Radiative Corrections Set Up Inflation: Dynamical Models

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Abstract

We establish the possibility that the initial configuration of fields for an inflationary period in the early universe can be set up by radiative corrections to the tree-level potential. A maximum of the radiatively-corrected potential can occur at an energy scale just above with the Planck scale; we consider that the scalar (inflaton) field rolls down from this maximum. In a model, we find that the radiative corrections also induce a minimum in the potential at a non-zero value of the energy scale somewhat below the Planck scale.

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1 Introduction

It is the purpose of this paper to illustrate with definite examples the possibility that the initial configuration from which a scalar field (inflaton) "rolls down" during an inflationary period in the early universe is set up by radiative corrections (at one loop) to the tree-level potential. The idea rests upon utilizing the possible, unusual properties for the development of the relevant couplings with energy scale via the renormalization-group equations. These properties allow for the occurrence of a maximum of the radiatively-corrected potential at an energy just above the Planck scale. We consider that the scalar field rolls down from this maximum toward zero.

2 Models and Results of Calculations

The first model is primarily meant to be illustrative of the general ideas which are involved in the dynamics. In this model we consider the tree-level potential, with quantum interactions, as given by

$$V(\phi) = \frac{\lambda}{4} \phi^4 + g \bar{\psi}_L \psi_L \phi + \sum_{n=1}^N g' \bar{\psi}_L \psi_L \phi_n \quad (1)$$

Here ϕ denotes the inflaton field and ψ_L denotes a primordial lepton field [2] with mass near to the Planck scale $M_P = 1.2 \times 10^{19}$ GeV. The ϕ_n denote possible additional (neutral) scalar quantum fields coupled to ψ_L (for simplicity with the same coupling g'). The “running” couplings $\lambda(\phi), g(\phi), g'(\phi)$ are dimensionless parameters taken, a priori, to be of the order of unity at a scale ¹ $\phi = \phi_i \lesssim M_P$ at which the computations are begun. The equations giving the development of the couplings with scale ϕ are as follows ², with $t = \ln(\phi/\phi_i)$

$$16\pi^2 \frac{d\lambda(\phi)}{dt} = 18\lambda^2 - 8g^4 = \beta_\lambda(\phi) \quad (2a)$$

$$16\pi^2 \frac{dg(\phi)}{dt} = 5g(g^2 + Ng'^2) = \beta_g(\phi) \quad (2b)$$

$$16\pi^2 \frac{dg'(\phi)}{dt} = 5g'(Ng'^2 + g^2) = \beta_{g'}(\phi) \quad (2c)$$

The first essential physical point is that the couplings $g(\phi)$ and $g'(\phi)$ run upward, that is they increase with increasing scale ϕ . The relevant β -functions in eqns. (2b,c) are positive because the couplings are leptonic. (Recall that in the familiar context [3,4] of running fermionic couplings, proportional to the running quark masses, the β -function in eq. (2b) is negative because of the necessary presence of a dominant negative term proportional to α_s , the QCD squared coupling. This is absent here.) Our second essential point is the imposition of the condition of an extremum [5] on $\beta_\lambda(\lambda(\phi), g(\phi))$ at the scale $\phi = \phi_i$, that is

$$\beta_\lambda(\lambda(\phi_i), g(\phi_i)) = 0 \quad (3)$$

Differentiation of eq. (2a), using eq. (3) gives

$$\left. \frac{d\beta_\lambda(\phi)}{dt} \right|_{\phi=\phi_i} < 0 \quad (4)$$

Therefore the extremum is a maximum of λ .³ For scales $\phi > \phi_i$, β_λ is negative. Consider now the first and second derivatives of the radiatively-corrected tree-level potential $V_c(\phi) = \frac{\lambda(\phi)}{4} \phi^4$.

$$\frac{dV_c(\phi)}{d\phi} = \left(\lambda(\phi) + \frac{\beta_\lambda(\phi)}{4} \right) \phi^3 \quad (5a)$$

$$\frac{d^2V_c(\phi)}{d\phi^2} = \left(3\lambda(\phi) + \frac{7}{4}\beta_\lambda(\phi) + \frac{3}{4} \frac{d\beta_\lambda}{dt} \right) \phi^2 \quad (5b)$$

