Gravitational Violation of R Parity
and its Cosmological Signatures

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Abstract

The discrete R-parity ($R_P$) usually imposed on the Supersymmetric (SUSY) models is expected to be broken at least gravitationally. If the neutralino is a dark matter particle its decay channels into positrons, antiprotons and neutrinos are severely constrained from astrophysical observations. These constraints are shown to be violated even for Planck-mass-suppressed dimension-five interactions arising from gravitational effects. We perform a general analysis of gravitationally induced $R_P$ violation and identify two plausible and astrophysically consistent scenarios for achieving the required suppression.
A discrete symmetry, called R-parity ($R_P$) is imposed on the Minimal Supersymmetric Standard Model (MSSM) [1] to ensure proton stability. This assumption makes the Lightest Supersymmetric Particle (LSP) stable. The most natural candidate for the LSP in SUSY models is a neutralino. Indeed, calculations in SUSY models with soft breaking terms and radiatively induced electroweak breaking lead to neutralino as LSP for a wide range of allowed parameters. Moreover, the relic neutralino density satisfies all requirements for being the cold dark matter over large parameter regions. The hypothesis of neutralino as Dark Matter (DM) particle is amenable to experimental verification [2]. Such neutralinos can be detected both directly through their elastic scattering off-nuclei [3] and indirectly through the products of neutralino annihilation [4],[5],[6].

Although R-parity may remain unbroken if it is a remnant of a gauge symmetry [7, 8, 9, 10, 11], there is no deep theoretical reason requiring it to be a symmetry of nature. In fact, many models of $R_P$ violation have been proposed [12]. These typically lead to large $R_P$ violation and thus do not allow the neutralino to be a dark matter particle. In contrast, if $R_P$ violation is very mild then the lightest neutralino is unstable, but very long-lived. Naively, one would think that a neutralino with a lifetime of the order of the age of the universe could be a viable dark matter candidate. However, most neutralino decays into visible channels, e.g. containing positrons, antiprotons, gamma's and neutrinos are severely constrained from observations. They typically require [13] neutralino lifetimes much larger than the age of the universe. This can only be realized if the violation of $R_P$ is extremely weak [13, 14]. Such a scenario was already considered in the context of a specific mechanism [15] for spontaneous $R_P$ violation driven by a vacuum expectation value (VEV) for the right-handed sneutrino. This mechanism requires very small Yukawa coupling, i.e. fine-tuning. A more natural way to obtain very small $R_P$ violation is to ascribe it to gravitational interactions. This violation will be described here by non-renormalizable terms suppressed by inverse powers of the Planck mass and should thus be naturally very small.

A somewhat surprising conclusion that emerges out of our study is that dimension-five interactions are in conflict with the astrophysical constraints by few orders of magnitude if the neutralino provides the cold DM.

Let us first quantitatively discuss constraints on the strength of $R_P$ violation. The lightest neutralino $\chi$ is given by a superposition of wino $\tilde{W}$, bino $\tilde{B}$ and two
Higgsinos, $\tilde{H}_1$ and $\tilde{H}_2$ as:

$$\chi = Z_{\tilde{\chi}W} \tilde{W}_3 + Z_{\chi\tilde{H}} \tilde{H} + Z_{\chi\tilde{H}_1} \tilde{H}_1 + Z_{\chi\tilde{H}_2} \tilde{H}_2.$$  \hspace{1cm} (1)

We can parameterize the effective $R_P$ violating interactions responsible for the neutralino decay in terms of the MSSM fields as follows:

$$W_{\epsilon eff} = \lambda_1 (U^c D^c)^c F + \lambda_2 (L L E^c)^c F + \lambda_3 (Q D^c L)^c F + \epsilon(L H^c T)$$ \hspace{1cm} (2)

The notation for the fields is standard. We suppressed the generation indices. Eq. (2) includes only renormalizable terms relevant for neutralino decay. However, as it will be understood below, this expression has wider generality.

