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**Separation energy dependence of hole
form factors**

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Abstract

Form factors of fragmented hole states are studied within the quasiparticle-phonon model, using the inhomogeneous equation method. Validity of this method is successfully checked by comparison with coupled equation solutions in schematic vibrational model cases. A systematic investigation of form factors is performed for neutron and proton hole states in the valence and first inner shells of ^{208}Pb . Large fluctuations of form factor radii are observed for individual levels superimposed on a general increase with separation energy. Average characteristics are introduced for groups of levels, namely the mean form factors, summed source terms and correction potentials, and their behaviour is presented. The role of the relative values of the interaction radius parameter and binding well radius is discussed in details.

1 Introduction

Fragmentation of hole or particle strengths has been extensively studied in the last 20 years. Most results rely on the analysis of pick-up or stripping reactions performed with unpolarized and also polarized beams. Proton hole states have more recently been studied with $(e, e'p)$ reactions with high resolution energy beams. In this latter case form factors of well separated, rather low-lying levels have been measured over a significant range of momentum transfers.

The behaviour of form factors of the many fragments which may be located several MeV away from the corresponding quasiparticle states is not known. The standard well depth procedure has been generally used in order to provide a correct slope of the wave function tail at large radius. It has long been recognized that such an approximation may induce significant errors on the extraction of spectroscopic factors. Instead, it has been suggested that the coupling with surface vibrations is mostly responsible of the fragmentation process, would better be taken into account through a modification of the

effective potential well radius [1] or by an additional surface potential the depth of which is adjusted to the separation energy.

Form factor theories were developed long ago [2] as a more general approach of nuclear structure calculations but the complexity has prevented their use for practical purposes, excepted in very few cases. I. Hamamoto [3] has described particle-vibration coupling states in odd nuclei around ^{208}Pb , by expanding the total wave function in terms of products of wave functions of the odd particle and of the ^{208}Pb core states. The coupling strength was adjusted to reproduce the experimental levels by solving a set of coupled differential equations. The admixed single-particle wave function of each weak coupling state resulting from such calculation is bound at the correct energy. Formally, the coupling strength being known the admixed wave functions are solutions of inhomogeneous differential equations obtained from the Schrödinger equation. Austern [4] has proposed to skip the solution of the coupled equation set and concentrate on the prediction of the sole shape of the inhomogeneous term via a structure calculation. The right hand side of such equation would not change much by using pure single-particle radial wave functions instead of radial functions resulting of coupled equations.

In the present paper, we address the question of form factors mainly in the framework of the quasiparticle-phonon model, hereafter QPM. This model has been extensively used to study the fragmentation or spreading of hole and particle strengths in many nuclei and for subshells not too far away from the Fermi level. It thus allows a rather systematic investigation of radial form factors. We will especially emphasize underlying mean features which may be useful for analyzing experimental results. Hole states in the doubly magic nucleus ^{208}Pb are considered for practical examples.

The general formalism and the tests of the inhomogeneous equation method performed with the vibrational model are presented in Sect. 2. The method and approximations used for calculating form factors in QPM are described in Sect. 3. The results on neutron and proton hole states in ^{207}Pb and ^{207}Tl are presented and discussed in Sect. 4. Summary and conclusion are given in Sect. 5

2 Form factors

2.1 Formulation

The Hamiltonian of the system of $A \pm 1$ nucleons is separated into three parts, as given by

$$H = H_A + H_{p(h)} + H_{int} \quad (1)$$

where H_A is the Hamiltonian for the internal motion of the core, $H_{p(h)}$ is that of the particle (hole) outside the core and H_{int} expresses the interaction between the odd particle (hole) and the core. In this framework, the total wave function is given by

$$|\Psi_J^M\rangle = \frac{1}{r} \left\{ \sum_{l_i j_i \lambda_i} i^{l_i} R_{\lambda_i l_i j_i}(r) [\mathcal{Y}_{l_i j_i} \Phi_{\lambda_i}]_J^M \right\} \quad (2)$$

where Φ_{λ_i} describes the wave functions of core states and $\mathcal{Y}_{l_i j_i}$ is a spin-angle function. $R_{\lambda_i l_i j_i}(r)/r$ is the form factor of the odd particle or hole. For the sake of clarity, $R_{\lambda_i l_i j_i}(r)$ will be named the radial function of the odd quasiparticle.

The radial functions are solutions of the following set of coupled equations

$$\begin{aligned} & \left\{ \frac{d^2}{dr^2} - \frac{l_k(l_k + 1)}{r^2} - \frac{2\mathcal{M}}{\hbar^2} V_{l_k j_k}(r) + \frac{2\mathcal{M}}{\hbar^2} (E - \eta \hbar \omega_{\lambda_k}) \right\} R_{\lambda_k l_k j_k}(r) \\ &= \frac{2\mathcal{M}}{\hbar^2} \sum_{i \neq k} i^{l_i - l_k} \langle [\mathcal{Y}_{l_k j_k} \Phi_{\lambda_k}]_J^M | H_{int} | [\mathcal{Y}_{l_i j_i} \Phi_{\lambda_i}]_J^M \rangle R_{\lambda_i l_i j_i}(r) \end{aligned} \quad (3)$$

Here $\eta=1$ or -1 for a particle or a hole state respectively. The radial functions $R_{\lambda_k l_k j_k}(r)$ and $R_{\lambda_i l_i j_i}(r)$ entering eqs. (3) have no definite number of nodes.