At scales $\phi > \phi_i$, $\beta_\lambda(\phi)$ and $d\beta_\lambda(\phi)/dt$ are negative; $\lambda(\phi)$ is decreasing for increasing ϕ . Therefore, it is possible to reach a point on the energy scale $\phi = \phi_m$, where

$$\left. \frac{dV_c(\phi)}{d\phi} \right|_{\phi=\phi_m} = 0 \quad (6a)$$

$$\left. \frac{d^2V_c(\phi)}{d\phi^2} \right|_{\phi=\phi_m} < 0 \quad (6b)$$

¹In the numerical work in this paper, the running scale is taken as approximately the (classical) field variable ϕ ; we study the range of ϕ from $\phi = 0$ to $\phi \gtrsim M_P$.

²In this paper, we study radiative corrections at the one-loop level, since we want to illustrate simply the essential dynamical points which are involved in the idea.

³In the calculations of ref. [5] relevant to the standard-model Higgs field, the extremum is a minimum of the self-coupling parameter $\lambda_H(\phi_H \cong M_P)$. The potential also has a minimum there if $\lambda_H(M_P) = 0$.

There is a maximum of the radiatively-corrected tree-level potential. Such a field configuration sets up a natural starting point for a period of inflation, that is the field “rolls down” from this maximum of the potential. It can do so either toward the origin or away from it [6]. In this paper, we give detailed numerical examples for roll-down toward the origin; we assume that the initial instability favours this possibility. We comment upon the other possibility below. In the first numerical illustration of the dynamics, we take $\lambda = 0.6$ at the scale $\phi_i = 0.01M_P$. The condition in eq. (3) determines $g^2 = \frac{3}{2}\lambda$ at ϕ_i ; for illustration we take $g' = 1$ at ϕ_i (note $g \cong g'$) and $N = 1$. The renormalization-group eqs. (2a-c) are solved numerically. The development of $\lambda(\phi)$ is shown in Fig. 1. The development of $g(\phi)$ and $g'(\phi)$ is shown in Fig. 2. The previously stated general features of the development of the coupling parameters is evident. In Fig. 3, we plot the radiatively-corrected potential $V_c(\phi) = \frac{1}{4}\lambda(\phi)\phi^4$. The maximum which is present in this potential at $\phi_m \cong 2M_P$, above ϕ_i , is the essential new feature. In Fig. 3 we also exhibit a second curve which illustrates what happens if we add to $V_c(\phi)$ a term containing a negative squared-mass, i. e. $-\frac{1}{2}|\mu^2|\phi^2$ with $\sqrt{|\mu^2|} \sim \frac{10^{13}}{\sqrt{2}}$ GeV. By addition of a constant, we have renormalized the calculated curve so that the minimum of the potential (at $\langle\phi\rangle = |\mu^2|^{1/2}/\sqrt{\lambda(\langle\phi\rangle)}$) has $V_c(\langle\phi\rangle) = 0$. The mass of ϕ is $m_\phi = 10^{13}$ GeV in this example.

The above numerical example is only illustrative of the idea. The value for $\lambda(\phi_i) \sim 0.6$ is too large ⁴ [7, 8]. A very small value for $\lambda(\phi_i)$ causes the condition in eq. (6a) to be reached only at very high scales ϕ_m , if ϕ_i is near M_P . This is because the necessary cancellation in the bracket in eq. (5a) requires that the negative difference between initially extremely small quantities in $\beta_\lambda(\phi)$ cancels the relatively large positive quantity $\lambda(\phi)$. It is worth noting however, that $\lambda(\phi)$ is going to zero as ϕ drops to zero. Also formally, at large scales $\phi > M_P$, $\lambda(\phi)$ drops sharply down through zero as $g'(\phi)$ and $g(\phi)$ formally become infinite at the approach to a Landau pole. Of course, very high scales are in a domain of excessive energy densities where gravitational effects become, in principle, relevant. We turn then to a second, potentially realistic model, which has a much richer dynamical structure.