The above interactions result in neutralino decay to three fermions. The width for this decay depends upon whether it proceeds through the Higgsino or gaugino component. In the former case,

$$\tau_\chi^{-1} = \lambda_i^2 Z_{\chi H}^2 \frac{G_F m_T^2}{192(2\pi)^3} \frac{m_\chi^5}{m_T^4},$$ \hspace{1cm} (3)

where $m_\chi$, $m_f$ and $m_j$ are neutralino, sfermion and fermion masses, respectively. In the case of quarks the width should be multiplied by the number of colours. When the decay proceeds through the bino component $\tilde{B}$ of the neutralino the decay width is

$$\tau_\chi^{-1} = \lambda_i^2 Z_{\chi H}^2 \frac{\alpha_m Y_{fR}^2}{192(2\pi)^3} \frac{m_\chi^5}{m_f^4 \cos^2 \theta_W},$$ \hspace{1cm} (4)

where $Y_{fR}$ is hyper-charge of the right fermion and $\theta_W$ is the weak mixing angle. Finally, the width of $\chi \to \nu + e^+ + e^-$ due to the last term of Eq. (2) is

$$\tau_\chi^{-1} = \epsilon^2 Z_{\chi H}^2 (\frac{1}{4} + \sin^2 \theta_W + \frac{4}{3} \sin^4 \theta_W) \frac{G_F m_\chi^3}{192 \pi^3},$$ \hspace{1cm} (5)

The condition $\tau_\chi > t_0$ (where $t_0$ is the age of the universe) leads to constraints on $\lambda_i$ and $\epsilon$. However, much more stringent limits follow from production of positrons in our Galaxy [13, 14], from the diffuse gamma-radiation [13, 14] and from neutrino-induced muons [16]. From the analysis given in [14] it follows that when the decay to positrons is unsuppressed as in the present case, the strongest constraints on both $\lambda_i$ and $\epsilon$ follow from the observed flux of positrons in our Galaxy. The lower limit on neutralino lifetime from this flux is [14]

$$\tau_\chi(\chi \to \epsilon^+ + \text{anything}) > 7 \times 10^{10} \sqrt{m_{100} t_0} h, \hspace{1cm} (6)$$

where $m_{100} = m_\chi/100$ GeV and $h$ is dimensionless Hubble constant.
Using this limit and keeping in mind indirect production of positrons through decay of other particles, we find

\[ \lambda_i < 4 \times 10^{-21} Z^{-1} \left( \frac{m_f}{1 \text{ TeV}} \right)^2 \left( \frac{100 \text{ GeV}}{m_{\chi}} \right)^{9/8} \left( \frac{1 \text{ GeV}}{m_f} \right)^{1/2} \]  

(7)

\[ \epsilon < 6 \times 10^{-23} Z^{-1} m_{100}^{-7/4} \text{GeV} \]  

(8)

The above estimates show that if the neutralino is a DMP the R-parity violating parameters \( \lambda_i \) and \( \epsilon \) are very strongly limited from above.

Let us now turn to a systematic dimensional analysis of R-parity-violating operators in the MSSM. We have already discussed the operators of dimension d=4. Extremely small couplings are needed for the neutralino to be a DMP, which requires fine tuning.

The relevant Planck-mass suppressed operators of dimension d=5 in the MSSM are the following ones:

\[ \frac{\beta_a}{M_p} (H_1 H_2 E^c)_D \quad \frac{\beta_a}{M_p} (Q L^c U^c)_D \quad \frac{\beta_a}{M_p} (U^c D^c E^c)_D \]

\[ \frac{\beta_a}{M_p} (Q QQ H_1)_F \quad \frac{\beta_a}{M_p} (L H_2 H_1 H_2)_F \]  

(9)

The first three terms in the above equation contain the F-terms of the anti-chiral fields \( H_2, L^c \) and \( D^c \) respectively. These are determined in the supersymmetric limit by the standard MSSM \( R_p \) conserving superpotential. Using this one can show that their contribution to the neutralino decay is suppressed either due to small Yukawa couplings or kinematically, when the charged Higgs is heavier than the neutralino. The contribution of the fourth term with three \( Q \) fields to neutralino decay can also be shown to be small.