For each predicted state, we are interested in the form factor $R_{0l_1 j_1}(r)/r$ of the quasiparticle with no excitation of the core ($\Phi_{\lambda_1} \equiv \Phi_0$), hereafter denoted by $R_{l_1 j_1}(r)/r$. As the right hand side of eqs. (3) becomes negligible at large distance, it is clear that the form factor of each fragment will have the correct asymptotic shape. Following the suggestion of ref. [4], approximate radial functions R^a of each level of interest can be conveniently obtained by solving an inhomogeneous equation

$$\left\{ \frac{d^2}{dr^2} - \frac{l_1(l_1 + 1)}{r^2} - \frac{2\mathcal{M}}{\hbar^2} V_{l_1 j_1}(r) + \frac{2\mathcal{M}}{\hbar^2} E \right\} R_{l_1 j_1}^a(r) = S(r) \quad (4)$$

where the source term $S(r)$ is

$$S(r) = \frac{2\mathcal{M}}{\hbar^2} \sum_{i \neq 1} i^{l_i - l_1} \langle [\mathcal{Y}_{l_1 j_1} \Phi_0]_J^M | H_{int} | [\mathcal{Y}_{l_i j_i} \Phi_{\lambda_i}]_J^M \rangle x_i \bar{R}_{n_i l_i j_i}(r) \quad (5)$$

The energy eigenvalues, the core states together with the amplitudes x_i of each one quasiparticle-phonon ($1qp \otimes 1ph$) component must be first calculated within a specific nuclear model. Eqs. (4,5) are very similar to eq. (3) with $k = 1$, excepted that the functions $\bar{R}_{n_i j_i}(r)$ in the source term $S(r)$ are the normalized solutions of the Schrödinger equation:

$$\left\{ \frac{d^2}{dr^2} - \frac{l_i(l_i + 1)}{r^2} - \frac{2\mathcal{M}}{\hbar^2} V_{l_i j_i}(r) + \frac{2\mathcal{M}}{\hbar^2} E_{j_i} \right\} \bar{R}_{n_i l_i j_i}(r) = 0 \quad (6)$$

where $n_i = 1, 2, \dots$

2.2 Test of the inhomogeneous equation results with the vibrational model

The vibrational model has often been used to explain the splitting of valence hole or particle states in odd nuclei, due to the coupling with a few low-lying collective states of the even core. Such one-phonon states are described by their energy $\hbar\omega$, their spin, parity and β value. The interaction Hamiltonian is chosen as

$$H_{int} = K(r) \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda}^{\mu*}(\theta, \phi) \quad \text{with} \quad K(r) = R \frac{dV(r, R)}{dR}$$

$$\text{and} \quad \alpha_{\lambda\mu} = \frac{i^{-\lambda} \beta_{\lambda}}{\sqrt{2\lambda + 1}} [c_{\lambda\mu}^{\dagger} + (-1)^{\lambda+\mu} c_{\lambda-\mu}] \quad (7)$$

where $c_{\lambda}^{\dagger}$ is the phonon creation operator, V is the central part of the Woods-Saxon binding potential and R is the well radius.

The energy eigenvalues of the few resulting levels in the odd nuclei together with the amplitudes of each $1qp \otimes 1ph$ component are easily obtained by solving the linear set of equations

$$(E_{j_1} - E) x_1 + \sum_{i=2}^N \langle \hat{0} | a_{j_1 m_{j_1}} H_{int} [a_{j_i}^{\dagger} c_{\lambda_i}^{\dagger}]_J^M | \hat{0} \rangle x_i = 0 \quad (8)$$

$$\begin{aligned} \langle \hat{0} | [a_{j_k}^\dagger c_{\lambda_k}^\dagger]_J^{M^\dagger} H_{int} a_{j_1 m_{j_1}}^\dagger | \hat{0} \rangle x_1 + (E_{j_k} + \eta \hbar \omega_{\lambda_k} - E) x_k = 0, \\ k = 2, \dots, N \end{aligned} \quad (9)$$

where $a_j^\dagger$ is the single-particle creation operator. The exact radial functions are obtained by solving the set of corresponding coupled equations (3).

Calculations have been performed for schematic cases describing neutron $1i_{13/2}$, $1h_{9/2}$ and $2f_{7/2}$ hole strength fragmentation in ^{207}Pb . Up to four levels among the 2^+ , 3^- , 4^+ and 5^- first levels in ^{208}Pb have been taken into account with standard β_λ values as given in refs. [3,5], and up to five valence state configurations. The separation energies obtained by solving Eqs. (8,9) or the coupled equations (3) are, as expected, nearly identical. This is also the case for the amplitudes of the different components.

