Forces between composite particles in QCD
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Starting from the meson-meson Green function in 3+1 dimensional quenched lattice QCD we calculate potentials between heavy-light mesons for various light-quark mass parameters. For the valence quarks we employ the staggered scheme. The resulting potentials turn out to be short ranged and attractive. A comparison with a tadpole improved action for the gauge fields is presented.

1. INTRODUCTION

For several decades nucleon-nucleon interactions have been parametrized by phenomenological potentials. Substantial effort, employing purely hadronic degrees of freedom, has lead to meson-exchange potentials [1]. Attempts to take into account quark and gluon degrees of freedom with hybrid models have also been made [2].

Today, QCD is believed to be the fundamental theory of strong interactions. Thus it has become a challenge for theoretical nuclear physics to extract a nucleon-nucleon potential from first principles. In the low energy regime of QCD non-perturbative tools have to be used. This leads us to study the forces in systems of two hadrons on the lattice.

Previous lattice calculations with static valence quarks have revealed an attractive force between two three-quark clusters [3]. When dynamical quark propagators are used antisymmetrization and the exchange of valence quarks become possible [4]. In 2+1 dimensional QED the potential between light mesons exhibits a repulsive hard core and is attractive at intermediate distances [5]. An extension of this formalism to 3+1 dimensional QCD using a hopping parameter expansion for the quark propagators is reported in [6].

In the present work we take another step toward the goal of calculating hadronic potentials from QCD. We study a system of two heavy-light mesons with dynamical quark propagators for the light valence quarks. Results from calculations with the Wilson action for the gauge field and a tadpole improved gauge field action are reported.

2. THEORY

2.1. Meson-meson correlator

The one-meson field is a product of staggered Grassmann fields \( \chi \) and \( \bar{\chi} \) with a heavy and a light external flavor \( h \) and \( l \), respectively,

\[
\phi_{\vec{x}}(t) = \bar{\chi}_{l}(\vec{x}t)\chi_{h}(\vec{x}t).
\]

The meson-meson fields with relative distance \( \vec{r} = \vec{y} - \vec{x} \) are then constructed by

\[
\Phi_{\vec{r}}(t) = V^{-1} \sum_{\vec{x}} \sum_{\vec{y}} \delta_{\vec{r},\vec{y} - \vec{x}} \phi_{\vec{x}}(t) \phi_{\vec{y}}(t).
\]

The information about the dynamics of the meson-meson system is contained in the time correlation matrix

\[
C_{\vec{r}\vec{r}'}(t, t_0) = \langle \Phi_{\vec{r}}(t) \Phi_{\vec{r}'}(t_0) \rangle - \langle \Phi_{\vec{r}}(t) \rangle \langle \Phi_{\vec{r}'}(t_0) \rangle,
\]

where \( \langle \rangle \) denotes the gauge field configuration average. On the hadronic level \( C \) is a 2-point correlator of a composite local operator describing a molecule-like structure. Working out the contractions between the Grassmann fields the following diagrammatic representation is obtained

\[
C = C^{(A)} - C^{(C)} = \bigg\langle \bigg\{ \bigg\} \bigg\rangle\bigg\} - \bigg\langle \bigg\{ x \bigg\}\bigg\} - \bigg\langle \bigg\{ y \bigg\}\bigg\}.
\]

Each contribution to the correlator comprises the exchange of gluons. Thus, even diagram
$C^{(A)}$ leads to forces, whereas diagram $C^{(C)}$ corresponds also to interaction mediated by the exchange of the light valence quark. Denoting contractions of the Grassmann fields by

$$
\ldots \chi \ldots \bar{\chi} \ldots = G_n, \quad \text{with} \quad n = 1 \ldots 4,
$$

we have for example

$$
C^{(A)} \sim 43^{12} 21^{34} \langle \phi_y^1, \phi_x^1, \phi_x^1, \phi_y^1 \rangle = \langle G_1^{(h)} G_2^{(h)} G_3^{(h)} G_4^{(h)} \rangle,
$$

with $\vec{r} = \vec{y} - \vec{x}$, $\vec{r}' = \vec{y}' - \vec{x}'$, and $\sim$ stands for the sums and factors that carry over from (2). Color indices are suppressed. The gauge configuration average is taken over the product of all four propagators $G$. The propagator of the light quark $G^{(l)}$ is obtained from inverting the staggered fermion matrix with a random source estimator. A standard conjugate gradient algorithm is used. The heavy-quark propagator is given by

$$
G_{\vec{x}, \vec{x}_0}^{(h)} = \left( \frac{1}{2m_h a} \right)^k \left| \Gamma_{\vec{x}4} \right|^k \prod_{j=1}^{k} U_{x=(\vec{x}, ja), \mu=4},
$$

where the phase factors $\Gamma_{\vec{x}4} = (-1)^{(x_1 + x_2 + x_3)/a}$ in the Kogut-Susskind formulation correspond to the Dirac matrices and $k = (t - t_0)/a$. In our calculations we set $m_h a = 1$.