The first four operators thus satisfy astrophysical constraints without need for extra suppression in the corresponding coefficient. In contrast, the decay induced by the last term cannot be suppressed kinematically. It leads to the effective \( R_p \) breaking parameter

\[ \epsilon \sim \beta_5 M_{\text{EW}}^2 / M_p \sim \beta_5 \times 10^{-15} \text{GeV} \]

This value of \( \epsilon \) is extremely small and leads to a neutralino lifetime longer than the age of the universe. But it is in conflict with the astrophysical constraints (8) unless \( \beta_5 \lesssim 10^{-5} - 10^{-7} \). This situation is similar to the gravitationally induced axion mass [17], [18] where the quantum gravitation corrections are not small enough to suppress it adequately.

If \( 1/M_p \) terms are forbidden (for example by some unbroken symmetry), then
terms (d=6 operators) become important. An example of such operator is

$$\frac{\beta}{M_{Pl}^2} ((LH_2)(H_1H_2)^*)_D$$

(10)

This term gives an $\epsilon$ which is about 10 orders of magnitude less than needed to produce observable effects.

Therefore, while in the MSSM $1/M_{Pl}^2$ terms are too small, the $1/M_{Pl}^2$ terms are too large and need additional suppression, i.e. small $\beta$.

If wormhole effects are responsible for the terms we are discussing, they can contain a topological suppression leading to very small $\beta$. Generically, this suppression is described by an $e^{-S}$ factor, where $S$ is an action of a wormhole which absorbs the $R_P$ charge. In the semi-classical approach $S \sim 10$. In the case of the Peccei-Quinn symmetry such estimates give $S \sim \ln(M_P/f_{PQ}) \approx 16$ resulting in a suppression factor $\beta \sim 10^{-7}$, which is needed in our case. A detailed discussion of such suppression factors in wormhole effects is given in [17]. It is shown that the action $S$ is connected with the size of the wormhole throat $\tilde{r}(0)$ and can vary from $S \approx 6.7$ for the naive estimate $\tilde{r}(0) \approx M_P^{-1}$, up to $8\pi^2/g_{swr}^2 \approx 190$ in string inspired models. Thus if the action is close to its semi-classical value the topological wormhole effects can provide the suppression needed for the long-lived neutralino $\beta \sim 10^{-7} - 10^{-5}$ to satisfy the astrophysical constraints.

Apart from topological wormhole effects, suppression of d=5 operators can occur due to some additional symmetry. Let us assume that there exists a singlet sector which communicates with the MSSM sector only gravitationally through non-renormalizable terms in the Lagrangian. R-parity can be broken spontaneously in this sector, for example, due to some $R_P$-odd field $\eta$ developing a non-zero VEV. $R_P$ violation can penetrate the MSSM sector through non-renormalizable interactions between $\eta$ and the MSSM fields. In contrast to the first case, gravity is not directly responsible for the breaking of $R_P$ but it leads to effective $R_P$ violation in the observable sector through the presence of non-renormalizable interactions. We now discuss this possibility of hidden R-parity violation in a model independent way and then provide an example.

Let us assume the existence of an $SU(3) \times SU(2) \times U(1)$ singlet field $\eta$ coupling to the MSSM fields only through non-renormalizable terms. This can be achieved by a proper symmetry as we shall discuss. There are four dimension-five operators involving $\eta$ which lead to $R_P$ violation:

$$O_1 = \frac{\alpha_1}{M_{Pl}^2} (U^c D^c D^c \eta)_F \quad O_2 = \frac{\alpha_2}{M_{Pl}^2} (LLE^c \eta)_F$$
$$O_3 = \frac{\alpha_3}{M_{Pl}^2} (Q D^c H_1 \eta)_F \quad O_4 = \frac{\alpha_4}{M_{Pl}^2} (LH_2 \eta^*)_D$$