We have checked that for each level, the norm of the inhomogeneous equation solution agrees (within typically 1%) with the corresponding squared amplitude x_1^2 . Approximate and exact form factor shapes can be conveniently compared via the deduced mean squared radii, hereafter named R_{FF} . The main conclusion resulting from all test calculations is that the radii obtained with the inhomogeneous equation approach reproduces very well (typically within few 0.1%) the radii predicted with the coupled equations. We notice that this remains true not only for the states with small admixture of one-hole strength, but also for the quasihole state with the largest part of the strength. Varying the assumed coupling strength β_λ and the number of components does not change the above conclusions.

3 Form factors in the QPM model

The inhomogeneous equation method has been extended to calculation of form factors in the framework of the QPM model.

3.1 Formulation

The QPM model and its related approximations have been described by Soloviev [6] and several review papers [7,8]. It is sufficient to say for the present purpose that in terms of the nucleon variables the model Hamiltonian contains average fields for protons and neutrons, proton-proton and neutron-neutron pairing interactions of the BCS type and separable multipole and spin-multipole interactions in the particle-hole channel. As it follows from the

consistency relation between a small deviation of a nuclear density and that of an average field [9], the long-range particle-hole interaction has to be of a surface character. So, for a spherical nucleus the radial shape of a separable particle-hole interaction is proportional to dV/dr where V is the central part of the Woods-Saxon well for the quasiparticle. The parameters of the well are given in Table (1). These parameters have often been used [10–12] to describe low-lying spectra, giant resonances, deep-hole and high-lying single-particle states in nuclei around the lead region. Other parameters of the model, in particular residual interaction constants, are fitted in each nucleus to describe experimental data about low-lying collective states and giant resonances. The prescriptions used to determine these parameters can be found in [7].

	r_0 (fm)	V_0 (MeV)	V_{SO} (MeV)	a (fm)	r_c (fm)
Neutrons	1.26	-44.83	34.00	0.63	–
Protons	1.24	-60.3	33.54	0.63	1.24

Table 1
Parameters of Woods-Saxon wells

After series of transformations, the QPM Hamiltonian takes the form of eq. (1) where the core term H_A is the Hamiltonian of non-interacting RPA-phonons. In the present paper, the simplest version of the QPM for odd nuclei is used and only one-phonon states are taken as excited states of the core.

The wave function of an odd nucleus in the coordinate space is similar to that in the vibrational model. The core wave functions Φ_{λ_i} are the one-phonon wave functions $|Q_{\lambda_i}^+\rangle$ calculated within the RPA with separable multipole or spin-multipole interactions. The matrix element $\frac{1}{\sqrt{2}}\Gamma(Jj\lambda i)$ of H_{int} within the QPM is the following

$$\langle \bar{R}_{LJ} [\mathcal{Y}_{LJ}\Phi_0]_J^M | H_{int} | [\mathcal{Y}_{LJ}\Phi_{\lambda_i}]_J^M \bar{R}_{Lj} \rangle = \sqrt{\frac{2\lambda+1}{2J+1}} \langle J || \frac{dV}{dr} Y_\lambda || j \rangle \frac{1}{\sqrt{2\mathcal{N}^{\lambda i}}} \quad (10)$$

Here, the number i numerates different one-phonon states with the same angular momentum λ according to their excitation energies. The value $\mathcal{N}^{\lambda i}$ is calculated knowing the RPA structure of the one-phonon state Φ_{λ_i} . It is a microscopic counterpart of the phenomenological dynamical deformation β_λ multiplied by the nuclear radius R . For the lowest vibrational states the following relation is valid: $\beta_\lambda R \simeq (2\lambda+1)/\sqrt{2\mathcal{N}^{\lambda 1}}$.

The inhomogeneous equation needed to calculate the radial form factor of the one-hole component of any fragment of a hole strength is given by eq. (4) with

the following expression for the source term $S(r)$:

$$S(r) = \frac{2\mathcal{M}}{\hbar^2} \frac{dV}{dr} \sum_{\lambda i j} x_j^{\lambda i}(J\nu) \langle J || Y_\lambda || j \rangle \left(\frac{2\lambda + 1}{2J + 1} \right)^{1/2} \frac{1}{\sqrt{2\mathcal{N}^{\lambda i}}} \bar{R}_{nlj}(r) \quad (11)$$

where $x_j^{\lambda i}(J\nu)$ stands for the amplitude of the corresponding $1h \otimes 1ph$ component in the wave function of the ν -th state with momentum J of an odd nucleus¹. The functions $\bar{R}_{nlj}(r)$ are pure single-hole radial functions calculated in the chosen Woods-Saxon potential.

It is easy to calculate the energies and structures of excited states of an odd nucleus, using a separable residual interaction and simple model wave functions which contain only one-hole and $1h \otimes 1ph$ components. For example, the energy eigenvalues are roots of an algebraic secular equation [7]. The main difference of the present version of QPM and the vibrational model is that the terms H_A and H_{int} of the Hamiltonian (1) are determined by the same residual interaction. So, the phonon characteristics $\hbar\omega_{\lambda i}$ and $\mathcal{N}^{\lambda i}$ are not taken from experiment or some phenomenological estimations but are calculated microscopically in the framework of the RPA with the separable residual interaction. The same interaction determines H_{int} and therefore appears in $S(r)$. The expression (11) for $S(r)$ is also valid for more complicated model wave functions of an odd nucleus which include terms like $1h \otimes 2ph$, $1h \otimes 3ph$ etc. if anharmonic corrections and/or Pauli principle corrections are not taken into account.