This model is, in essence, one that has sought popularity through its use as an illustrative example in a whole series of papers by Linde [9, 10] and others ⁵ [6]. The potential and quantum interactions, now involve a second scalar field which we denote [2, 4] ⁶ by b

$$V(\phi, b) = \lambda\phi^4 + \lambda_b\phi^2b^2 + \tilde{\lambda}_bb^4 + g\bar{\psi}_L\psi_L\phi + g_b\bar{\psi}_L\psi_Lb \quad (7)$$

We use [1, 7, 8, 10] $\lambda(\phi_i) = 10^{-12}$ with $\phi_i = M_P$. ⁷ For all of the other dimensionless couplings in eq. (7) we have examined, in our numerical work below, a range of values at ϕ_i from about 0.0001 to 0.1. The equations giving the development of the

⁴See, in particular, the discussions of the relation between acceptably small values for empirical density fluctuations ($\sim \sqrt{\lambda(\phi)}$) and a very small value of $\sqrt{\lambda}$, given in chapter 8, “Inflation”, of ref. [8]

⁵Ref. [6] contains a listing of many additional recent references.

⁶We have ourselves recently studied (in ref. [4]) the effect of an interaction $\lambda_b\phi_H^2b^2$ upon the prediction (in ref. [5]) for the mass of the standard-model Higgs scalar (ϕ_H). We speculated that the additional spin-zero field b is in fact a pseudoscalar whose non-zero vacuum expectation value is related to a spontaneous breakdown of CP invariance at a high scale.

⁷We note that if $\lambda(\phi_i) = 0$, the value to which λ evolves at the minimum of the potential discussed below is $|\lambda(\phi_c)| = O(10^{-12})$.

couplings with ϕ are

$$16\pi^2 \frac{d\lambda(\phi)}{dt} = 72\lambda^2 + 2\lambda_b^2 - 2g^4 = \beta_\lambda(\phi) \quad (8a)$$

$$16\pi^2 \frac{d\lambda_b(\phi)}{dt} = \lambda_b(24\lambda + 16\lambda_b + 24\tilde{\lambda}_b) - 4g^2g_b^2 = \beta_{\lambda_b}(\phi) \quad (8b)$$

$$16\pi^2 \frac{d\tilde{\lambda}_b(\phi)}{dt} = 72\tilde{\lambda}_b^2 + 2\lambda_b^2 - 2g_b^4 = \beta_{\tilde{\lambda}_b}(\phi) \quad (8c)$$

$$16\pi^2 \frac{dg(\phi)}{dt} = 5g(g^2 + g_b^2) = \beta_g(\phi) \quad (8d)$$

$$16\pi^2 \frac{dg_b(\phi)}{dt} = 5g_b(g_b^2 + g^2) = \beta_{g_b}(\phi) \quad (8e)$$

Formally, one notices that this model differs from the first model (with $N = 1$) by starting with a very small (or zero (see footnote 7)) $\lambda(\phi_i)$ but a significant $\lambda_b(\phi_i)$; significant corrections to $\lambda(\phi)$ are generated at the one-loop level. In the first model, one starts with a significant $\lambda(\phi_i)$ but zero $\lambda_{\phi_n}(\phi_i)$ and $\tilde{\lambda}_{\phi_n}(\phi_i)$; (the analogues of $\lambda_b(\phi_i)$ and $\tilde{\lambda}_b(\phi_i)$); these are then generated by the fermionic loops.