(11)
where $\alpha_{1,2,3,4}$ are parameters of order one. These operators conserve $R_P$ if the field $\eta$ is chosen odd. The non-zero VEV $\langle \eta \rangle$ then breaks $R_P$ and leads to the effective interactions in eq.(2) with couplings given by $(i=1,2,3)$:

$$\lambda_i = \alpha_i \langle \eta \rangle / M_P \quad \epsilon = \alpha_4 \langle F_{\eta} \rangle / M_P$$

The effective $R_P$ violation among the MSSM fields is governed by two distinct scales. One scale is $\langle \eta \rangle$ and determines the trilinear interactions of eq.(2), while the other scale $\langle F_{\eta} \rangle$ characterizes SUSY breaking in the singlet sector and determines the bilinear term $\epsilon$. In general, these two scales could be quite different. The constraints derived in ((8)) imply

$$\langle \eta \rangle \lesssim 10^{-1}\text{GeV}; \langle F_{\eta} \rangle \lesssim 10^{-2}\text{GeV}^2$$

If SUSY is broken through the usual soft terms, the constraint on $\langle F_{\eta} \rangle$ can easily be satisfied, as we will show in a specific example. In contrast, the constraint on the trilinear coupling implies very small $\eta$ VEV which may be unnatural. To avoid this one should forbid the corresponding dimension-5 operators, in which case the dominant $R_P$ violation would arise from dimension-six interactions. For example, the operator

$$\frac{1}{M_P} [(LH_2)(H_3H_2)\eta]_F$$

leads to $\epsilon \sim 10^{-34}\langle \eta \rangle$ GeV. The effect of this term could be observable only if $R_P$ violation in the singlet sector occurs close to the unification scale.

Let us now consider a concrete realization of the above scenario. We will impose a gauged discrete symmetry in order to prevent gravitational breaking. $R_P$ by itself can be gauged in a suitable extension of the MSSM but it cannot prevent the occurrence of a term like $LH_2\eta$ which lead to a large $R_P$ violation when $\eta$ develops a vev. We therefore look for a more general symmetry which leads to effective $R_P$ conservation at the renormalizable level. Gauged $Z_N$ symmetry of the type considered in [9] provides an example. The $\theta$ is assumed to carry the $Z_N$-charge $-1$. The $Z_N$ charge of one of the observable superfields is set to zero by appropriate redefinition of the $Z_N$ generators. Requiring that the standard $R_P$ conserving couplings of the MSSM fields are allowed by the $Z_N$ symmetry we determine the charges of the remaining fields in terms of two parameters $x$ and $y$ according to:

$$
\begin{array}{ccccccc}
Q & U^c, H_1 & D^c, H_2 & L & E^c & Y & \eta \\
0 & x & 2-x & y & 2-(x+y) & 2 & \frac{y}{2}
\end{array}
$$

5
where we have introduced a singlet field $Y$ in addition to $\eta$, in order to obtain $R_P$ violation. With these charge assignments dimension-4 terms respect $R_P$ and the $\eta, Y$ do not couple to the MSSM fields in the renormalizable Lagrangian, as long as $x \neq 2$ and $x - y \neq 0, -2, N/2$. The most general $Z_N$-invariant renormalizable superpotential in this case can be written as \[ W = W_{MSSM} + \delta Y(\eta^2 - f^2) \] (15)

The above superpotential gives rise to a VEV for $\eta$ at the supersymmetric minimum, leading to effective $R_P$ violation for the MSSM fields through operators of dimensionality $\geq 5$. The choice $2 + y - x = N/2$ allows the dimension-6 operator of (14). The allowed higher dimensional terms are given in this case by

\[ \mathcal{L}_{NR} = \frac{\beta_5}{M_p} (LH_2\bar{\eta})_D + \frac{1}{M_p^2} [\delta_1 (LLE^c\eta^*)_D + \delta_2 (QD^c\eta^*)_D + \delta_3 (LH_2\bar{\eta}Y)_D + \delta_4 (LH_2H_1H_2\eta)_F] \] (16)

