Liquid-Gas Coexistence in ^{208}Pb -Emulsion Interactions at 160A GeV

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Abstract

The charge distribution of multifragments of the ^{208}Pb beam at 160A GeV in nuclear emulsion has been fitted with a power-law. The moments of the resulting nuclear charged fragment distribution provide strong evidence that nuclear matter possesses critical point observables. The values of the critical exponents (γ , β and τ) extracted from the ^{208}Pb beam are compared with the values for the ^{197}Au beams at 10.6A GeV and 1A GeV. These values are very close to those for a liquid-gas system.

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In heavy-ion interactions, the production of multifragments has been frequently associated with a liquid-gas phase transition of nuclear matter. Power-law behavior observed in the charge distribution of the fragments was considered a critical phenomenon occurring in a small system [1]. Recently, the method of scaled factorial moments (SFM) was used to study the fluctuations in the nuclear fragmentations of the ^{197}Au beam at 10.6A GeV [2]. Intermittent behaviour was observed only in the midcentral events and these results are supported by several theoretical studies [3,4], which indicated that critical events give a power-law for SFM versus the bin size. Further progress (theoretically [5-9] and experimentally [6] in the subject of multifragmentation) has indicated that the power-law behavior is only suggestive and not conclusive of critical behavior. Critical behavior is characterized by a finite number of critical indices which characterize the sharp change in the properties of a substance. So far, Gilkes *et al.* [10] have tried to find the critical indices from a liquid-gas coexistence region located in the mul-

tifragmentation reactions of Gold nuclei at $\approx 1\text{A}$ GeV. How good is this technique at higher beam energies for different nuclear systems with different sizes? If the indices values are the same then, in a sense, we have determined the critical exponents of nuclear matter. In order to answer this question, we present here the exclusive data from the determination of critical exponents associated with the multifragmentation of a ^{208}Pb beam at the highest energy 160A GeV *for the first time* and compare these values at low energies with 10.6A GeV [11] and 1A GeV [10] ^{197}Au beams and with a liquid-gas system [6]. The present experiment enables us to find the liquid-gas coexistence region and locate the critical point in the neighborhood of nuclear multifragmentation reactions in the heaviest projectile mass with the highest beam energy available. This also sheds some light on the unanswered questions raised in Ref. [10] about the thermal equilibrium of the system and the fragment multiplicity proportionality to the temperature.

Recently [2] the universality of the fragment production was tested by studying the decays of ^{84}Kr , ^{131}Xe , ^{238}U at low energy $\approx 1\text{A}$ GeV and with the ^{197}Au beam at 10.6A GeV from BNL (Expt. No. 875). For all four beams, the correlation among the average number of intermediate-mass-fragments (IMF), normalized by their maximum values, was plotted against the released number of projectile protons (N_p) normalized by the charge of their respective beams, where $N_p = Z_{\text{beam}} - Z_{\text{bound}}$ [12] and Z_{bound} is the sum of all projectile fragments (PFs) with $Z \geq 2$. All four beams, independent of their collision dynamics, have a general characteristic of a rise and fall passing through their maxima as shown in Fig. 1(b) of Ref. [2] which was first reported in

Ref. [11]. Thus, it is quite appropriate to state that the decaying spectator nuclei are sufficiently equilibrated such that the memory of the entrance channel is lost apart from the deposited energy reflected through the measured value of Z_{bound} . Previously [12] it was shown that the energy deposition in the projectile fragmentation of the ^{197}Au beams at 10.6A GeV and 1A GeV [11] is not much different. The same is true with the ^{197}Au beam at 10.6A GeV and the ^{208}Pb beam at 160A GeV [13]. This one can understand by knowing that a system of lower density has smaller Coulomb barriers for fragment formation and this requires a smaller temperature to breakup into fragments. During the expansion to lower density additional particles are lost and some of the internal energy may be converted into radial flow. Therefore, the low density source should have less mass and excitation energy. The partition space explored in these spectator decays is well described by statistical multifragmentation models [14].