Eqs. (10,11) are valid for nuclei where pairing correlations vanish. In nuclei far from closed shells the term $H_{p(h)}$ has to include the pairing field as well. The problem how to include the BCS pairing in nuclear structure calculations within the coordinate space is quite hard and longstanding. To avoid these difficulties one may follow the standard empirical way by changing single-particle (hole) state occupation numbers only. In this case, the phonon properties have to be calculated from the quasiparticle RPA equations and the superfluid factor $v_{Jj}^{(\pm)} = u_J u_j \pm v_J v_j$ (u, v are the Bogoliubov transformation coefficients) has to be included in the expression of the interaction matrix element $\Gamma(Jj\lambda i)$ and hence in the channel amplitudes in eq. (11).

The important ingredient of the calculation of single-particle (hole) strength fragmentation within the QPM is the proper choice of the basis of complex components ($1h \otimes 1ph$ in our case). Formally, this basis has to be formed by the whole set of bound and quasi-bound single-particle states in the Woods-Saxon well and all phonons that can be generated within this set of single-particle

¹ The value $x_j^{\lambda i}(J\nu)$ is equal to the product of the one-hole amplitude $C_{J\nu}$ and $1h \otimes 1ph$ amplitude $D_j^{\lambda i}(J\nu)$ in the standard QPM notation [7]

states. But in practice the basis has to be truncated. Usually, the $1h \otimes 1ph$ components in the energy interval $[0, E_J - 10]$ MeV are taken into account in calculating the fragmentation of the single-hole state with energy E_J . More than 90% of the single-hole strength is exhausted in this energy interval as a general rule. For the valence subshells all possible complex configurations can be taken into account. For deep hole states with $E_J \leq -15$ MeV an additional truncation of the basis is needed because of a very large number of complex states. This truncation is based on the values of $\Gamma(Jj\lambda i)$. In principle, this procedure decreases the number of states among which the single-particle strength is shared and, hence, decreases the fragmentation. But in fact, the weakly coupled configurations mainly produce states with negligibly small one-hole amplitudes, so that the truncation at the level $\Gamma(Jj\lambda i) \geq 0.01\Gamma_{max}(Jj\lambda i)$ is enough.

3.2 Truncation of the source term and test of approximations in QPM

All calculations discussed in the following have been performed with the FFQPM [13] code. The wave function of every state has several hundred components and formally all of them have to be included in the source term $S(r)$ of eq. (4). Of course, this is intractable and we have studied the dependence of the form factor properties on the number of terms in the right-hand side of the inhomogeneous equation. The channels were ordered according to the coefficients multiplying the single-hole radial functions $\bar{R}_{nlj}$ in eq. (11). The values of the channel amplitudes are determined mainly by two factors: an amplitude $x_j^{\lambda i}$ of the particular component $1h \otimes 1ph$ and its “phonon factor” $1/\sqrt{N^{\lambda i}}$. The latter is especially large for the low-lying collective phonons, and due to it, the configurations built with these phonons give sizeable contributions to the right-hand side even if they correspond to very small values of the amplitudes $x_j^{\lambda i}$.

The form factor norm has to coincide with the hole amplitude squared of the wave function of the state. The number of channels affects the norm, but one additional problem is the proper determination of the absolute energy scale of the fragments as calculated by the QPM code PHOQUS [14]. In the QPM calculations the scale is given by the phenomenological energies of the single-hole states in the valence shell (they have been fitted in [12]). A small energy shift (200-300 keV) of all fragments with the same orbital momentum and parity was needed in the left hand side of eq. (4) in order to recover the correct norms. The value of the shift depends only slightly on the number of channels in $S(r)$, if this number is sufficiently large. The inclusion, in $S(r)$ of a particular state, of all the channels with absolute values of amplitudes larger than 0.01 of the maximal amplitude seems to be appropriate in all cases. This condition is reasonably fulfilled in the present calculations using fifty channels.

Systematic calculations have been performed for hole subshells in ^{207}Pb with the chosen basis of complex configurations in order to evaluate the role of the approximations made and the conditions for consistent predictions of form factor shapes. One may notice, following Eq. (4), that these shapes only depend on the energy and the corresponding source term shapes, independently of a global amplitude. In the remaining part of the paper, we will only refer to normalized form factors $F(r)$, as used in reaction calculations. Most tests have been performed on hole state fragments with quantum number $n = 1$. The deduced form factors can thus be conveniently characterized by their separation energy E_{sep} and mean squared radius R_{FF} .

A most important conclusion of all the above tests is that the fragment form factor radii don't change noticeably with the source term truncation and by less than 1% with typical energy shifts.

4 QPM form factors of neutron and proton hole states in ^{207}Pb and ^{207}Tl

A systematic investigation of form factors has been performed on neutron hole states in the valence and first inner shell of ^{208}Pb . A few calculations have also been performed for proton hole states. The standard QPM parameters have been used in most cases.