Note that the dimension-5 operator above cannot be forbidden if the dimension-6 term in (14) is to be allowed. As discussed above it does not lead to large $R_P$ violation as long as SUSY remains unbroken in the singlet sector, as happens with the superpotential choice in (15). In a realistic situation soft SUSY breaking introduces terms which make $F_\eta, Y$ non-zero. If $\langle \eta \rangle \lesssim M_{SUSY} \sim \text{TeV}$ then $\langle F_\eta \rangle \sim \frac{2m_\eta M_2^2 A}{m_\eta}$ where $m_\eta \sim A$ characterizes soft SUSY breaking. Choosing $\delta \sim 10^{-2}$, $m_\eta \sim 10^3 \text{ GeV}$, $A \sim 10^2 \text{ GeV}$ and $\langle \eta \rangle \lesssim 100 \text{ GeV}$ we have $F_\eta \lesssim 10^{-2} \text{GeV}^2$ in agreement with the constraint of (13). If $\langle \eta \rangle$ is much larger than $M_{SUSY}$ then $\langle F_\eta \rangle$ would also be large and induce large $\epsilon$. This can be prevented by a symmetry. Specifically, if the kinetic energy terms for the singlet fields are chosen to be no-scale type [20] then $\langle F_{\eta,Y} \rangle$ vanish at the minimum of the potential and effective breaking of $R_P$ would arise only from the dimension-6 operator. This operator could lead to observed signatures if $\langle \eta \rangle$ is very large, near the unification scale.

The $Z_N$ symmetry introduced above can be gauged if it satisfies discrete gauge anomaly constraints discussed in [10]. In our case these are given as:

\[ -2N_y + 6 = k_1 N \]
\[ N_y(y - 4) + 4 = k_2 N \]
\[ N_y(-7 + y - x) + N/2 - 9 = k_3 N + \kappa k_4 N/2 \] (17)

where $\kappa$ is 1 (0) for even (odd) $N$ and $k_{1,2,3,4}$ are integers. The first constraint is automatically satisfied for three ($N_y = 3$) generations. The remaining constraints
can also be satisfied for appropriate choices of $x$, $y$ and $N$. A choice which satisfies all the anomaly constraints and leads to the required interactions in Eq. (15) and (16) is given by $N = 3$, $x = 1/6$ and $y = -1/3$. Clearly many other choices would be possible.

In summary, there is no deep theoretical motivation for R-parity to be absolutely conserved. In this paper we have analyzed the possibility that R-parity can be weakly violated by gravitationally induced terms suppressed by inverse powers of the Planck mass. Due to astrophysical constraints (mainly to the positron production in our Galaxy) this extremely weak R-parity violation is compatible with hypothesis that the neutralino is a DM particle only if its lifetime is about $10^{10}$ times longer than the age of the universe. We analyzed gravitationally induced dimension-5 operators and demonstrated that they break R-parity too strongly to comply with this constraint. We showed that the unstable neutralino as a DM particle is possible if dangerous d=5 terms are strongly suppressed. For the MSSM case, the required suppression could be provided by the classical wormhole action $S \approx 7$.

Another possibility for very weak R-parity breaking can be provided by existence of additional gauge discrete symmetry. We constructed a model with a gauge $Z_N$ symmetry and $SU(3) \times SU(2) \times U(1)$ singlet field $\eta$, which communicates with the MSSM fields only through gravity. R-parity violation is driven by VEV of this field.

The decaying neutralino has interesting astrophysical signatures. In some models [14] the neutralino decay to the Majoron $\eta, \chi \rightarrow \nu + J$ may be dominant, resulting in a detectable isotropic flux of mono-energetic neutrinos. In the more general case of $R_{BP}$ breaking by d=5 operators discussed above, the neutralino decay signature is weaker and is given by the ratio of the signals from the Sun and Earth to that from the Galactic halo. The signal from annihilation of neutralinos in the Earth and the Sun is the same as for a stable neutralino, while the positron and antiproton fluxes from the Galactic halo could be strongly enhanced due to neutralino decay.

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References

[1] The MSSM is defined here as a supersymmetrized $SU(3) \times SU(2) \times U(1)$ model.
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[19] The $\eta^2$ term allowed by the symmetry can be removed by a redefinition of the field $Y$.