In our Expt. No. EMU11 at CERN SPS, a couple of emulsion stacks were exposed to the ^{208}Pb beam at 160A GeV. By following along the primary tracks of the ^{208}Pb beam, we observed 10^3 nuclear interactions with at least one projectile fragment (PF) of charge $Z \geq 2$ in emulsion, which has the highest spatial resolution, and hence is quite suitable for investigating these nuclear reactions. The charges of all the PFs were determined by a combination of several methods: grain, gap and δ -density and track widths (as discussed previously [2,12,15]), along with their charge resolution. The accuracy in the angular measurements in emulsion is better than 0.1 mrad for angles $\theta \leq 1$ mrad and the angles of all PFs were measured very accurately. Fig. 1 shows the charge determination of PFs from the ^{208}Pb beam up to IMFs of charges $3 \leq Z \leq 20$ by the δ -ray technique. The doubly ionized tracks are identified by grain density which is four times the minimum ionization [15] and are most easily identified in emulsion. The PFs of higher charges, $20 < Z \leq 80$, which are not shown in Fig. 1, were determined by the track widths technique. The IMFs of the ^{208}Pb beam at 160A GeV were considered between $2 \leq Z \leq 20$ in peripheral collisions. We first look at the growth process of fragments for the charge distribution $\ln N(Z)$ as a function of $\ln Z$ for an inclusive data, where $N(Z)$ stands for the total number of projectile fragments corresponding to a given charge $2 \leq Z \leq 20$. This distribution is presented in Fig. 2(a), where a straight line shows an inverse power-law ($Z^{-\tau}$) with an exponent value of

$\tau = -2.17 \pm 0.15$ with $\chi^2 = 2.79$. The error bars in all of the figures are of statistical origin unless otherwise stated. We may point out that although the fragments of charge $2 \leq Z \leq 80$ are formed over a wide range of remnant excitation energies [15], the power-law behaviour is exhibited only over a limited charge range, $2 \leq Z \leq 20$. This is a characteristic of critical phenomenon.

In more violent collisions, the yield of heavier fragments is strongly reduced and for events with charges $3 \leq Z \leq 12$, an exponential charge distribution is found and these reactions are expected to have a high probability of being thermally and chemically equilibrated. We divide the alpha-producing events with PFs of charges $3 \leq Z \leq 12$ into two categories of alphas according to their momenta $P_t < 200$ MeV/c and $P_t \geq 200$ MeV/c as shown in Ref. [15]. Fig. 2(b) shows the normalized probabilities of fragment formation from the two categories of alphas. These distributions are nearly equal. This shows that the thermalized excitation energies are not too different in both types of interactions. The ^{208}Pb beam data is further compared with the ^{197}Au beam at much lower energy of 10.6A GeV and at a lower interaction temperature, but with identical cut-off values of alpha-producing events with momenta $P_t < 200$ MeV/c and $P_t \geq 200$ MeV/c. Their distributions are also shown in Fig. 2(b). Filled and open symbols are for $P_t \geq 200$ MeV/c and $P_t < 200$ MeV/c, respectively. The distributions of the ^{208}Pb and ^{197}Au beams can be expressed through the same exponential function with $\alpha = 0.74 \pm 0.11$ with $\chi^2 = 1.5$. Hence, the data for the ^{208}Pb and ^{197}Au beams scale at different temperatures. Instead of tagging the events of charges $3 \leq Z \leq 10$ with two categories of alphas, we now separate the data sample into peripheral events with projectile proton number $N_p \leq 28$ and mid-central events with $28 < N_p \leq 60$ [12]. The plotted multiplicity distribution of PFs of charge $3 \leq Z \leq 10$ is shown Fig. 2(c). Both the distributions have approximately the same slope and are shifted by a constant factor. Figs. 2(b) and 2(c) indicate that the decaying nuclei are sufficiently equilibrated and leave the charge distribution unaltered; this answers the question raised earlier in Ref. [10]. Later, it is also shown that the presence of prompt particles among the excited remnant fragments does not affect the extraction of exponents γ and β from Eq. (1).