We impose the extremum conditions

$$\beta_\lambda(\phi_i) = 0 \quad (9a)$$

$$\beta_{\lambda_b}(\phi_i) = 0 \quad (9b)$$

For a given value of $\lambda_b(\phi_i)$, these conditions determine $g(\phi_i)$ and $g_b(\phi_i)$. The renormalization-group eqns. (8a-e) are solved numerically. In the figures, we use $\lambda_b(\phi_i) = \tilde{\lambda}_b(\phi_i) = 0.00028$. We state a reason for this number below. The initial values for the leptonic couplings are $g(\phi_i) = 0.017$ and $g_b(\phi_i) = 0.053$. The development of $\lambda(\phi)$, and of $\lambda_b(\phi)$ and $\tilde{\lambda}_b(\phi)$, is shown in Figs. 4-5 and Fig. 6, respectively. The development of $g(\phi)$ and $g_b(\phi)$ is shown in Fig. 7. Again we have the essential general features:

- (1) The upward movement of $g(\phi)$ and $g_b(\phi)$ with increasing scale.
- (2) The maximum for $\lambda(\phi)$ at $\phi = \phi_i \cong M_P$.

The evolution of $\lambda_b(\phi)$ is exceedingly slow, on both the low-scale and the high-scale side. This is because of very small values of $\beta_{\lambda_b}(\phi)$ which evolve from $\beta_{\lambda_b}(\phi_i) = 0$. The upward evolution of $g(\phi)$ and of $g_b(\phi)$ is also slow, because of the nature of the dependence in eqs. (8d,e) upon the relatively small initial values $g(\phi_i)$ and $g_b(\phi_i)$. In Fig. 8, we plot the radiatively-corrected potential, $V_c(\phi) = \lambda(\phi)\phi^4$. There is a maximum a little above the Planck scale at $\phi_m \cong 7.9M_P \gtrsim \phi_i$. This occurs for the general reasons stated before eq. (6a,b), but now the cancellation in eq. (5a) is immediately brought about by the negative difference between initially sizable quantities in $\beta_\lambda(\phi)$ (note eq. (8a)) cancelling the very small positive quantity $\lambda(\phi)$. There is an interesting second effect shown in Fig. 8: the radiatively-corrected tree-level potential acquires a minimum⁸ somewhat below the Planck scale at non-zero $\phi = \phi_c \cong 0.077M_P$. The minimum occurs because $\lambda(\phi)$ is driven to negative values as ϕ goes down; the cancellation in eq. (5a), leading to eq. (6a), can now take place

⁸By addition of a constant, we have again renormalized the calculated curve so that the minimum has $V_c(\phi_c) = 0$.

between a negative quantity $4\lambda(\phi)$ and a positive quantity $\beta_\lambda(\phi)$. This means that the radiative corrections produce effectively [11] a squared-mass; in our numerical example this squared mass is $m_\phi^2 = d^2V(\phi)/d\phi^2 \cong (1.5 \times 10^{12} \text{ GeV})^2$.⁹ The value of $\lambda_b(\phi_i) = 0.00028$ was chosen so as to have the minimum of the potential occur at a scale which is about 0.01 of the scale at which the maximum occurs. This probably allows for inflation to result in a very large increase in the dimension-scale $R(t)$ for the universe [8,12]. A smaller value for $\lambda_b(\phi_i)$ gives a larger time (scale) interval for inflation; a larger value gives a smaller interval.¹⁰ Quite small density perturbations may naturally occur proportional to $\sqrt{\lambda}$ or to a ratio of mass scales [1,10] involving ϕ_c and M_P , i. e. $O(\phi_c/M_P)^3 \sim 10^{-4}$. These points require further investigation. It is not our thought here to attempt to represent all dynamical aspects quantitatively (see footnote 9)

To conclude, we comment briefly upon unusual formal behaviour in the domain of ϕ above the maximum of the potential at $\phi_m \gtrsim M_P$; although physics in this domain is not considered in this paper. In the above numerical example ($\lambda_b = \tilde{\lambda}_b = 0.00028$; $\phi_m \cong 7.9M_P$), $\lambda(\phi)$ again drops through zero at $\phi \cong 10M_P$. The potential is thus negative and is not bounded as ϕ increases. From the point of view of this paper, inflation is then limited to starting at a scale ϕ that is infinitesimally to the left of ϕ_m , which is $\sim 7.9M_P$ if ϕ_i is taken as M_P as in our example. Of course, ϕ_i can be somewhat above M_P , this has the effect of moving the disallowed region further to the right.