With the large configuration space, even valence subshell strengths are fragmented into a very large number of levels. However, one must emphasize that a one-to-one correspondence between theoretical and experimental levels can be established only for a few, relatively low-lying states. At higher excitation energy theory can predict only some gross features of hole strength distributions. It is not only due to a quite simple form of the model wave functions but also to uncertainties in the model parameters. Nobody knows the model parameters which would predict for example a particular level energy within less than 100 keV. This latter value is already larger than average distances between calculated states at $E_x \sim 8$ MeV. Changing the model parameters within natural limits thus produces drastic changes in energy and structure of a particular state at this excitation energy. The same effect appears because of minor changes in the model basis of complex configurations. However the large scale (of the order of several hundred keV) features of strength distributions is much more stable. This was, by the way, the reason to use the strength function method in calculating the fragmentation of deep hole or high-lying single particle states [6,7]. Some averaging procedure is obviously also needed for form factors. Moreover, if systematic calculations of form factor radii are achievable for valence levels, such calculations become intractable for deep hole states. This is also the case for the large number of level form factors

with the purpose of using them in reaction calculations.

We will discuss in sect. 4.1 systematic results on $i_{13/2}$ and $h_{9/2}$ neutron hole QPM levels in some details. Mean semi-empirical form factors will be presented in sect. 4.2 and averaged properties in sect. 4.3.

4.1 $i_{13/2}$ and $h_{9/2}$ QPM levels

Form factor radii of all $i_{13/2}$ and $h_{9/2}$ levels with spectroscopic amplitudes larger than 0.01 are shown in Fig. 1. It is striking that they exhibit rather strong fluctuations superimposed over a general increase with separation energy E_{sep} . Of the two groups of states identified in the figure for each hole state, the one with the smallest hole amplitudes exhibits the largest fluctuations. In this group the levels have one or a few dominant ($1h \otimes 1ph$) components. The form factor radii of the two groups have been fitted separately by quadratic functions in order to extract the underlying variation with E_{sep} . The corresponding curves are similar for the two groups. The variation with E_{sep} is clearly larger for $h_{9/2}$ than for $i_{13/2}$ levels (see Fig. 1). Calculations have also been performed with higher values of the minimal interaction matrix element $\Gamma(Jj\lambda i)$. This reduces the number of levels and increases the average value of one-hole amplitudes of every state. The fluctuations of radii become weaker but the overall tendency is not changed.

It is worthwhile to notice that in spite of the large fluctuations the overlap of most renormalized radial functions R_{lj} with single-hole wave function $\bar{R}_{nlj}(r)$ is generally larger than 0.98.

The respective role of E_{sep} and the detailed structure of the levels on radial form factors and source terms is illustrated in Fig. 2a-b for three typical $h_{9/2}$ levels.

The lowest level is the $1h_{9/2}$ quasihole and concentrates the main part of the hole strength ($\sim 86\%$). High lying levels are mainly of $1h \otimes 1ph$ type. The two fragments selected in Fig. 2 have nearly identical separation energies. Their form factors are shifted to larger radius compared with the first quasihole level, but this shift is much smaller for the second fragment. The structure of the first fragment is dominated by one $1h \otimes 1ph$ component. Nevertheless three channels with amplitudes of the same order of magnitude and the same signs give the largest contributions to the source term. The wave function of the second fragment is dominated by two $1h \otimes 1ph$ components with amplitudes of opposite signs. The amplitudes of the corresponding items in the source term are also of the same order and with opposite signs. Moreover, a hole state of one of the components is $3p_{1/2}$ hole with two nodes. As a result the source term of that fragment does not exhibit the typical surface peaked shape observed

Fig. 1. Form factor radii for $1i_{13/2}$ and $1h_{9/2}$ levels and groups of levels in ^{207}Pb calculated with QPM standard parameters. Dots: Levels with amplitudes $0.01 < |c| < 0.03$. Large dots: Levels with $|c| > 0.03$. Squares: One MeV wide groups of levels (see text). Large crosses: Pure hole state. The quadratic fits obtained for the low amplitude levels, the larger amplitude levels and the groups are indicated by full lines, dotted lines and dashed lines respectively (see text).

for the first level and many other fragments and R_{FF} is drastically anomalous as well. This example gives an insight into the nature of so strong dispersion of R_{FF} values.

Eq. (4) can be reduced to an homogeneous equation by adding to the normal Woods-Saxon potential a correction potential Δ_{pot} :

$$\left\{ \frac{d^2}{dr^2} - \frac{l_1(l_1 + 1)}{r^2} - \frac{2\mathcal{M}}{\hbar^2}(V_{l_1 j_1}(r) + \Delta_{pot}(r)) + \frac{2\mathcal{M}}{\hbar^2}E \right\} R_{l_1 j_1}^a(r) = 0 \quad (12)$$

$$\Delta_{pot}(r) = \frac{\hbar^2}{2\mathcal{M}} S(r)/R_{l_1 j_1}(r) \quad (13)$$

Fig. 2. Form factors , source terms and correction potentials for three $h_{9/2}$ QPM levels in a), b), c). Full line: Quasi-hole level at $E_{sep}=10.55$ MeV with $c=0.86$; dashed line: Level at $E_{sep}=15.7$ MeV with $c=0.0145$; dotted line: Level at $E_{sep}=15.8$ MeV with $c=0.0575$.