In order to check further on the thermal nature of the PFs produced in the ^{208}Pb beam, one needs to

know the excitation energy of the event or at least a quantity proportional to it. Here we assume that the transverse energy ($E_t = \sum E_i \sin \theta_i$, where E_i and θ_i are the kinetic energy and the polar angle of each fragment, respectively) is proportional to the excitation energy E^* . The excitation energy was found to be independent of the specific target and even of the bombarding energy [16]; it depends upon the value of Z_{bound} or on the projectile proton number (N_p) in an event, where N_p has a monotonic relationship with the impact parameter (b) [15]. The smaller the value of b , the larger the centrality and hence the larger the multiplicity.

Assuming a degenerate Fermi gas, nuclear temperature and excitation energy are related, $E_t \propto T^2$. We used a subsample of 337 events over a broad range of charge $2 \leq Z \leq 20$. Fig. 2(d) depicts $\ln N$ of such fragment emissions when plotted against $1/E_t^{1/2}$ (Arrhenius plot) which gives a straight line. The decay probability of N PFs is fitted with an exponential function $e^{-\alpha/E_t^{1/2}}$. The linear dependence is strongly suggestive that there was a large degree of thermal equilibrium in the decay system of the excited spectators (PFs) at different impact parameters over the measured E_t range. There is further correlation between $E_t^{1/2}$ vs Z_{bound} , which is proportional to the impact parameter (b) and thus to the charged multiplicity (N) [12]. The relation between T and fragment multiplicity (N) justifies that nuclear fragmentation is a thermally-driven phenomenon and this sheds some further light on the questions raised earlier in Ref. [10]. In the following section, we shall try to extract the critical exponents from the liquid-gas coexistence region.

Originally, Campi [9] suggested the behavior of a large percolation lattice which might be relevant to the analysis of multifragmentation data. If the fragments were produced in a system near its critical point, then the moments of the fragment distribution should exhibit features characteristic of critical phenomena. According to Stauffer [6], in percolation theory, the moments of the cluster distribution contain the signals for critical behavior. Fragment distributions are characterized by the k moments of the distribution [8] $S_k = \sum_Z Z^k n(Z)$. Here, $n(Z)$ is the number of PFs of charge Z in an event. The sum runs over all the fragments of $Z \geq 1$, excluding the heaviest one produced in an event, and multiplicity n is the number of charged fragments - Z_{max} . Once we know the Z_{bound} of an event then the projectile number of the spectator proton (N_p) of that

event is fixed irrespective of the produced proton in the intranuclear cascade, which are difficult to separate out from N_p . Here, we assume that all the projectile fragments are associated with multifragmentation. For a finite system, one has to find the region in probability where the power-law behavior of moments of charge distribution holds. If we perform an event-by-event analysis of the moments of the charge distribution of PFs of the ^{208}Pb beam at 160A GeV, the results of the emulsion data suggest that nuclei break-up similar to percolation clusters [9]. In Fig. 3(a) we show the behavior of the second moments $\langle \ln S_2 \rangle$ vs n for the ^{208}Pb beam at 160A GeV. The S_2 value reaches a maximum near $n_c \approx 38$, which is a manifestation of a phase transition for an intermediate value of multiplicity. Near the critical point (n_c), S_2 shows large fluctuations and then decreases very slowly. These fluctuations near the critical point, as expected, can be interpreted as a signal that the event spectrum contains, 'liquid', 'liquid-gas' and 'gas' phases corresponding to the region, below, at and beyond n_c , respectively. The broad maximum in place of a singularity in Fig. 3(a) is due to the finite size of the system and for which we have omitted the largest fragment (Z_{max}) from the liquid side in all of the following calculations.