3 Conclusions

The results of this paper represent, at the least, a provocative exercise in the examination of the possible role played by radiative corrections which arise from the application of the renormalization-group equations to specific dynamical models, in setting up inflation in the early universe. The results may provide an indication of the answers to certain open questions concerning inflation which are listed explicitly at the end of chapter 8 in reference [8]:

- (1) What determines the starting point on the energy scale? We suggest that this point is (just below) the value $\phi_m \gtrsim M_P$ where the radiatively-corrected tree-level potential $V_c(\phi) = \lambda(\phi)\phi^4$ has a maximum.
- (2) Why is λ a very small number? We have shown here that the condition $\beta_\lambda(M_P) = 0$, supplemented with $\lambda(M_P) \approx 0$, results in an extremum not only for $\lambda(\phi)$, but also for $V_c(\phi)$ just above the Planck energy scale. For the inflaton these extremums correspond to maxima. The extremum conditions in eq. (3) and eq. (6a) bear a marked similarity to the conditions (see footnote 3) used in

⁹Formally then, the ϕ_c^2 generates a b -mass via the $\lambda_b\phi^2b^2$ term in eq. (7). There is a trilinear coupling [2,10] $(2\lambda_b\phi_c)\phi b^2$, as well as probably also a trilinear coupling $(2\lambda_b b_c)b\phi^2$. We consider that there can be an explicit negative squared-mass term present for the b -field. This is not dealt with in this paper and can have the effect of placing the b -mass at a lower value. At scales well below the b -mass, one should decouple the b -field from the evolution eqns. (8a-e). We do not do this in the present illustrative calculation with the model, which deals with the unusual new effects: the maximum and minimum of the potential near to the Planck scale.

¹⁰For $\lambda_b(\phi_i) \gtrsim 0.001$ the sensitivity to the value of λ_b becomes very slight. The potential has a maximum almost exactly at M_P and a minimum at $\phi_c \cong 0.6M_P$.

the calculations of reference [5] in the context of a discussion of the behaviour of the Higgs potential in the standard model. We shall later present results on the possibility that the maximum can occur before the minimum as ϕ goes up, if $\lambda(\phi_i)$ is taken to be negative and $\beta_{\lambda_b}(\phi_i) \neq 0$. There is then a minimum of $\lambda(\phi)$ at ϕ_i .

- (3) Does the spin-zero inflaton field ϕ (and also the spin-zero field b in eq. (7)) have any other role? In references [2,4], we have speculated that b is a pseudoscalar field involved in a spontaneous breakdown of CP invariance at a high scale. We have shown that a scalar field could be involved in an effectively spontaneous breakdown of CPT invariance near the origin [2], with an additional reason for the smallness of $\sqrt{\lambda}$ being its relation to a small failure of CPT invariance. This breakdown would give slightly different masses (at the level of 10^{-9}) to a primordial very heavy neutral lepton L and antilepton $\bar{L}$. Subsequent decays result in an asymmetry between light neutral leptons and antileptons. This asymmetry can drive the creation of the baryon asymmetry near the electroweak scale [8]. A result of the present calculations is relevant to this problem [2]: the quantity ¹¹ $\frac{\sqrt{\lambda(\phi_c)}\phi_c^2}{M_P^2} \cong 3 \times 10^{-9}$ is the same order of magnitude as the baryon asymmetry. It is noteworthy that the primordial fermions with mass near to the Planck scale are leptons, an assumption which is utilized also in the present work.

In summary, our calculations show that specific quantum effects can be involved in an essential way in setting up inflation. It would be a remarkable physical situation if quantum effects at the smallest distances establish conditions for a cosmological phenomenon which involves the largest distance-scales.