Fig. 2c shows the shape of the correction potential Δ_{pot} as function of E_{sep} for the quasi-hole level and the two high-lying fragments. The correction potential shape is drastically anomalous for the second fragment.

As a general rule, most levels especially with relatively large hole amplitude have surface-peaked source terms and correction potentials, both with quite stable position of their maximum.

4.2 Mean semi-empirical form factors

A mean form factor shape has been attributed to a group of levels, by solving, at the centroid energy of that group strength, the inhomogeneous equation whose source term is the sum of the source terms of the contributing levels. Form factor radii have been calculated for successive groups of levels within one MeV energy slices and their variation with E_{sep} has been parametrized. As shown in Fig. 1, the behaviour of the corresponding curve is very similar to those obtained for individual level fits. The fluctuations around the quadratic fit are even smaller than those of the levels with the largest amplitudes. Note that in the case of the mean form factor the contribution of the large number of levels with very small amplitudes is automatically included.

The variation of form factor radii with E_{sep} over a small energy interval is much smaller than the fluctuations from level to level. The summed form factor over a group of levels is thus expected from Eq. (4) to exhibit nearly the same shape as the above described mean form factor. An additional evaluation of the validity of the mean form factor approach has been performed on pick-up cross-sections. In a first approximation, one may assume that the differential cross section per nucleon of each level within a group (thus neglecting the variation with E_{sep}) depends linearly on the corresponding form factor radius. Under this condition, the cross section summed over the individual levels is well approximated by the cross section calculated with the mean form factor and the summed spectroscopic factors. This assumption has been successfully checked, within less than 5%, in the case of the (d, t) reaction at $E_d = 200$ MeV.

4.3 Average properties

Systematic calculations have also been performed for $h_{11/2}$ and $g_{7/2}$ neutron deep hole states and for $h_{11/2}$ and $g_{7/2}$ proton valence hole states. For deep hole states, fragmentation occurs in many more states than for the valence strengths, so that only mean form factors of groups of levels have been calculated. Among hole states with n larger than one, only $2f_{7/2}$ neutron hole levels have been investigated, as an example.

4.3.1 Form factor radii

The form factor radii of the quasihole level and of the groups of levels have been fitted by a quadratic function

$$R_{FF} - R_h = a(E_{sep} - E_{sep-h}) + b(E_{sep} - E_{sep-h})^2 \quad (14)$$

Here, E_{sep-h} and R_h are respectively the separation energy and form factor radius of the pure hole state. The deduced parameters a and b are summarized in table (2) for neutron and proton hole states. The behaviour of the form factor radii with E_{sep} , as given by these parameters, are very similar for neutron and proton holes with the same type of spin-orbit coupling, either $j_+ = \ell + 1/2$ or $j_- = \ell - 1/2$. The dependence on E_{sep} is strikingly different for the two different hole state types. It is much larger for hole states with $j_- = \ell - 1/2$. Mean form factor radii decrease slightly with E_{sep} for the $2f_{7/2}$ strength. The relative location of the node and the source term is critical in this result, and for the variation of pick-up cross-sections. The latter ones increase somewhat with E_{sep} , contrary to the form factor radius.

	E_{sep-h} (MeV)	R_h (fm)	a (fm MeV ⁻¹)	b (fm MeV ⁻²)
$n^{-1} i_{13/2}$	9.2	6.52	0.079	-0.005
$n^{-1} h_{11/2}$	15.6	6.23	0.091	-0.004
$p^{-1} h_{11/2}$	9.2	6.09	0.085	-0.005
$n^{-1} h_{9/2}$	11.2	5.95	0.137	-0.007
$n^{-1} g_{7/2}$	18.7	5.67	0.124	-0.004
$p^{-1} g_{7/2}$	11.9	5.53	0.120	-0.005

Table 2
Parameters a and b fitting form factor radii (see section 4.3.1)

4.3.2 Source term

As discussed in section 4.1, the source term may exhibit rather different shapes for individual levels. Summed source terms of different groups of levels have surface peaked shapes with rather stable positions and widths. They exhibit very similar shapes for the different hole states. This behaviour is qualitatively expected as the source term shape depends mainly on the interaction radial shape dV/dr , while all channel radial functions with $n = 1$ contributing to $S(r)$ vary smoothly in this radial region.

We find it interesting to fit the summed source term calculated over the whole set of levels of each type with a first derivative Woods-Saxon shape. Typical results obtained for the $1h_{9/2}$ neutron hole levels and groups are shown in Fig. 3. The fits are rather good except toward the lower radius values. Very good fits are achieved by multiplying with a second degree polynomial. Such fits give a same value for the source term maximum ($1.21 \pm 0.002 A^{1/3}$ fm) and for the surface thickness (0.57 ± 0.005 fm), for all the four studied neutron holes.

Fig. 3. Fits of the summed $1h_{9/2}$ source term. Full line: QPM source term; Dotted line: Fit with a first derivative Woods-Saxon shape; Dashed line: Fit with a first derivative Woods-Saxon shape multiplied by a second degree polynomial.

It is worthwhile to notice that at least qualitatively, the source term gives a positive or a negative contribution to the fragment form factor radius for values of r respectively larger and smaller than the pure hole form factor radius. This contributes to larger effect of similar source terms for the holes with smallest R_{FF} of the pure state.