In the critical region, we shall try to find in an independent manner three critical exponents (γ , β and τ), and determine whether these exponents obey the well known scaling law. Critical exponents for a large system are defined as:

$$(i) S_2 \approx |n - n_c|^{-\gamma}, \quad (ii) Z_{max} \approx |n - n_c|^\beta$$

$$\text{and (iii) } N(Z) \approx Z^{-\tau}, \quad (1)$$

where $|n - n_c|$ is the linear measured distance from critical multiplicity. Eq. (1) can be applied to the thermal as well as the percolation model. In most statistical and percolation models, there are two independent exponents, and the three exponents (γ , β and τ) are not independent, but obey the scaling relation $\tau = 2 + \beta/(\beta + \gamma)$. In a fluid system, S_2 describes the isothermal compressibility which diverges at the critical point but in the percolation method it is the mean size. For the nuclear system, Z_{max} is the order parameter of a phase transition. $N(Z)$ describes the cluster distribution at the critical point. The singular behavior in each part (i), (ii) and (iii) of the Eq. 1 is scale invariant, which is given by a power-law, and holds not just at the critical point but also in the neighborhood of the critical point. For a

finite system the second moment S_2 goes through a maximum (instead of infinity), and τ goes through a minimum near the critical point. Thus, τ and S_2 can be used to identify the critical point in a finite system within a certain nuclear density range. On the average, nuclear density can be determined from the secondary (shower) particles produced in the nuclear reactions. Nuclear density varies from event to event, as does their temperature. The higher the density, the higher the temperature at which the critical point is observed. Hence it is justified to use the inclusive data from a thermalized nuclear system to determine the critical exponents at the point where either maximum in S_2 or a minimum in τ is achieved.

After establishing the thermal nature of PFs, we can now extract the critical exponents from the dependence of the moments of the charge distribution upon the charge multiplicity (n) which is typically associated with the temperature and/or excitation energy as discussed earlier. First we discuss the exponent γ in (i) $S_2 \approx |n - n_c|^{-\gamma}$ from Eq. (1). The value of γ depends on the region of $|n - n_c|$ used since finite size distortions dominate as $|n - n_c| \rightarrow 0$, and the signature of critical behavior vanishes for a large $|n - n_c|$ (i.e. in the mean field regime). We choose a trial value of n_c and plot the quantity $\Delta\gamma = |\gamma_g - \gamma_l|$ in Fig. 3(b) and find that $\Delta\gamma$ is minimum near $n_c \approx 38$ and when we plot $\langle \gamma_{gl} \rangle = (|\gamma_g + \gamma_l|/2)$, this quantity is maximum near $n_c \approx 38$. The distribution of $\langle \ln S_2 \rangle$ vs $\ln |n - n_c|$ is shown in Fig. 3(c) for the liquid side ($n < n_c$) with $\gamma_l = 1.054 \pm 0.256$ and for the gas side ($n > n_c$) in Fig. 3(d) with $\gamma_g = 1.077 \pm 0.078$. The γ values are the same within the statistical errors and the difference between γ_l and γ_g is $< 2.2\%$. The average value of γ is given in Table I. The straight lines are the best fits to the data points. These values differ slightly from that predicted by percolation theory [6]. We use the same value of $n_c \approx 38$ to get the value of β for the distribution of (ii) $Z_{max} \approx |n - n_c|^\beta$ on the liquid side from Eq. (1) and this is depicted in Fig. 4(a) in the plot of $\langle \ln(Z_{max}) \rangle$ vs $\ln |(n - n_c)|$. The value of β is 0.264 ± 0.018 . Remembering that there will always be some uncertainties in experimental data due to contamination from pre-equilibrium particle spectators etc., we find that the average total multiplicity and average second stage multiplicities exhibit strict proportionality over the whole multiplicity range as seen in Fig. 2(c). Hence, for the extraction of exponents (γ, β), the presence of prompt particles does not effect the values. Furthermore, Campi [9] has

pointed out that one can determine the value of the exponent τ from the slope of $\langle \ln S_3 \rangle$ vs $\ln S_2$, where we use the gas branch of the plot and obtain a value of $\tau_g = 2.169 \pm 0.059$, as shown in Fig. 4(b). S_2 and S_3 assume their maxima around the critical value of n_c . Above the critical zone, points fall closer to the origin representing fragments which are generally produced with relatively small impact parameters. The τ value obtained from Fig. 2(a) and 4(a) are the same. The values of β , γ and τ determined by using the above method obey the scaling relation given above.

