilde{F}(x) = x^{\nu} F(x^{\Delta}) \) to

\[
\xi = |h|^{-\nu / \Delta} \tilde{F} \left( \frac{|h|^{1/\Delta}}{|t|} \right).
\]

(8)

Assuming \( \tilde{F}(\infty) < \infty \) this means \( \tilde{\nu} = \nu / \Delta \).

The mean-field exponents \( \beta = 1/2, \gamma = 1 \) and \( \nu = 1/2 \) correspond to \( \tilde{\nu} = 1/3 \) and \( \tilde{\alpha} / \tilde{\nu} = 2 \).

Typically, the most precise way to determine the correlation length critical exponent numerically is the measurement of the finite size scaling of the edge singularity in the complex coupling plane (Fisher zeros). From scaling arguments for the free energy we expect for the first zero \( z_0 \):

\[
\text{Im} z_0(L) \Big|_{t=0} = A \cdot L^{-1/\tilde{\nu}}.
\]

(9)

Because all directions non-parallel to the 1st order PT are equivalent, we expect the same exponent \( \tilde{\nu} \) also if we fix \( \beta = \beta_{E_{\infty}} \) or \( \kappa = \kappa_{E_{\infty}} \).

We did a numerical simulation at \( \beta = 0.848 \approx \beta_{E_{\infty}} \) on lattices from \( 4^4 \) to \( 16^4 \) at 5 to 10 \( \kappa \) values with a statistic per point between 32000 and 192000 measurements, each separated by 4 Metropolis sweeps. For the determination of the zeros we use a multihistogram reweighting with 10000 bins.

Fig. 2 shows nice scaling for all lattice sizes with a critical exponent \( \tilde{\nu} = 0.3250(10) \). This exponent is quite near to the mean-field exponent \( \tilde{\nu} = 1/3 \). The small deviations outside the error bars might be due to logarithmic corrections, which are to be expected at a Gaussian fixpoint. We intend to check this by methods as described in [6] for the 4d Ising model.

A similar exponent has also been measured in the SU(2)-Higgs model [7,8], but it was not realized, that this is compatible with a Gaussian fixpoint.

To check this scaling, we have also determined the specific heat and a fourth order cumulant:

\[
\begin{align*}
\langle c_V \rangle &= \frac{1}{6V} \langle (E - \langle E \rangle)^2 \rangle, \\
\langle V_{\text{CLB}} \rangle &= -\frac{1}{3} \langle (E^2 - \langle E^2 \rangle)^2 \rangle.
\end{align*}
\]

(10)

(11)

Here \( E \) can be any linear combination of \( E_L \) and \( E_P \) that picks up also the orthogonal component \( E_{\perp} \) (for def. of \( E_{\perp} \) see [2]). We checked this for \( E_L, E_P \) and \( E_{\perp} \).

We expect the following scaling relation to hold:

\[
\begin{align*}
c_{V, \text{max}}(L) &= A \cdot L^{\tilde{\alpha}/\tilde{\nu}}, \\
V_{\text{CLB,min}}(L) &= A \cdot L^{\tilde{\alpha}/\tilde{\nu} - 4}.
\end{align*}
\]

(12)

(13)
Figure 2. Scaling of the edge singularity for $\beta = 0.848 \simeq \beta_{E_{\infty}}$. The value for $\beta_{E_{\infty}}$ has been taken from [4].

Figure 3. Scaling of the pseudocritical couplings with lattice size. All data have been fitted with one $\kappa_{c}$. Only lattices with $L \leq 6$ have been considered in the fit.

Figure 4. Scaling of the edge singularity for three different $\kappa$. From top to bottom: $\kappa = 0.267$, $\kappa = 0.263 \simeq \kappa_{E_{\infty}}$ and $\kappa = 0.26$. There might be a regular contribution to $c_{V,\text{max}}$, but our data show no indication for this, and so we do not add any constant in our fits.

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The first two observables show nice scaling for all lattice sizes with exponents in good agreement with those determined by the Fisher zeros. Small deviations indicate systematic uncertainties a little larger than the statistical errors.

We also checked the finite size scaling of the real part of the Fisher zeros, as we do for the pseudocritical coupling, determined by the extrema of the cumulants (fig. 3). The value of the shift exponent $\lambda$ is compatible with $1/\tilde{\nu} \simeq 3$.

To check the claim, that $\tilde{\nu}$ is independent of the direction, we also made some runs for fixed $\kappa$. The results are shown in fig. 4. The critical endpoint corresponds to $\kappa = 0.263$. The critical exponent $\tilde{\nu} = 0.3206(15)$ is in good agreement with that for fixed $\beta = 0.848$.

In fig. 4 we also show the scaling for two $\kappa$ a little bit off from the endpoint. $\kappa = 0.267$ is beyond the endpoint, where no critical behavior.
is expected. $\kappa = 0.26$ is on the 1st order line, corresponding to $\bar{\nu} = 0.25$. In both cases clear deviations from a linear scaling can be observed, with the right tendency for increasing lattice size. The straight lines are just plotted to visualize the deviations.

Both observations support the reliability of this finite size scaling method.

4. Spectrum in the Nambu phase at the tricritical point $E$

A rich spectrum is to be expected near the tricritical point $E$, significantly different from that of the second order chiral PT line $NE$. This is discussed in [1,3] and here we want to show some new results. We fix $\kappa = 0.30 \simeq \kappa_E$, because the $\kappa$ of the endpoint (for small fixed $am_0$) turned out to be less dependent on $am_0$ than $\beta$.

As shown in [3] we observe strong finite size effects. The boson mass $am_S$ (scalar bound state of $\phi$'s) shows a pronounced dip at the critical point, which shifts with the volume and $am_0$. It turned out, that those dips fall together, if one plots $am_S$ as a function of the fermion mass $am_F$.

To investigate the scaling, we look for the mass ratio $am_S/am_F$ (fig. 5). The sudden increase of this ratio for small $am_F$ indicates the symmetric phase, with $am_F$ vanishing (up to finite size effects). The nearly constant value of this ratio in the broken phase may indicate that both particles survive in the continuum limit with a mass ratio around 1/2.

We also measure different ‘mesons’ (fermion-antifermion bound states), defined in [2]. The mass of the $\sigma$ particle is very hard to measure, due to its vacuum quantum numbers and the nonvanishing background expectation value connected with this. So we show here the results for the $\rho$-meson (fig. 6). We interpret this data as an indication for $\rho$ surviving in the continuum limit as a resonance.

5. Conclusions

The investigation of the tricritical point in the $\chi U_4\phi_4$-model makes further progress. The mass ratio $am_S/am_F$ approaches a constant value,
supporting the expectation, that the continuum limit is substantially different from that of the NJL model. This strengthens the hope, that the theory might be renormalizable and a model for dynamical mass generation. We hope to get a deeper understanding of the scaling behavior by means of the finite size scaling analysis.

As a by-product we confirm the observation in [4], that the endpoint of the U(1)-Higgs model show mean-field like scaling, probably with logarithmic corrections.

REFERENCES