Since the heavy valence quarks are fixed in space the relative distance between the mesons is the same at the initial and final time of the propagation, $\vec{r}' = \vec{r}$. The effective ground-state energy of the meson-meson system $W(\vec{r})$ can then be extracted from the large euclidean time behavior of $C$ following quantum-mechanical reasoning for composite particles [6],

$$
C_{\vec{r}\vec{r}}(t, t_0) = \sum_n \left| \langle r | n \rangle \right|^2 e^{-E_n (t-t_0)}
$$

$$
\sim \bar{c}(\vec{r}) e^{-W(\vec{r})(t-t_0)}. \quad (8)
$$

The residual meson-meson potential is

$$
V(r) = W(r) - 2m,
$$

with the mass $2m$ of two free mesons subtracted.

### 2.2. Improved action

Discretization errors due to finite lattice spacing $a$ can be reduced by including terms of higher order in $a$ with the action. It has been shown that at the classical level adding a term with six-link rectangular plaquettes $U_{rt}$ to the usual Wilson gauge field action removes $O(a^2)$ errors [7]. A significant improvement is obtained by introducing tadpole factors in the six-link term [8]. We use the improved action

$$
S[U] = \beta_{pl} \sum_{pl} \frac{1}{3} \text{Re} \text{Tr}(1 - U_{pl})
$$

$$
+ \beta_{rt} \sum_{rt} \frac{1}{3} \text{Re} \text{Tr}(1 - U_{rt}), \quad (10)
$$

where the first term is the Wilson action with four-link plaquettes $U_{pl}$. The coupling parameter

$$
\beta_{rt} = -\frac{\beta_{pl}}{20u_0^2}
$$

and the mean link

$$
u_0 = \left( \frac{1}{3} \text{Re} \text{Tr}(U_{pl}) \right)^{1/4} \quad (12)
$$

are determined self-consistently for a given $\beta_{pl}$.

### 3. RESULTS

We considered a periodic $8^3 \times 16$ lattice. Each potential is the result of a measurement on 100 independent gauge field configurations which were separated by 200 sweeps. The inversion of the fermion matrix was performed with 32 random sources.

The effective energy $W(r)$ was extracted from the correlator $C$ by a four parameter (Levenberg-Marquardt) fit with the function

$$
C_{rr}(t) = B(r) \cosh \left[ W(r) (t - 8a) \right]
$$

$$
+ (-1)^{f/a} B(r) \cosh \left[ \bar{W}(r) (t - 8a) \right]. \quad (13)
$$

The second term alternating in sign is a peculiarity of the staggered scheme. In order to improve the quality of the fits 15 time slices were used. The extracted potentials are thus between meson states which are contaminated by excitations. To be numerically consistent with $W(r)$ at large distances, the mass $2m$ of two noninteracting mesons was extracted from the square of the meson two-point function $[C^{(2)}]_2^2$. 

Figure 1: Potentials between heavy-light mesons obtained from simulations with conventional Wilson action compared to results with improved action. The potentials are attractive and become stronger with decreasing light-quark mass. Error bars are smaller when using tadpole improved action. Improvement was restricted to the gauge field action.
3.1. Wilson action

For the update with the usual Wilson action we chose the inverse gluon coupling constant $\beta = 5.6$. This corresponds to a lattice spacing of approximately 0.2fm. The left column of Fig. 1 shows the resulting potentials for different values of the light valence quark mass $m_a = 0.05, 0.1$ and $0.2$, respectively. In all cases the potential between two heavy-light mesons is short ranged and attractive. Calculations from inverse scattering theory propose a similar shape for KK-potentials [9]. Experimental scattering data contain inelastic channels which are not accounted for in our QCD calculation. At distances $r/a > 1$ essentially no interaction energy can be resolved and the effective total energy approaches the value of two noninteracting mesons $2m$. With decreasing light-quark mass the interaction becomes stronger.

3.2. Improved action

Here the coupling was set to $\beta_{pl} = 7.6$. This value corresponds to $a \approx 0.2fm$, as for the Wilson action [8]. The right column of Fig. 1 shows the resulting potentials, again for different values of the light valence quark mass $m_a = 0.05, 0.1$ and $0.2$. Again we obtain attractive potentials which become deeper with decreasing quark masses. Those exhibit larger anisotropies than in the Wilson case. The likely reason is that only the gluonic action but not the staggered fermion matrix has been improved. In any case, it is interesting to observe that the shape of the potential remains stable.

4. CONCLUSIONS

We applied a practical method to extract effective hadron-hadron potentials from lattice QCD to a system of two heavy-light mesons. For the light valence quark we employed dynamical quark propagators. The resulting potentials are short ranged and attractive. Smaller quark mass parameters lead to stronger interaction. A preliminary analysis of simulations with improved action showed consistent results. Further insight into the interaction mechanism is expected from calculations with mesons consisting of two light valence quarks.

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