An empirical source term with a surface Woods-Saxon shape has been used in ref. [15] to obtain radial form factors entering finite range DWBA calculations of the reaction $^{208}\text{Pb}(\vec{d}, t)^{207}\text{Pb}$ at $E_d=200$ MeV. The position of this source term was chosen 10% inside of the well radius instead of 4% found in the present calculation.

4.3.3 Correction potential

As for the source terms, individual level correction potentials strongly fluctuate in shape (see Fig. 2). The correction potential for groups of levels however exhibits quite stable surface peaked shape for the different neutron holes. Such shape could also be fitted by a first derivative Woods-Saxon function. The depth D_{pot} varies with E_{sep} from slightly positive values for the quasihole level to increasingly negative values for the fragments. This behaviour is illustrated in Fig. 4 for the $1i_{13/2}$ and $1h_{9/2}$ quasihole levels and high-lying groups of levels. The change of sign occurs as expected at the pure hole separation energy. One notices that the potential depth mean slope with respect to E_{sep} is larger for the hole states with $j_- = \ell - 1/2$ than for $j_+ = \ell + 1/2$.

It has been suggested that a surface-peaked potential could be added to the

Fig. 4. Depths of $1i_{13/2}$ and $1h_{9/2}$ correction potentials of groups of levels as function of E_{sep} .

normal Woods-Saxon potential, and its depth fitted to reproduce the fragment separation energies, or alternatively that the radius of the well could be increased [1]. An increase of potential well radius cannot simulate the total potential shape obtained with the present correction potential. Our results for $n=1$ hole states showing a steady position and shape of the correction potential support the first approach. This latter one remains empirical without an underlying structure calculation. The correction potential presents a singularity at node radial positions for $n > 1$ hole states. It has been however shown in ref. [16]) that a rather well approximated solution of the Schrödinger equation could be obtained by interpolating the potential across the discontinuity. This indicates the possibility to recover a surface-peaked correction potential beyond the $n=1$ case.

4.4 Discussion

A natural question arises concerning a dependence of the above results on the QPM parameters. To answer it, two types of calculations were performed for neutron-hole valence states. Firstly, a new radial parameter was adopted both for the Woods-Saxon well and the interaction: $r_0=1.22$ fm [17] was chosen instead of $r_0=1.26$ fm, with a consistent change of well depth to preserve the neutron binding energy. Other model parameters and matrix elements were determined with the new neutron single-particle scheme according to the standard QPM procedure, then the states of odd-mass nucleus were calculated. The same overall behaviour of R_{FF} with E_{sep} was obtained. This result was qualitatively expected as both the pure hole state radius and the source term radius decrease. Switching off the collective effects by drastic decreasing of the coupling constants of residual multipole - multipole interaction for $\lambda = 2, 3, 4, 5, 6$ does not influence qualitatively the above results.

The dV/dr shape of the QPM effective interaction (see Sect. 3.1) is a result of the consistency conditions of Bohr – Mottelson type. Nevertheless, other radial shapes of this separable interaction (e.g., $\sim r^\lambda$) have also been used in theoretical studies and appropriate descriptions of properties of nuclear excited states have been obtained. So, we have studied the validity of the above conclusions if only the radial shape parameter of the separable residual interaction is changed. We have decreased the value of this radial parameter to $r_0 = 1.1$ fm keeping the parameters of the Woods-Saxon potential fixed. The whole procedure of form factor calculations was repeated under these conditions. Of course, phonons generated with the new interaction parameter have a structure and properties which differ to some extent from those deduced with the standard parameters. In particular, much larger fractions of $E\lambda$ transition strengths appear at low excitation energy but an overall agreement with experimental data remains still achievable. The above value of r_0 is however a lower limit beyond which the phonon excitations of the core cannot be described properly.

The new form factors were calculated for fragments of $1h_{9/2}$ and $1i_{13/2}$ neutron hole states. They exhibit similar characteristics as found with the standard parameters. The main difference is a smaller increase of R_{FF} with E_{sep} . For the $1i_{13/2}$ hole, mean form factor radii are almost independent on E_{sep} in spite of the large fluctuations of R_{FF} of particular states. The separation energy dependence of form factor radii is typically two times smaller than with the standard parameters in the case of $1h_{9/2}$ hole state.

Fig. 5. Comparison of the radial shape of the QPM standard interaction (full line) with the radial shape deduced from Ref. [18] (see text) (dotted line).

The effect of collective modes on single-particle states and the nucleon optical potential in ^{208}Pb have been studied in [18] using the Skyrme III force. In this approach, the product of the coupling strength $\tilde{v}(r)$ derived from the effective Skyrme III interaction with transition densities of the collective states

would play the same role as the dV/dr interaction. It was shown in ref. [18] that transition density radial shapes could be approximated by the derivative of the density $d\rho/dr$. The $\tilde{v}(r) d\rho/dr$ product is compared in Fig. 5 with the standard QPM interaction radial shape. The two distributions have their maximum nearly at the same radius. QPM and Hartree-Fock hole state radii also agree within typically 1%, so that a form factor calculation performed by just replacing the standard interaction shape by the $\tilde{v}(r) d\rho/dr$ shape gives an insight on the results expected with the approach of ref. [18]. The deduced variation of R_{FF} is nearly the same as for the standard QPM calculation.

