Comment on Covariant Duality Symmetric Actions

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Abstract

We demonstrate that an action proposed by A. Khoudeir and N. R. Pantoja in Phys. Rev. D53, 5974 (1996) for endowing Maxwell theory with manifest electric–magnetic duality symmetry contains, besides the Maxwell field, additional propagating vector degrees of freedom. Hence it cannot be considered as a duality symmetric action for a single abelian gauge field.

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The action proposed in [1] to describe abelian vector fields in four–dimensional Minkowski space has the following form⁴:

\[ I = -\frac{1}{2} \int d^4 x \left( u^m \mathcal{F}^\alpha_{mn} \Phi_{mp}^\alpha + \Lambda^\alpha_{mp} \Phi_{mp}^\alpha \right), \]

where \( \alpha = 1, 2, \mathcal{L}_{\alpha\beta} \) is the antisymmetric unit tensor,

\[ \Phi_{mp}^\alpha \equiv F_{mp}^\alpha + \mathcal{L}_{\alpha\beta} F_{mp}^\beta, \]

is a self–dual tensor \( \Phi_{mn}^\alpha \equiv \frac{1}{2} \epsilon_{mpq} \mathcal{L}_{\alpha\beta} \Phi^{\beta pq} \) constructed out of the field strengths of two abelian gauge fields \( A_m^\alpha \)

\[ F_{mn}^\alpha = \partial_m A_n^\alpha - \partial_n A_m^\alpha, \quad \mathcal{F}^{\alpha mn} = \frac{1}{2} \epsilon^{mpq} F_{pq}^\alpha, \]

\( u_m(x) \) is an auxiliary vector field satisfying the condition

\[ u_m u^m = -1, \]

and

\[ \Lambda^\alpha_{mn} \equiv -\frac{1}{2} \epsilon_{mpq} \mathcal{L}_{\alpha\beta} \Lambda^{\beta pq} \]

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⁴for details of notation and convention see [1]
is anti–self–dual Lagrange multiplier, since $\Phi_{mn}^\alpha$ (2) is self–dual.

The equations of motion one gets from (1) reduce to

$$\frac{\delta}{\delta \Lambda_{mn}^\alpha} I = 0 \implies \Phi_{mn}^\alpha = 0,$$

(6)

$$\frac{\delta}{\delta A_m^\alpha} I = 0 \implies \varepsilon^{mnpq} \partial_p \Lambda_{nq}^\alpha = 0.$$

(7)

From (6) it follows [2, 3] that the field strength of one of the gauge fields $A_m^\alpha$ is dual to another one. Thus on the mass shell only one of $A_m^\alpha$ remains independent and the latter satisfies the free Maxwell equations of motion (see [2, 3] for details).

At the same time the general solution of Eq. (7) is

$$\Lambda_{mn}^\alpha = \partial_{[m}B_{n]}^\alpha,$$

(8)

where $B_m^\alpha(x)$ are vector fields which, because of anti–self–duality of $\Lambda_{mn}^\alpha$ (5), satisfy the Maxwell equations

$$\partial^m \partial_{[m}B_{n]}^\alpha = 0.$$

(9)

Eq. (9) is the point which demonstrates that the statement of Ref. [1] that on the mass shell $\Lambda_{mn}^\alpha = 0$ fails. It might be so if the action (1) had a local symmetry under which $\Lambda_{mn}^\alpha$ transformed as

$$\delta \Lambda_{mn}^\alpha = \partial_{[m}\phi_{n]}^\alpha - \mathcal{L}^\alpha_{\beta\gamma} \varepsilon_{mn}^{\rho\sigma} \partial_{[\rho} \phi_{\sigma]}^\beta \delta_{\gamma]},$$

(10)

with a vector parameter $\phi_n^\alpha(x)$. Then on the mass shell one might use this symmetry to eliminate $B_m^\alpha$. [Note that simpler transformations of $\Lambda_{mn}^\alpha$ of the form $\delta \Lambda_{mn}^\alpha = \partial_{[m}\phi_{n]}^\alpha$, cannot be considered as a nontrivial local symmetry of the model since they leave the action invariant only if $\phi_n^\alpha(x)$ a priori (because of anti–self–duality of $\Lambda_{mn}^\alpha$) satisfies the dynamical Maxwell equations the same as $\Lambda_{mn}^\alpha$ on the mass shell (9). The action of any theory possesses such kind of trivial invariance]. One can see that if other fields of the model are inert under transformations with $\phi_n^\alpha(x)$ the action is not invariant under (10).

Thus one should try to find appropriate transformations of $u_m$ and $A_m^\alpha$ which would cancel that of $\Lambda_{mn}^\alpha$ in the action. An argument against the existence of such transformations is that for the local symmetry to be present there must be first–class constraints on dynamical variables of the model which generate this symmetry. However there are no relevant constraints in the case at hand. Analogous situation takes place in simpler case of chiral bosons [4, 5, 6] where an action proposed in [6] has a Lagrange multiplier term linear in derivatives of physical fields (like in (1)). There the Lagrange multiplier is a propagating degree of freedom which causes the problem with unitarity of the model of [6] (see [7] for detailed discussion of these points).

By the same reasons the model of [1] contains additional propagating vector degrees of freedom $B_n^\alpha$ and cannot be considered as a covariant version of a duality–symmetric free Maxwell action [2, 3]. A consistent Lorentz covariant way of constructing duality–symmetric actions was proposed in [8], and alternative formulations, based on an infinite number of auxiliary fields, were considered recently in [9].

References