Quenched Light Hadron Spectrum and Decay Constants using Improved Wilson Fermion Actions

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We compare results obtained using the Sheikholeslami-Wohlert (SW) fermion action with tree-level and tadpole-improved coefficients for $5.7 \leq \beta \leq 6.2$.

1. LATTICE PARAMETERS

The use of a tadpole-improved coefficient [1] for the clover term in the SW fermion action,

\[ S_F = \sum_{x,y} \bar{q}(x) \left\{ \left[ 1 - \frac{i\kappa}{2} \sigma_{\mu\nu} F_{\mu\nu}(x) \right] \delta_{xy} \right. \]

\[ -\kappa \left[ (1 - \gamma_\mu) U_\mu(x) \delta_{x+\hat{\mu},y} \right. \]

\[ + \left. (1 + \gamma_\mu) U_\mu(y) \delta_{x-\hat{\mu},y} \right\} q(y), \tag{1} \]

where $c = 1/u_0^3$, $u_0 = (\frac{1}{3} \text{Tr} \Omega)^{1/4}$, can be regarded as a step towards full $O(a)$ improvement [2]. We compare the resulting light hadron quantities with those obtained using the tree-level coefficient, $c = 1$, for which discretisation errors are $O(g^a)$ in perturbation theory [3], and also with data from GF11 [4] corresponding to $c = 0$, at a range of $\beta$ values, to seek indications of better scaling behaviour.

Our data set comprises: at $\beta = 5.7$, Jacobismeared quark propagators at two $\kappa$ values with $c = 1/u_0^3$ and $c = 1$ on $142 \times 16^3 \times 32$ configurations; at $\beta = 6.0$, fuzzed propagators [5] at three $\kappa$ values with $c = 1/u_0^3$ on $499 \times 16^3 \times 48$ configurations; and, at $\beta = 6.2$, fuzzed (local) propagators at three (two) $\kappa$ values with $c = 1/u_0^3$ ($c = 1$) on $130 \times 24^3 \times 48$ configurations. We construct meson correlators from all $\kappa$ combinations, but baryon correlators only from degenerate combinations. Consequently, we do not have enough baryon data to perform a reliable chiral extrapolation. Hadron masses are obtained from multi-exponential fits to various combinations of smeared and local correlators. The results presented are from a preliminary analysis of our current data (see also [6]); higher statistics at $\beta = 6.2$ will be available soon.

2. HADRON SPECTRUM

The ratio of the nucleon to vector meson mass, $m_N/m_V$, at a fixed ratio of the pseudoscalar to vector meson mass, $m_{PS}/m_V$, has a noticeable dependence on $c$ at our largest lattice spacing. The trend is towards improved scaling for this ratio as $c$ is increased to the tadpole-improved value. Evidence for the latter is given in the Edinburgh plot in Figure 1.

Linear extrapolation of data for the pseudoscalar meson mass to $m_{PS}(\kappa_s, \kappa_c) = 0$ for the tadpole-improved action gives $u_0\kappa_c = 0.12347(3)$, $0.12224(1)$, and $0.12205(2)$, for $\beta = 5.7$, 6.0 and 6.2 respectively, in reasonable agreement with the tree-level value of 0.125. We define $\kappa_{ud}$, corresponding to degenerate $u$ and $d$ quarks, at $m_{PS}/m_V = m_{\pi}/m_\rho$. Then, as shown in Figure 2, our estimates for $m_\rho$ in units of the square root of the string tension, $\sqrt{K}$, have decreasing dependence on the lattice spacing, $a$, as $c$ is increased from 0 to $1/u_0^3$, although the dependence is not removed entirely.

The $\kappa$ value corresponding to the strange quark mass, $\kappa_s$, may be fixed from any one of the ratios $m_K/m_\rho$, $m_\phi/m_\rho$, and $m_{K^*}/m_\rho$. In Figure 3 we show the results of fixing one of the first two ratios and calculating the third. Evidently, the $K^*$ mass scales with the $\rho$ mass in both cases, independently of whether $c = 1$ or $1/u_0^3$, but dif-

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different values are obtained for $m_{K^*}/m_\rho$. This is an indication that the strange quark mass cannot be determined consistently in the quenched approximation [7]. We also find that $J$ [8] is not changed by tadpole improvement, and scales at a value inconsistent with experiment.

Finally, we observe that, at all three $\beta$ values, the magnitude of the meson spin splitting, $m_V^2 - m_{PS}^2$ in units of $m_K$ or $m_\rho$, is insensitive to whether $c = 1$ or $1/\omega_0^3$.

3. DECAY CONSTANTS

Our quark propagators are not ‘rotated’, so $O(a)$-improved matrix elements are constructed from improved quark fields:

$$q^f = \sqrt{2\kappa u_0} \left[1 - \frac{\vec{p}}{2u_0}\right] q$$

$$= \sqrt{2\kappa u_0} \left[1 + \frac{1}{2u_0} \left(\frac{1}{2\kappa} - \frac{1}{2\kappa_c}\right)\right] q$$

(2)

where we have used the equations of motion and employed the tadpole-improvement prescription [1]. Thus, we take the tadpole-improved pion decay matrix element to be [9]

$$\langle 0| q^f \gamma_4 \gamma_5 q^f | \pi \rangle = Z_A 2\kappa u_0 \left[1 + \frac{1}{u_0} \left(\frac{1}{2\kappa} - \frac{1}{2\kappa_c}\right)\right]$$

(3)

where we obtain $\kappa_c$ from $m_{PS}(\kappa_c, \kappa_c) = 0$ and the current normalisation, $Z_A$, from tadpole-improved one-loop perturbation theory.

Comparison of the pion decay constant values obtained with different values of $c$ is complicated by systematic effects inherent in the current normalisation prescription. Given this caveat, we show our tadpole-improved results along with those of GF11 [4] in Figure 4. Both sets of data suggest scaling may set in above $\beta = 6.0$.

The ratio of decay constants, $f_K/f_\pi$, is independent of the current normalisation and so may be determined more reliably. Our results, shown in Figure 5, are insensitive to whether $c = 1$ or $1/\omega_0^3$. For $\beta \geq 6.0$ the ratio agrees with experiment, but a weak lattice spacing dependence remains below this $\beta$ value.

4. CONCLUSIONS

Using a tadpole-improved SW fermion action in quenched QCD, we conclude the following.

1. For $\beta \geq 5.7$, $m_\rho/\sqrt{K}$ has a weaker dependence on the lattice spacing (Figure 2), $m_N/m_V$ and $m_{K^*}/m_\rho$ scale (Figures 1 and 3), although
the latter clearly shows that the strange quark mass cannot be determined consistently.

2. Some quantities, such as $m_{K^*}/m_{\rho}$, $m_V^2 - m_{PS}^2$, $J$ and $f_K/f_\pi$, are insensitive to whether $c$ has its tree-level or tadpole-improved value (Figures 3 and 5).

3. Provided tadpole-improved perturbation theory for $Z_A$ is reliable, there is an indication that $f_\pi/m_\rho$ may scale for $\beta \geq 6.0$ (Figure 4).

REFERENCES
9. P.A. Rowland (UKQCD), these proceedings.
Figure 5. $f_K/f_x$ versus lattice spacing.