References

- [1] A. Linde, *Lectures on Inflationary Cosmology* SU-ITP-94-36, hep-th/9410082
A. Linde, *Recent Progress in Inflationary Cosmology*, SU-ITP-96-01, astro-ph/961004
- [2] S. Barshay, *CPT Noninvariance: A Model for an Initial Lepton Asymmetry due to Unequal Masses for Lepton and Antilepton*, Aachen preprint PITHA 96/8, March 1996 (to appear in Z. Phys. C)
- [3] M. Sher, Phys. Lett. **B317** (1993) 159
addendum: Phys. Lett. **B331** (1994) 448
- [4] S. Barshay and G. Kreyerhoff, *Another calculation of the Higgs mass and the top mass from the principles of H. B. Nielsen: $m_H \cong 163$ GeV for $M_t \cong 190$ GeV* Aachen preprint PITHA 96/15, April 1996 (to appear in Z. Phys. C)
- [5] C. D. Froggatt and H. B. Nielsen, Phys. Lett. **B368** (1996) 96
- [6] D. H. Lyth and E. D. Stewart, *More varieties of hybrid inflation*, LANCS-TH/9613, RESCEU-20/96, hep-ph/9606412
- [7] L. A. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Lett. **B157** (1985) 361

¹¹In ref. [2], the essential hypothesis is that CPT noninvariance near the origin involves the square of the ratio of a somewhat lower energy scale to the Planck scale.

- [8] E. W. Kolb and M. S. Turner, *The Early Universe* Addison-Wesley Publishing Co. 1990, chapter 8 for inflation
- [9] A. Linde, Phys. Lett. **B351** (1995) 99
- [10] A. Linde, Phys. Lett. **B259** (1991) 38
A. Linde, Phys. Rev. **D49** (1994) 748
A. Linde, Phys. Lett. **B249** (1990) 18
A. Linde, Phys. Lett. **B202** (1988) 194
A. Linde, Phys. Lett. **B162** (1985) 281
L. Kofman and A. Linde, Nucl. Phys. **B282** (187) 555
L. Kofman, A. Linde and A. A. Starobinsky, Phys. Rev. Lett. **73** (1994) 3195
L. Kofmann, *The Origin of Matter in the Universe: Reheating after Inflation* preprint, Institute of Astronomy, University of Hawaii, 1996
- [11] S. Coleman and E. Weinberg, Phys. Rev. **D7** (1973) 1888
Q. Shafi and A. Vilenkin, Phys. Rev. Lett. **52** (1984) 691
- [12] A. R. Liddle, Phys. Rev. **D49** (1994) 739

Figures

Figure 1: Renormalization-group evolution of the quartic self-coupling parameter $\lambda(\phi)$ for the model in eqs. (1,2). The scale is set by ϕ_i , taken here as $0.01M_P$.

Figure 2: Renormalization-group evolution of the leptonic couplings $g(\phi)$ and $g'(\phi)$ for the model in eqs. (1,2). The solid curve is g and the dashed curve is g' .

Figure 3: The radiatively-corrected tree-level potential $V(\phi) = \frac{1}{4}\lambda(\phi)\phi^4$ for the model in eqs. (1,2). For illustration, a minimum in the potential is induced by addition of a term containing a negative squared-mass $-\frac{1}{2}|\mu^2|\phi^2$, with $\sqrt{|\mu^2|} = \frac{10^{13}}{\sqrt{2}}$ GeV. The scale is set by ϕ_i , taken here as $0.01M_P$.

Figure 4: Renormalization-group evolution of the quartic self-coupling parameter $\lambda(\phi)$ for the model in eqs. (7,8). The scale is set by $\phi_i = M_P$.

Figure 5: As in Fig. 4, over an extended scale.

Figure 6: Renormalization-group evolution of the bosonic coupling parameters $\lambda_b(\phi)$ and $\tilde{\lambda}_b(\phi)$ for the model in eqs. (7,8).

Figure 7: Renormalization-group evolution of the leptonic coupling $g(\phi)$ and $g_b(\phi)$ for the model in eqs. (7,8).

Figure 8: The radiatively-corrected tree-level potential $V(\phi) = \lambda(\phi)\phi^4$ for the model in eqs. (7,8)