The values of the critical exponents γ , β and τ are listed in Table I for the ^{208}Pb beam at 160A GeV as well as for the ^{197}Au beams at 10.6A GeV [15] and ≈ 1 A GeV [12]. These values for the beams of different masses and energies are approximately similar within their statistical errors. Exponents from these two experiments are further compared with three-dimensional systems, possessing a scalar order parameter, liquid-gas system [7], percolation [6] and mean field limit of liquid-gas system [17] which are also listed in Table I. We find that our values are close to the point for liquid-gas systems and are significantly away from either the percolation or liquid-gas mean field results. Critical exponents describe the power-law behavior of thermodynamical quantities as the critical point is approached.

We thus conclude that despite the large difference in beam mass (size effect), beam energies, detection technique, different critical multiplicity (n_c) cuts and different targets, nearly identical exponent values are observed and they are the critical exponents of nuclear matter. The thermal nature of N fragments is illustrated by the nearly universal linear Arrhenius plot observed in the ^{208}Pb projectile at 160A GeV. The technique used here for the extraction of critical exponents is reasonably valid and should be tested with other nuclear systems.

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Figure Captions

Fig. 1 Frequency distribution of δ -rays per $100\mu\text{m}$ of the PFs of charge $3 \leq Z \leq 20$ in nuclear events of ^{208}Pb at 160A GeV.

Fig. 2 (a) Relationship between $\ln N(Z)$ and $\ln Z$. Solid line represents a polynomial fitting, $Z^{-\tau}$. (b) A plot of $\ln[N(Z)/N_{ev}]$ versus charge, Z for the ^{208}Pb and ^{197}Au data. The data points of ^{197}Au beam are displaced along x-axis for the sake of clarity. Filled symbols are for the data points with $P_t \geq 200$ MeV/c, whereas open symbols are for those with $P_t < 200$ MeV/c. Solid line shows an exponential fit, $e^{-\alpha Z}$ through the data points. (c) Normalized charge distribution of PFs with $3 \leq Z \leq 10$ in events with: (i) $N_p \leq 28$ (filled circles) and (ii) $28 < N_p \leq 60$ (open circles). Dotted line is an exponential fit to each data set (see text). (d) A plot of $\ln N$ as a function of $E_t^{-1/2}$. Solid line is an exponential fit to the data (see text).

Fig. 3 (a) Variation of $\langle \ln S_2 \rangle$ with n for ^{208}Pb at 160A GeV. (b) A plot of $|\gamma_g - \gamma_l|$ or $\langle \gamma_{gl} \rangle$ vs n . The values of $|\gamma_g - \gamma_l|$ are multiplied by a factor of 3.5 to accommodate the data in this figure. Solid and dotted curves are free hand drawn through the respective data points for $|\gamma_g - \gamma_l|$ and $\langle \gamma_{gl} \rangle$. The variation of $\langle \ln S_2 \rangle$ as a function of $\ln |(n - n_c)|$ for the ^{208}Pb events in: (c) liquid zone and (d) gas zone. The solid lines correspond to the best fits.

Fig. 4 For ^{208}Pb data: (a) $\langle \ln(Z_{max}) \rangle$ as a function of $\ln |(n - n_c)|$ in the liquid phase and (b) $\langle \ln S_3 \rangle$ as a function of $\ln S_2$ in the gas phase. The solid lines are the best fits to the data points. Error shown in Fig. 4(a) is the standard deviation from the mean value.

Table I. Critical multiplicity and exponents for the ^{208}Pb and ^{197}Au projectile fragments and other three dimensional systems.

Parameter	This Expt.	Ref. [12]	Ref. [10]	Liquid-gas	Percolation	Liquid-gas mean field
n_c	38 ± 1	22 ± 1	26 ± 1	—	—	
γ	1.066 ± 0.082	1.17 ± 0.19	1.4 ± 0.1	1.23	1.8	1.0
β	0.264 ± 0.018	0.241 ± 0.011	0.29 ± 0.02	0.33	0.41	0.5
τ	2.17 ± 0.15	2.038 ± 0.056	2.14 ± 0.06	2.21	2.18	2.33

Fig. 1

Fig. 2

Fig. 3

Fig. 4