Pick-up reaction studies do not give direct access to experimental form factors. Momentum form factors can be measured via $(e, e'p)$ reactions for proton hole states. However, in heavy nuclei such measurements for known spin and parity levels have only been performed, to our knowledge, for quasihole levels (with the exception of doublets populated from odd targets). Several $d_{5/2}$ proton hole levels have been studied in ^{39}K and ^{47}K (see ref. [19]). The form factor radii have been found constant or slightly increasing with E_{sep} while a vibrational model calculation with similar parameter choice as in QPM would give larger slopes. Further experiments in heavy nuclei are clearly needed, for comparison with QPM predictions.

5 Summary and Conclusions

We have used the inhomogeneous equation method suggested in [4] to calculate form factors of fragmented hole states in the region of ^{208}Pb in the framework of the quasiparticle-phonon model QPM. In this approach, it is assumed that the radial function of the odd particle with no excitation of the core, solution of a set coupled equations, is well approximated by the solution of an inhomogeneous equation. The corresponding source term is obtained using pure hole radial functions, the amplitudes of each contributing configuration and the surface-peaked interaction. The above approach has been validated in the case of few particle-vibration coupling levels described by the vibrational model. We have concluded from the schematic tests, that approximated and exact form factors radii differ at most by a few 0.1%.

The behaviour of fragment form factors have been studied in details for $i_{13/2}$ and $h_{9/2}$ levels in ^{207}Pb , generated by QPM using standard parameters. Large fluctuations of form factor radii are observed from level to level. The level density given by the model becomes very large at 2 or 3 MeV beyond the quasihole levels and a one-to-one correspondence between theoretical and experimental levels cannot be established. More meaningful for analyzing experimental data is the overall trend of such radii with separation energy. They are found to increase quite significantly with E_{sep} independently of the spectroscopic am-

plitudes. The observed slopes are systematically larger for $j_- = \ell - 1/2$ than for $j_+ = \ell + 1/2$ orbits, about 2% per MeV compared with 1%.

As average characteristics, a mean radial form factor, a summed source term and a mean correction potential for groups of levels were introduced. We have checked that differential cross sections obtained for groups of levels via the mean form factor approach for the $(\vec{d}, t)$ reaction at 200 MeV compare rather well with the corresponding summed cross sections of individual levels. The positions and surface thicknesses of summed source terms and mean correction potentials are quite stable and nearly independent of the considered excitation energy and hole state. Their shapes are rather well fitted by a derivative of Woods-Saxon well.

We have shown that QPM calculations with a (13%) smaller dV/dr interaction shift the source term position to a lower radius compared with the standard one. Consequently, the separation energy dependence of form factor radii is significantly reduced. Smaller interaction radii would not be acceptable for describing phonon states in the core. It is also worthwhile to notice that the standard dV/dr interaction shape compares rather well with estimations from [18], of the product of a Skyrme III interaction coupling strength with transition densities of the collective states in ^{208}Pb . This supports qualitatively the dependence of form factor radii given by QPM calculations with standard parameters.

Very few measurements of momentum form factors have been performed via $(e, e'p)$ reactions for well identified fragments of a same hole state. Systematic studies would be especially helpful in heavy nuclei, for comparison with QPM predictions.

The QPM model adopted a dV/dr interaction shape for the strongly collective transitions but also for non collective and/or higher multipolarity transitions. As the number of the latter transitions increases with excitation energy, the above approximation may become more and more questionable as concerns the calculation of form factors. One would expect that a tail of the interaction shape at smaller radius would decrease the separation energy dependence of form factor radii. The coupling with two phonon levels occurs in QPM via $(1h \otimes 1ph)$ levels. The interaction radial shape would thus not be modified, with expected small change in form factor radii behaviour. Pauli principle corrections, phonon interaction and ground state correlations in the odd nucleus have been studied in QPM calculations of hole strength fragmentation (Refs. [7,20]). It would be quite interesting to investigate their influence on the source term and form factors.

As a general rule, the QPM calculations in the $1qp \otimes 1ph$ approximation and even $1qp \otimes 2ph$ approximation predict too pronounced structures at high exci-

tation energy, in particular for deep-hole states. Many-body damping processes have been recently included in the description of deep hole state spreading in particle-hole basis, dropping collective effects [21]. Significantly wider strength distributions are predicted. Investigation of form factors under this approximation would be most valuable.

We would like to emphasize the following main conclusions. We have studied the separation energy dependence of hole form factors for the first time in a systematic and quantitative way. The inhomogeneous equation method used with the QPM model, has proved quite useful for this purpose. The deduced effects on form factor radii are quite significant especially for hole states with $j_- = l - 1/2$. The summed source term positions and shapes are very weakly dependent on energies and structure of the groups of levels. This is a most interesting point for the purpose of calculating form factor shapes needed in transfer or knock-out reaction analyses. A source term shape deduced from few calculations and conveniently parametrized would be directly used in the inhomogeneous equation to derive form factor shapes as function of separation energy.

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